Mark Scheme (Result)

November 2021

Pearson Edexcel GCE Further Mathematics
Advanced Level in Further Mathematics
Decision 1
Paper 9FM0/3D

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

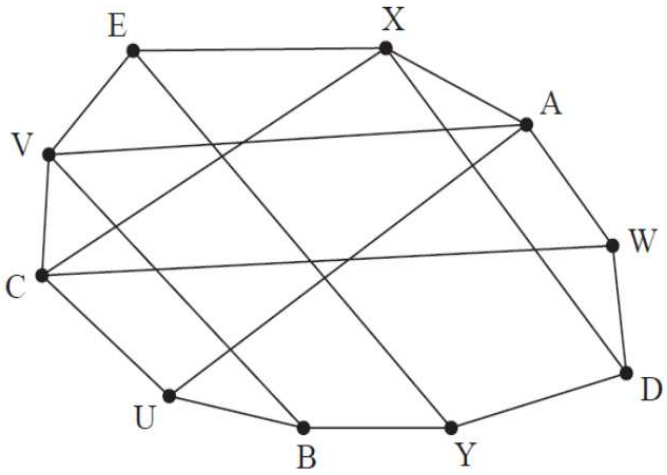
- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\surd$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- * The answer is printed on the paper
- ⌐ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

Question	Scheme	Marks	AOs
1(a)	…, D, Y, B, U, C	B1	1.1b
		(1)	
(b)	 Or list of arcs: AU, AV, BV, CW, CX, DX, EY	M1	2.1
	e.g., select (and label) AU (as I) and the arcs that intersect AU are BV, EY, CW and DX (so label them O so AU(I), AV, BV(O), CW(O), CX, DX(O), EY(O))	A1	1.1b
	Edges BV and CW intersect and so the graph is not planar (oe)	A1	2.2a
		(3)	
			(4 marks)

Notes:

(a)

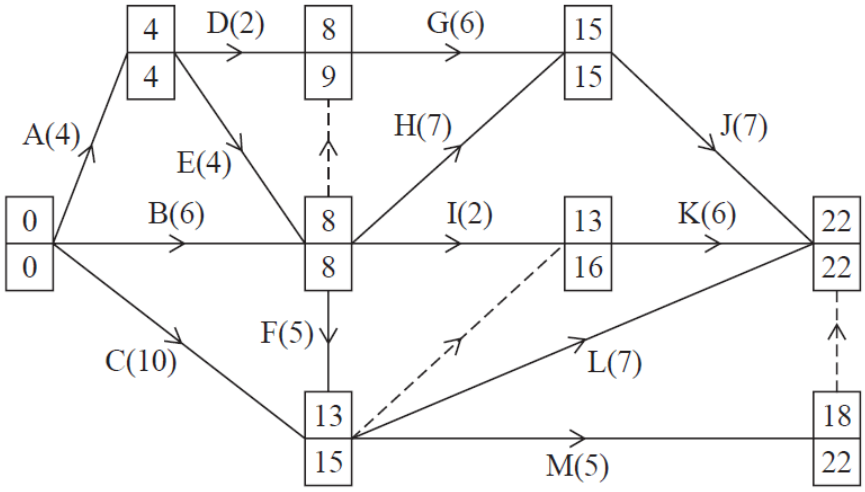
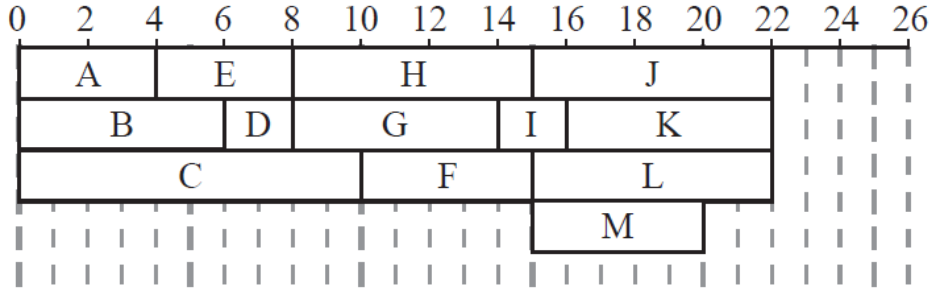
B1: CAO (CVEXAWDYBUC) – must return to C

(b)

M1: Either draws their Hamiltonian cycle from part **(a)** as the edges of a polygon and shows the remaining arcs intersecting inside **OR** lists the arcs that are not part of the Hamiltonian cycle

A1: Selects any arc (that is not part of the Hamiltonian cycle) and lists/references the correct arcs that intersect with this selected arc – dependent on any correct Hamiltonian cycle and correct arcs that are not part of this cycle

A1: cao – based on their initial arc selection, states the two arcs that are unlabelled (or that are labelled with the same label) which intersect each other (e.g., if CX chosen as the initial selection then this arc intersects with BV, EY and AV but EY and AV intersect) **and** concludes that the graph is not planar. This mark is dependent on the correct Hamiltonian cycle stated in either **(a)** or **(b)**

Question	Scheme	Marks	AOs
2(a)		M1 A1 M1 A1	2.1 1.1b 1.1b 1.1b
		(4)	
(b)	$\frac{71}{22} = ...$	M1	1.1b
	…= 3.22… therefore 4 workers	A1	2.2a
		(2)	
(c)	e.g. 	M1 A1 A1	2.1 1.1b 1.1b
		(3)	
			(9 marks)

Notes:

(a)
M1: All top boxes completed, number generally increasing L to R (condone one "rogue")
A1: cao - top boxes (including zero at the source node)
M1: All bottom boxes completed, numbers generally decreasing R to L (condone one "rogue")
A1: cao - bottom boxes (including zero at the sink node)

(b)

M1: Attempt to find the lower bound (71 ± 10) / their completion time (a value of 3.2… seen with no working can imply this mark)

A1: cso - correct calculation seen or 3.2 followed by 4. An answer of 4 with no working scores **M0A0**

(c)

M1: Not a cascade chart, 4 'workers' used at most and at least 9 different activities placed

A1: 4 workers. All 13 activities present (just once – so if an activity appears for two different workers and is happening at the same time this is A0). Condone at most two errors. An activity can give rise to at most three errors; one on duration, one on time interval and only one on IPA

A1: 4 workers. All 13 activities present (just once). No errors

Activity	Duration	Time Interval	IPA
A	4	0 – 4	-
B	6	0 – 8	-
C	10	0 – 15	-
D	2	4 – 9	A
E	4	4 – 8	A
F	5	8 – 15	B, E
G	6	8 – 15	B, D, E
H	7	8 – 15	B, E
I	2	8 – 16	B, E
J	7	15 – 22	G, H
K	6	13 – 22	C, F, I
L	7	13 – 22	C, F
M	5	13 – 22	C, F

Question	Scheme	Marks	AOs
3(a)	Prim's starting at A: AH, AB, DH; CD, CE; DG, EF	M1 A1 A1	1.1b 1.1b 1.1b
		(3)	
(b)	Weight of MST is 183 (miles)	B1	2.2a
		(1)	
(c)	NNA: J – F – E – C – D – H – A – B – G – J	B1	1.1b
	Upper bound is 267 (miles)	B1	2.2a
		(2)	
(d)	183 + 27 + 25 =…	M1	3.1b
	… = 235 (miles)	A1	2.2a
		(2)	
			(8 marks)

Notes:

(a)
M1: First three arcs correctly chosen in order {AH, AB, DH, …} or first four nodes correctly chosen in order{A, H, B, D,…}. If any rejections seen at any point then **M1** (max) only. Order of nodes may be seen at the top of the matrix {1, 3, -, 4, -, -, -, 2} so please check the top of the matrix carefully
A1: First five arcs correctly chosen in order {AH, AB, DH, CD, CE, …} or all eight nodes correctly chosen in order{A, H, B, D, C, E, G, F}. Order of nodes may be seen at the top of the matrix so for the first two marks accept {1, 3, 5, 4, 6, 8, 7, 2} (do not condone any missing numbers e.g. the number 8 must be above F)
A1: cso – all arcs correct stated and chosen in the correct order. Candidates must be considering arcs for this final mark (do not accept a list of nodes or numbers across the top of the matrix unless the correct list of arcs (in the correct order) is also seen)

(b)
B1: cao (183)

(c)
B1: cao (for route – must return to J)
B1: cao (for upper bound of 267)

(d)
M1: Their answer to **(b)** + 27 + 25 (the two smallest arcs incident to J)
A1: cao (235)

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td>4(a)</td><td>Initial time matrix
<table><tr><th></th><th>A</th><th>B</th><th>C</th><th>D</th><th>E</th><th>F</th></tr><tr><td>A</td><td>-</td><td>57</td><td>95</td><td>150</td><td>63</td><td>230</td></tr><tr><td>B</td><td>57</td><td>-</td><td>72</td><td>∞</td><td>132</td><td>∞</td></tr><tr><td>C</td><td>95</td><td>72</td><td>-</td><td>289</td><td>160</td><td>125</td></tr><tr><td>D</td><td>150</td><td>∞</td><td>289</td><td>-</td><td>84</td><td>∞</td></tr><tr><td>E</td><td>63</td><td>132</td><td>160</td><td>84</td><td>-</td><td>191</td></tr><tr><td>F</td><td>230</td><td>∞</td><td>125</td><td>∞</td><td>191</td><td>-</td></tr></table></td><td>B1</td><td>1.1b</td></tr>
<tr><td></td><td></td><td>(1)</td><td></td></tr>
<tr><td>(b)</td><td>1st iteration:
<table><tr><th></th><th>A</th><th>B</th><th>C</th><th>D</th><th>E</th><th>F</th></tr><tr><td>A</td><td>-</td><td>57</td><td>95</td><td>150</td><td>63</td><td>230</td></tr><tr><td>B</td><td>57</td><td>-</td><td>72</td><td>207</td><td>120</td><td>287</td></tr><tr><td>C</td><td>95</td><td>72</td><td>-</td><td>245</td><td>158</td><td>125</td></tr><tr><td>D</td><td>150</td><td>207</td><td>245</td><td>-</td><td>84</td><td>380</td></tr><tr><td>E</td><td>63</td><td>120</td><td>158</td><td>84</td><td>-</td><td>191</td></tr><tr><td>F</td><td>230</td><td>287</td><td>125</td><td>380</td><td>191</td><td>-</td></tr></table>
<table><tr><th></th><th>A</th><th>B</th><th>C</th><th>D</th><th>E</th><th>F</th></tr><tr><td>A</td><td>A</td><td>B</td><td>C</td><td>D</td><td>E</td><td>F</td></tr><tr><td>B</td><td>A</td><td>B</td><td>C</td><td>A</td><td>A</td><td>A</td></tr><tr><td>C</td><td>A</td><td>B</td><td>C</td><td>A</td><td>A</td><td>F</td></tr><tr><td>D</td><td>A</td><td>A</td><td>A</td><td>D</td><td>E</td><td>A</td></tr><tr><td>E</td><td>A</td><td>A</td><td>A</td><td>D</td><td>E</td><td>F</td></tr><tr><td>F</td><td>A</td><td>A</td><td>C</td><td>A</td><td>E</td><td>F</td></tr></table></td><td>M1

A1</td><td>1.1b

1.1b</td></tr>
<tr><td></td><td></td><td>(2)</td><td></td></tr>
<tr><td>(c)</td><td>Route must start and finish at B, E therefore need to consider pairings of the other four odd nodes (A, C, D and F)</td><td>M1</td><td>3.1b</td></tr>
<tr><td></td><td>AC + DF = 95 + 275 = 370
AD + CF = 147 + 125 = 272*
AF + CD = 220 + 242 = 462</td><td>A1
A1</td><td>1.1b
1.1b</td></tr>
<tr><td></td><td>Repeat arcs: AE, DE and CF</td><td>A1</td><td>2.2a</td></tr>
<tr><td></td><td></td><td>(4)</td><td></td></tr>
<tr><td>(d)</td><td>Length: 1648 + 272 = 1920 (minutes)</td><td>B1</td><td>2.2a</td></tr>
<tr><td></td><td></td><td>(1)</td><td></td></tr>
<tr><td colspan="4">(8 marks)</td></tr>
</table>

Notes:

(a)

B1: Correct distance table (condone dashes, crosses, etc. for infinity but do not condone a 'large' number in these cells or these cells left blank)

(b)

M1: No change in the first row and first column of both tables with at least two values in the distance table correctly reduced and two letters in the route table correctly changed – all cells complete

A1: cao

(c)

M1: Either the correct three pairings of the correct four nodes A, C, D and F **or** recognises that as the route begins at B and finishes at E that only the nodes A, C, D and F need to be considered

A1: Two rows correct including pairings **and** totals

A1: All three rows correct including pairings **and** totals

A1: selecting the shortest pairing, and stating that these arcs (AE, DE and CF) should be repeated. Must be these arcs and not e.g., AED or AD via E, etc.

(d)

B1: cao

Question	Scheme	Marks	AOs
5(a)	e.g., middle right pivot(s) 30 12 5 2 23 **18** 36 10 15 24 30 23 **36** 24 18 12 5 **2** 10 15 36 30 **23** 24 18 12 5 **10** 15 2 36 30 **24** 23 18 12 **15** 10 5 2 36 30 24 23 18 15 12 10 5 2 Therefore, the sort is complete	M1 A1 A1ft A1	1.1b 1.1b 1.1b 1.1b
		(4)	
(b)	The 5 has been put in Bin 2 rather than Bin 1 which indicates that the size of the bins is less than 30 + 12 + 5 = 47 and so therefore $(42 \leq n) \leq 46$	B1	3.1a
	The fact that there is still room for the 2 in Bin 1 indicates that $n \geq 44$	B1	2.4
	The 18 cannot fit in Bin 2 and so therefore $n < 5+23+18 = 46$ which implies that n is either 44 or 45	B1	2.2a
		(3)	
(c)	Bin 1: 36 5 2 Bin 2: 30 15 Bin 3: 24 18 Bin 4: 23 12 10	M1 A1 A1	1.1b 1.1b 1.1b
		(3)	
			(10 marks)

Notes:

(a)

M1: Quick sort, pivot, p, chosen (must be choosing middle left or right – choosing first/last item as the pivot is M0). After the first pass the list must read (values greater than the pivot), pivot, (values less that the pivot). **If only choosing one pivot per iteration then max of M1 only** – Bubble sort is not a MR and scores **M0**

A1: First pass correct **and** next pivots chosen correctly for the second pass (but the second pass does not need to be correct)

A1ft: Second and third passes correct (follow through from their first pass and choice of pivots). They do not need to be choosing a pivot for the fourth pass for this mark

A1: cso (correct solution only – all previous marks in this part **must** have been awarded) including 'sort complete' statement

If list sorted into ascendng order then mark as a MR

(b)

B1: Correct reasoning why $n \leq 46$ or $n < 47$ or $n \leq 45$ or $n < 46$ - condone an argument which is mathematical in nature only e.g., $n < 30 + 12 + 5$ or $n < 5 + 23 + 18$

B1: Correct reasoning why $n \geq 44$ - condone 'largest bin filled is 44 so $n \geq 44$ ' or other similar mathematical argument. This mark can be awarded for an argument which is mathematical in nature only e.g., $n \geq 30+12+2$

B1: Completely correct reasoning for why n is either 44 or 45 only – this mark is dependent on the two previous **B** marks and must give sufficient detail (so an argument that contains no clear explanation of why certain inequalities hold cannot score this mark)

(c)

No MR or follow through in this part

M1: First five values placed correctly - with at least eight values placed (the squared values)

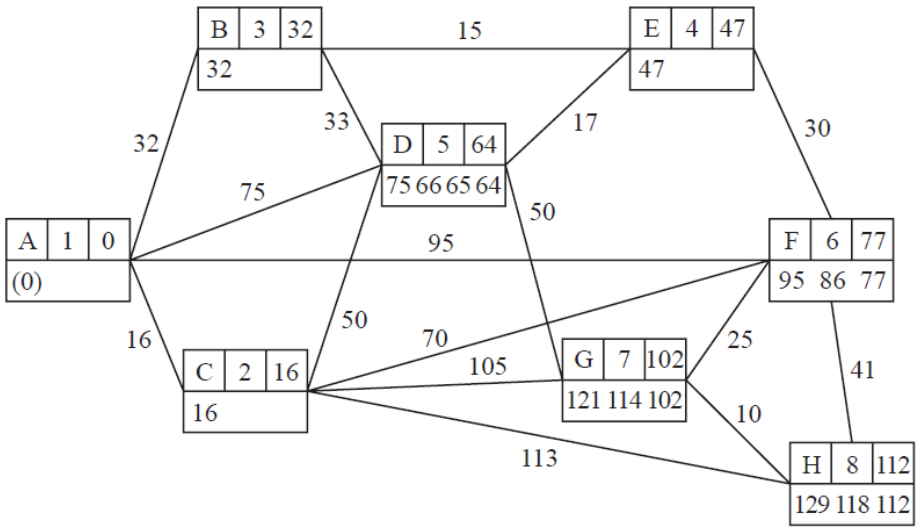
A1: First eight values places correctly with no additional/repeated values (the squared and underlined values)

A1: cso - no additional/repeated values

Middle left pivot(s)

30	12	5	2	**23**	18	36	10	15	24
30	**36**	24	23	12	5	**2**	18	10	15
36	**30**	24	23	12	5	**18**	10	15	2
36	30	24	23	18	12	**5**	10	15	2
36	30	24	23	18	12	**10**	15	5	2
36	30	24	23	18	**12**	15	10	5	2
36	30	24	23	18	15	12	10	5	2

Therefore, the sort is complete

Question	Scheme	Marks	AOs
6(a)(i)		M1 A1 A1 A1ft	1.1b 1.1b 1.1b 1.1b
	Shortest path from A to H: ABEFGH	A1	2.2a
(a)(ii)	Length of shortest path from A to H is 112	A1ft	2.2a
		(6)	
(b)	Applying Dijkstra repeatedly to n nodes implies that the order is $n\left(n^2\right)=n^3$	B1	3.1b
	$t=0.082\left(\frac{200}{10}\right)^3$	M1	3.4
	= 656 (seconds)	A1	2.2a
		(3)	
(c)	e.g. order of n^3 does not mean that the order is proportional to n^3 (which is the assumption behind the answer in (b)) but merely means that the dominant term is of order n^3	B1	3.2b
		(1)	
			(10 marks)

Notes:

In (a) it is important that all values at each node are checked very carefully – the order of the working values must be correct for the corresponding A mark to be awarded e.g. at H the working values must be 129 118 112 in that order (so 129 112 118 is incorrect)

It is also important that the order of labelling is checked carefully – some candidates start with a label of 0 at A (rather than 1) – which is fine. Also the order of labelling must be a strictly increasing sequence – so 1, 2, 3, 3, 4, … will be penalised once (see notes below) but 1, 2, 3, 5, 6, … is fine. Errors in the final values and working values are penalised before errors in the order of labelling

(a)

M1: A larger value replaced by a smaller value in at least two of the working boxes at either D or F or G or H

A1: All values in A, B, C and E correct. Condone lack of 0 in A's working value

A1: All values D and F correct and the working values in the correct order. Penalise order of labelling only once per question (D and F must be labelled in that order and D must be labelled after A, B, C and E)

A1ft: All values in G and H correct on the follow through and the working values in the correct order. Penalise order of labelling only once per question. To follow through G check that the working value at G follows from the candidate's final values from their feeds into G (which will most likely come from nodes C, D and/or F (in the order in which the candidate has labelled them)) and that the final value, and order of labelling, follows through correctly. Repeat this process for H (which will possibly have working values from C, F and/or G with the order of these values determined by the candidate's order of labelling at C, F and G)

A1: cao - correct path from A to H (ABEFGH)

A1ft: Follow through their final value at H only (if 112 stated and 112 is not the final value at H then **A0**)

(b)

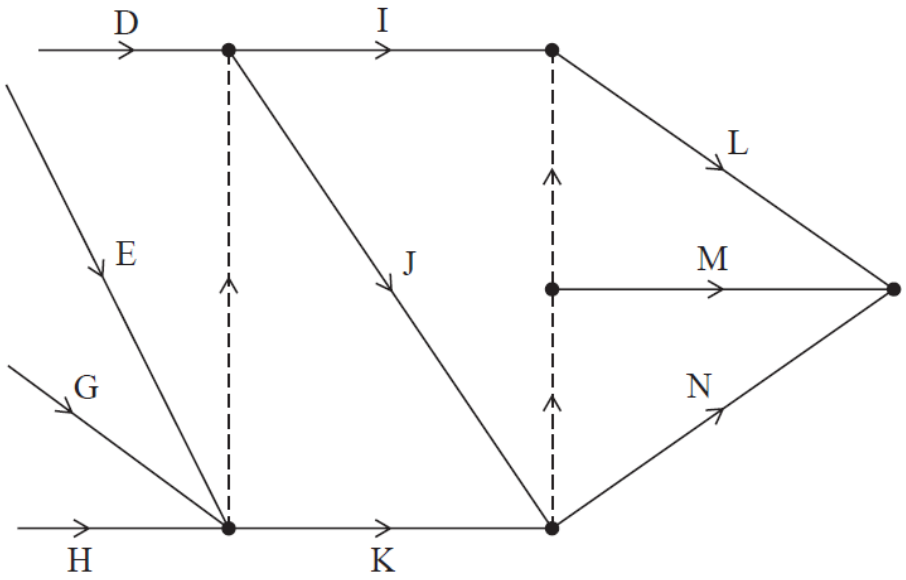
B1: Any indication that repeated application of Dijkstra has cubic order

M1: Complete method – allow 10/200 – allow slips in values e.g. 0.82 for 0.082 and accept 200/10 (or 10/200) either squared or cubed **only**

A1: cao

(c)

B1: Any indication that the run-time is not <u>exactly</u> proportional to n^3 e.g., may suggest that there are other terms ($n^3 + \ldots$), or that n^3 is the dominant term, or that order does not imply proportionality. Do not accept only that 'n^3 is not exact'. Condone use of n^2 (oe) for n^3

Question	Scheme	Marks	AOs
7(a)	<table><tr><th>Activity</th><th>Immediately preceding activities</th></tr><tr><td>A</td><td>-</td></tr><tr><td>B</td><td>-</td></tr><tr><td>C</td><td>-</td></tr><tr><td>D</td><td>A</td></tr><tr><td>E</td><td>A</td></tr><tr><td>F</td><td>A</td></tr><tr><td>G</td><td>B, F</td></tr><tr><td>H</td><td>B, C, F</td></tr></table>	B1 B1	1.1b 1.1b
		(2)	
(b)	e.g.  	M1 A1 A1 A1	2.1 1.1b 1.1b 1.1b
		(4)	
(c)	If all activities have the same duration then any critical path must contain 5 activities. All paths that pass-through D have only 4 activities and so therefore D cannot be critical.	B1 B1	2.4 2.4
		(2)	
			(8 marks)

Notes:

(a)
B1: Either row G or H correct
B1: All rows correct (condone blanks in A, B and C rows)

(b)

Condone lack of, or incorrect, numbered events throughout. 'Dealt with correctly' means that the activity starts from the correct event but need not necessarily finishes at the correct event, e.g. 'K dealt with correctly' requires the correct precedences for this activity, i.e. E, G and H labelled correctly and leading into the same node and K starting from that node but do not consider the end event for K. **Activity on node is M0**

Assume that a solid line is an activity which has not been labelled rather than a dummy (even if in the correct place for where a dummy should be)

Ignore incorrect or lack of arrows on the activities for the first three marks only

(b) M1: At least five activities (labelled on arc), at least two dummies placed

A1: Activities I, J, K and first dummy + arrow dealt with correctly

A1: Activities L, M, N and a second dummy + arrows dealt with correctly

A1: cso – all arrows present for every activity with one finish and exactly three dummies. Note that this is not a unique solution e.g. M and N could be interchanged so please check these carefully.

Please check all arcs carefully for arrows – if there are no arrows on any dummies then M1 only

Note that additional (but unnecessary) 'correct' dummies that still maintain precedence for the network should only be penalised with the final A mark if earned

For reference:

Activity	**I**	**J**	**K**	**L**	**M**	**N**
IPA	D, E, G, H	D, E, G, H	E, G, H	I, J, K	J, K	J, K

(c)

B1: Explains that all critical paths must contain 5 activities (oe method e.g., attempting a forward and backward pass with each activity having the same duration)

B1: cao that D cannot be critical with mention of all paths through D only contain 4 activities (oe method e.g., showing that the total float on activity D is not zero)

SCB1 – stating or implying that D has a float of 1 (oe) by considering a forward pass (which may or may not be done mathematically) up to at least activity D

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td>8(a)</td><td>$x+y+z\leq 39$</td><td>B1</td><td>3.3</td></tr>
<tr><td></td><td>$\frac{2}{5}(x+y+z)\leq x \quad (\Rightarrow -3x+2y+2z\leq 0)$</td><td>M1
A1</td><td>3.3
1.1b</td></tr>
<tr><td></td><td>$x+z\geq 28$</td><td>B1</td><td>1.1b</td></tr>
<tr><td></td><td>Maximise $P=y+z \quad (\Rightarrow P-y-z=0)$</td><td>B1</td><td>3.3</td></tr>
<tr><td></td><td></td><td>(5)</td><td></td></tr>
<tr><td>(b)</td><td>$x+y+z\leq 39 \Rightarrow x+y+z+s_1=39$
$-3x+2y+2z\leq 0 \Rightarrow -3x+2y+2z+s_2=0$</td><td>M1
A1</td><td>2.1
1.1b</td></tr>
<tr><td></td><td>$x+z\geq 28 \Rightarrow x+z-s_3+a_1=28$</td><td>B1</td><td>2.5</td></tr>
<tr><td></td><td>$I=-a_1 \Rightarrow I-x-z+s_3=-28$</td><td>M1</td><td>2.1</td></tr>
<tr><td></td><td>e.g.
<table><tr><td>b.v</td><td>x</td><td>y</td><td>z</td><td>s_1</td><td>s_2</td><td>s_3</td><td>a_1</td><td>Value</td></tr><tr><td>s_1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>0</td><td>0</td><td>0</td><td>39</td></tr><tr><td>s_2</td><td>−3</td><td>2</td><td>2</td><td>0</td><td>1</td><td>0</td><td>0</td><td>0</td></tr><tr><td>a_1</td><td>1</td><td>0</td><td>1</td><td>0</td><td>0</td><td>−1</td><td>1</td><td>28</td></tr><tr><td>P</td><td>0</td><td>−1</td><td>−1</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td></tr><tr><td>I</td><td>−1</td><td>0</td><td>−1</td><td>0</td><td>0</td><td>1</td><td>0</td><td>−28</td></tr></table></td><td>M1
A1</td><td>3.3
2.2a</td></tr>
<tr><td></td><td></td><td>(6)</td><td></td></tr>
<tr><td>(c)</td><td>The only negative in the objective row is the -1 so the pivot is from the z-column</td><td>B1</td><td>2.4</td></tr>
<tr><td></td><td>The 5 in the s_2 row is the pivot because $\frac{62}{5}$ is less than $\frac{28}{1}$</td><td>B1</td><td>2.2a</td></tr>
<tr><td></td><td></td><td>(2)</td><td></td></tr>
</table>

(d)			

b.v.	x	y	z	s_1	s_2	s_3	Value	Row Ops
y	0	1	0	1	0	1	11	R1
z	0	0	1	$-\frac{2}{5}$	$\frac{1}{5}$	–1	$\frac{62}{5}$	$\frac{1}{5}$ R2
x	1	0	0	$\frac{2}{5}$	$-\frac{1}{5}$	0	$\frac{78}{5}$	R3 – R2
P	0	0	0	$\frac{3}{5}$	$\frac{1}{5}$	0	$\frac{117}{5}$	R4 + R2

		B1	1.1b
		M1	2.1
		A1	1.1b
		A1	1.1b
	Spend 15.6 hours swimming, 11 hours cycling and 12.4 hours running	A1	3.2a
		(5)	
			(18 marks)

Notes:

(a)

B1: cao ($x + y + z \leq 39$)

M1: $\frac{2}{5}(x + y + z) \square x$ where $\square$ is any inequality or equals

A1: cao

B1: cao ($x + z \geq 28$)

B1: Correct objective function ($P = y + z$) plus 'maximise' or 'max' but not 'maximum'

(b)

M1: One $\leq$ constraint re-formulated as an equation using slack variables – dependent on either the first **B** mark in **(a)** or the **M** mark in **(a)**

A1: cao (both $\leq$ constraints)

B1: $\geq$ constraint re-formulated as an equation using one surplus and one artificial variable

M1: Formulates second objective with $I = -a_1$ and their expression for a_1

M1: Setting up the initial tableau – all five rows complete with two correct rows (but ignore b.v. column for this mark)

A1: cao (any equivalent correct form)

(c)

B1: Correct reasoning that the pivot is a value from the z-column – condone any mention of negative value in P row

B1: Correct justification of why the 5 in the s_2 row is the next pivot – so must compare or state that 12.4 is less than 28 (not sufficient to just say that 12.4 (oe) is the least)

(d)

B1: Pivot row correct including change of b.v.

M1: All values in one of the non-pivot rows correct **or** one of the non zero and one columns (s_1, s_2 or value) correct (from their choice of pivot)

A1: Row operations used correctly at least twice, i.e. **two** of the non zero and one columns (s_1, s_2 or value)

A1: For all values and row operations correctly stated – do not penalise lack of correct b.v. in pivot row twice. Condone blank Row Ops in the first row only

A1: Correct allocation of training times – must be in context (so not just in terms of x, y and z)