

Please write clearly in block capitals.		
Centre number	Candidate number	
Surname		
Forename(s)		
Candidate signature		

A-level **MATHEMATICS**

Unit Pure Core 3

Wednesday 13 June 2018 Morning Time allowed: 1 hour 30 minutes

Materials

For this paper you must have:

the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer all questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question.
 If you require extra space, use an AQA supplementary answer book; do not use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
TOTAL	



Answer all questions.

Answer each question in the space provided for that question.

1 (a) Find
$$\frac{dy}{dx}$$
 when $y = (5 + 3x^2)^{\frac{1}{2}}$.

[2 marks]

(b) Find
$$\int (1 + \sin 4x) \, dx$$
.

[3 marks]

QUESTION PART REFERENCE	Answer space for question 1



QUESTION PART REFERENCE	Answer space for question 1
REFERENCE	



2 (a) By writing $\cot x$ as $\frac{\cos x}{\sin x}$, use the quotient rule to show that $\frac{\mathrm{d}}{\mathrm{d}x}(\cot x) = -\csc^2 x$.

[2 marks]

- **(b)** The curve with equation $x = \frac{\pi}{12} y + \cot 3y$ is defined for $0 < y < \frac{\pi}{3}$.
 - (i) Find $\frac{\mathrm{d}x}{\mathrm{d}y}$ in terms of y.

[2 marks]

(ii) Hence find the exact equation of the tangent to the curve at the point $\left(1, \frac{\pi}{12}\right)$, giving your answer in the form y = mx + c, where m is a rational number.

[4 marks]

QUESTION PART REFERENCE	Answer space for question 2



QUESTION PART REFERENCE	Answer space for question 2
REFERENCE	



3 (a) (i) Sketch the graph of $y = \ln(2x)$, stating the coordinates of any point where the curve crosses the coordinate axes.

[2 marks]

(ii) Describe a sequence of two geometrical transformations that maps the graph of $y = \ln(2x)$ onto the graph of $y = \ln(3 + 4x)$.

[4 marks]

(b) (i) Use Simpson's rule with five ordinates (four strips) to find an approximate value for $\int_{1}^{3} \ln(3+4x) \, dx$, giving your answer to five significant figures.

[4 marks]

(ii) Given that the value of $\int_1^3 \ln(3+4x) \, dx$ is A, show that the value of $\int_1^3 \ln(3e+4ex) \, dx$ is A+2.

[2 marks]

swer space for question 3



QUESTION PART REFERENCE	Answer space for question 3
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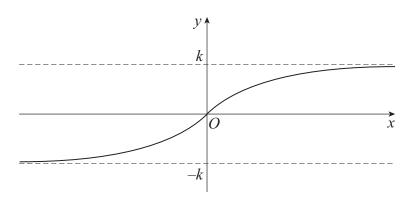
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QUESTION PART REFERENCE	Answer space for question 3
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4 (a) The function f, given by $f(x) = \tan^{-1} x$, is defined for all values of x. The graph of y = f(x) is sketched below.



(i) The range of f is -k < f(x) < k. State the exact value of k.

[1 mark]

(ii) On Figure 1 below, sketch the graph of $y = |\tan^{-1} x|$.

[1 mark]

(iii) By drawing a suitable straight line on your sketch on **Figure 1**, show that the equation $|\tan^{-1} x| - 7x - 3 = 0$

has exactly one real root.

[2 marks]

(b) The real root of the equation $|\tan^{-1} x| - 7x - 3 = 0$ is α . Show that α lies between -0.4 and -0.3.

[2 marks]

(c) Use the iterative formula

$$x_{n+1} = \frac{1}{7}(|\tan^{-1} x_n| - 3)$$
 with $x_1 = -0.4$

to find the values of x_2 and x_3 , giving your answers to three decimal places.

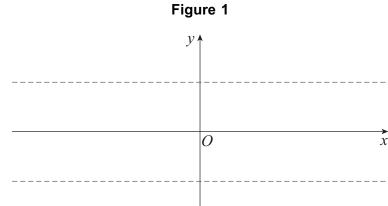
[2 marks]

QUESTION PART REFERENCE

Answer space for question 4

(a)(i)

 $k = \dots$



QUESTION PART REFERENCE	Answer space for question 4
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5	The functions f and g are defined with their respective domains	s by
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$$f(x) = \sqrt{2x+3}$$
 for $x \ge -1$
 $g(x) = x^2 + 4x$ for all real values of x

- (a) The inverse of f is f^{-1} .
 - (i) Find $f^{-1}(x)$.

[3 marks]

(ii) State the domain of f^{-1} .

[1 mark]

(b) Find the range of g.

[3 marks]

(c) (i) Find gf(x).

[1 mark]

(ii) Solve the equation gf(x) = 21.

[5 marks]

PART REFERENCE	Answer space for question 5



QUESTION PART REFERENCE	Answer space for question 5
REFERENCE	



QUESTION PART REFERENCE	Answer space for question 5



QUESTION PART REFERENCE	Answer space for question 5
REFERENCE	



6 Use the substitution $u = 2 + \ln x$ to show that

$$\int_{1}^{e} \frac{\ln x}{x(2+\ln x)^{2}} dx = p + \ln q$$

where p and q are rational numbers.

[7 marks]

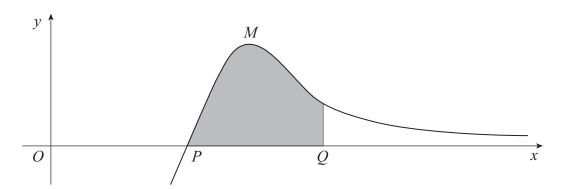
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7 Part of a curve is sketched below.



The curve has equation $y = (x - 1) e^{-3x}$.

(a) The curve has a stationary point M. Show that the x-coordinate of M is $\frac{4}{3}$. [3 marks]

(b) (i) By using integration by parts twice, find

$$\int (x-1)^2 e^{-6x} \, \mathrm{d}x$$

[6 marks]

(ii) The curve crosses the x-axis at the point P. The point Q has coordinates (2,0). The shaded region bounded by the curve, the x-axis from P to Q and the line x=2 is rotated through 2π radians about the x-axis. Find the exact value of the volume of the solid generated.

[3 marks]

QUESTION PART REFERENCE	Answer space for question 7

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QUESTION PART REFERENCE	Answer space for question 7



QUESTION PART REFERENCE	Answer space for question 7



8 (a)	Show that the expression	$\sec \theta$	$\sec \theta$	can be written as $2 \cos^2 \theta$
o (a)		$\sec \theta - 1$	$\overline{\sec \theta + 1}$	can be written as 2 cosec

[3 marks]

(b) Hence solve the equation

$$\frac{\sec(2x+0.4)}{\sec(2x+0.4)-1} + \frac{\sec(2x+0.4)}{\sec(2x+0.4)+1} = 8 - \cot(2x+0.4)$$

giving your answers in radians to three significant figures in the interval $0 < x < \pi$.

[7 marks]

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END OF QUESTIONS

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