

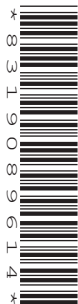


Oxford Cambridge and RSA

Thursday 08 October 2020 – Afternoon

A Level Further Mathematics A Y541/01 Pure Core 2

Time allowed: 1 hour 30 minutes



You must have:

- the Printed Answer Booklet
- the Formulae Booklet for A Level Further Mathematics A
- a scientific or graphical calculator

INSTRUCTIONS

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION

- The total mark for this paper is **75**.
- The marks for each question are shown in brackets [].
- This document has **8** pages.

ADVICE

- Read each question carefully before you start your answer.

Answer **all** the questions.

1 In this question you must show detailed reasoning.

Solve the equation $4z^2 - 20z + 169 = 0$. Give your answers in modulus-argument form. [5]

2 In this question you must show detailed reasoning.

The roots of the equation $3x^3 - 2x^2 - 5x - 4 = 0$ are α , β and γ .

(a) Find a cubic equation with integer coefficients whose roots are α^2 , β^2 and γ^2 . [4]

(b) Find the exact value of $\frac{\alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2}{\alpha\beta\gamma}$. [2]

3 In this question you must show detailed reasoning.

(a) Use partial fractions to show that $\sum_{r=5}^n \frac{3}{r^2 + r - 2} = \frac{37}{60} - \frac{1}{n} - \frac{1}{n+1} - \frac{1}{n+2}$. [5]

(b) Write down the value of $\lim_{n \rightarrow \infty} \left(\sum_{r=5}^n \frac{3}{r^2 + r - 2} \right)$. [1]

4 The equations of two intersecting lines l_1 and l_2 are

$$l_1: \mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ a \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix} \quad l_2: \mathbf{r} = \begin{pmatrix} 7 \\ 9 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix}$$

where a is a constant.

The equation of the plane Π is

$$\mathbf{r} \cdot \begin{pmatrix} 1 \\ 5 \\ 3 \end{pmatrix} = -14.$$

l_1 and Π intersect at Q .

l_2 and Π intersect at R .

(a) Verify that the coordinates of R are $(13, 3, -14)$. [2]

(b) Determine the exact value of the length of QR . [7]

- 5 A capacitor is an electrical component which stores charge. The value of the charge stored by the capacitor, in suitable units, is denoted by Q . The capacitor is placed in an electrical circuit.

At any time t seconds, where $t \geq 0$, Q can be modelled by the differential equation

$$\frac{d^2Q}{dt^2} - 2\frac{dQ}{dt} - 15Q = 0.$$

Initially the charge is 100 units and it is given that Q tends to a finite limit as t tends to infinity.

- (a) Determine the charge on the capacitor when $t = 0.5$. [6]
- (b) Determine the finite limit of Q as t tends to infinity. [1]
- 6 The equation of a curve in polar coordinates is $r = \ln(1 + \sin \theta)$ for $\alpha \leq \theta \leq \beta$ where α and β are non-negative angles. The curve consists of a single closed loop through the pole.
- (a) By solving the equation $r = 0$, determine the smallest possible values of α and β . [2]
- (b) Find the area enclosed by the curve, giving your answer to 4 significant figures. [2]
- (c) Hence, by considering the value of r at $\theta = \frac{\alpha + \beta}{2}$, show that the loop is **not** circular. [2]

- 7 The matrix \mathbf{A} is given by $\mathbf{A} = \begin{pmatrix} 0.6 & 2.4 \\ -0.8 & 1.8 \end{pmatrix}$.

- (a) Find $\det \mathbf{A}$. [1]

The matrix \mathbf{A} represents a stretch parallel to one of the coordinate axes followed by a rotation about the origin.

- (b) By considering the determinants of these transformations, determine the scale factor of the stretch. [2]
- (c) Explain whether the stretch is parallel to the x -axis or the y -axis, justifying your answer. [1]
- (d) Find the angle of rotation. [2]

8 In this question you must show detailed reasoning.

The complex number $-4 + i\sqrt{48}$ is denoted by z .

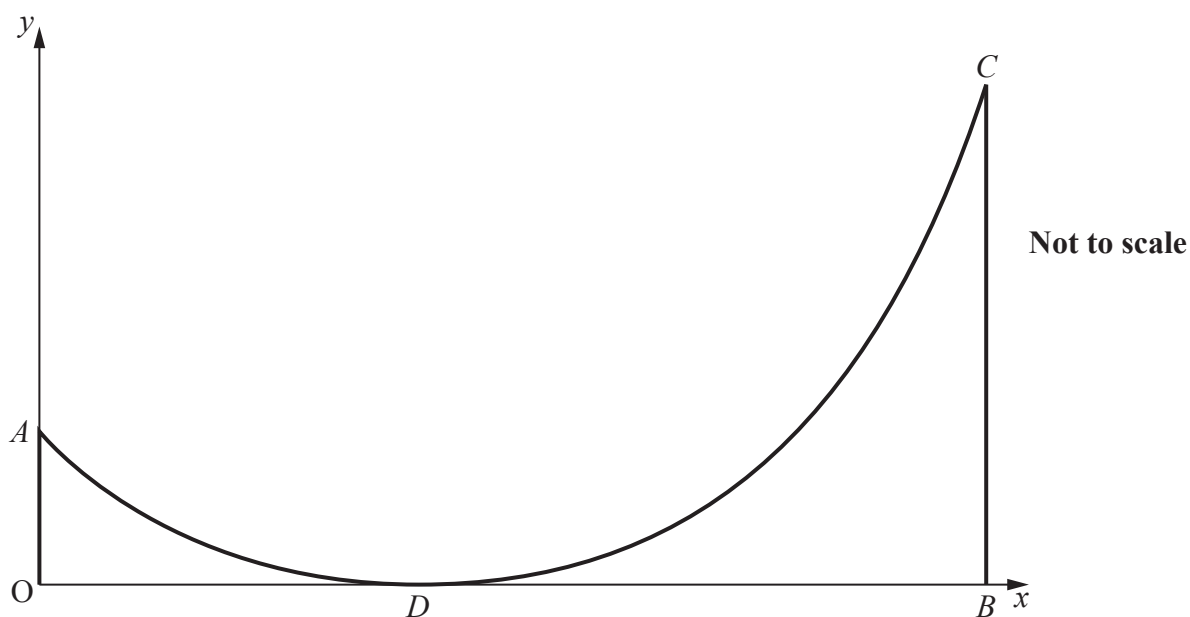
- (a) Determine the cube roots of z , giving the roots in exponential form. [6]

The points which represent the cube roots of z are denoted by A , B and C and these form a triangle in an Argand diagram.

- (b) Write down the angles that any lines of symmetry of triangle ABC make with the positive real axis, justifying your answer. [3]

- 9 Two thin poles, OA and BC , are fixed vertically on horizontal ground. A chain is fixed at A and C such that it touches the ground at point D as shown in the diagram.

On a coordinate system the coordinates of A , B and D are $(0, 3)$, $(5, 0)$ and $(2, 0)$.



It is required to find the height of pole BC by modelling the shape of the curve that the chain forms.

Jofra models the curve using the equation $y = k \cosh(ax - b) - 1$ where k , a and b are positive constants.

- (a) Determine the value of k . [2]

- (b) Find the exact value of a and the exact value of b , giving your answers in logarithmic form. [5]

Holly models the curve using the equation $y = \frac{3}{4}x^2 - 3x + 3$.

- (c) Write down the coordinates of the point, (u, v) where u and v are both non-zero, at which the two models will agree. [1]

- (d) Show that Jofra's model and Holly's model disagree in their predictions of the height of pole BC by 3.32 m to 3 significant figures. [3]

10 Let $f(x) = \sin^{-1}(x)$.

- (a) (i) Determine $f''(x)$. [2]
- (ii) Determine the first two non-zero terms of the Maclaurin expansion for $f(x)$. [3]
- (iii) By considering the first two non-zero terms of the Maclaurin expansion for $f(x)$, find an approximation to $\int_0^{\frac{1}{2}} f(x) dx$. Give your answer correct to 6 decimal places. [2]
- (b) By writing $f(x)$ as $\sin^{-1}(x) \times 1$, determine the value of $\int_0^{\frac{1}{2}} f(x) dx$. Give your answer in exact form. [3]

END OF QUESTION PAPER

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