

4751 (C1) Introduction to Advanced Mathematics

1	$[a =]2c^2 - b$ www o.e.	3	M1 for each of 3 complete correct steps, ft from previous error if equivalent difficulty
2	$5x - 3 < 2x + 10$ $3x < 13$ $x < \frac{13}{3}$ o.e.	M1 M1 M1	condone '=' used for first two Ms M0 for just $5x - 3 < 2(x + 5)$ or $-13 < -3x$ or ft or ft; isw further simplification of $13/3$; M0 for just $x < 4.3$
3 (i)	(4, 0)	1	allow $y = 0, x = 4$ bod B1 for $x = 4$ but do not isw: 0 for (0, 4) seen 0 for (4, 0) and (0, 10) both given (choice) unless (4, 0) clearly identified as the x -axis intercept
3 (ii)	$5x + 2(5 - x) = 20$ o.e. (10/3, 5/3) www isw	M1 A2	for subst or for multn to make coeffs same and appropriate addn/subtn; condone one error or A1 for $x = 10/3$ and A1 for $y = 5/3$ o.e. isw; condone 3.33 or better and 1.67 or better A1 for (3.3, 1.7)
4 (i)	translation by $\begin{pmatrix} -4 \\ 0 \end{pmatrix}$ or 4 [units] to left	B1 B1	0 for shift/move or 4 units in negative x direction o.e.
4 (ii)	sketch of parabola right way up and with minimum on negative y -axis min at (0, -4) and graph through -2 and 2 on x -axis	B1 B1	mark intent for both marks must be labelled or shown nearby
5 (i)	$\frac{1}{12}$ or $\pm \frac{1}{12}$	2	M1 for $\frac{1}{144^{\frac{1}{2}}}$ o.e. or for $\sqrt{144} = 12$ soi
5 (ii)	denominator = 18 numerator = $5 - \sqrt{7} + 4(5 + \sqrt{7})$ = $25 + 3\sqrt{7}$ as final answer	B1 M1 A1	B0 if 36 after addition for M1, allow in separate fractions allow B3 for $\frac{25 + 3\sqrt{7}}{18}$ as final answer www

6 (i)	<p>cubic correct way up and with two turning pts</p> <p>touching x-axis at -1, and through it at 2.5 and no other intersections</p> <p>y- axis intersection at -5</p>	<p>B1</p> <p>B1</p> <p>B1</p>	<p>intns must be shown labelled or worked out nearby</p>
6 (ii)	$2x^3 - x^2 - 8x - 5$	<p>2</p>	<p>B1 for 3 terms correct or M1 for correct expansion of product of two of the given factors</p>
7	<p>attempt at $f(-3)$ $-27 + 18 - 15 + k = 6$</p> <p>$k = 30$</p>	<p>M1</p> <p>A1</p> <p>A1</p>	<p>or M1 for long division by $(x + 3)$ as far as obtaining $x^2 - x$ and A1 for obtaining remainder as $k - 24$ (but see below)</p> <p>equating coefficients method: M2 for $(x + 3)(x^2 - x + 8) [+6]$ o.e. (from inspection or division) eg M2 for obtaining $x^2 - x + 8$ as quotient in division</p>
8	$x^3 + 15x + \frac{75}{x} + \frac{125}{x^3}$ www isw or $x^3 + 15x + 75x^{-1} + 125x^{-3}$ www isw	<p>4</p>	<p>B1 for both of x^3 and $\frac{125}{x^3}$ or $125x^{-3}$ isw</p> <p>and</p> <p>M1 for 1 3 3 1 soi; A1 for each of $15x$ and $\frac{75}{x}$ or $75x^{-1}$ isw</p> <p>or</p> <p>SC2 for completely correct unsimplified answer</p>

<p>9</p> <p>$x^2 - 5x + 7 = 3x - 10$</p> <p>$x^2 - 8x + 17 [= 0]$ o.e or $y^2 - 4y + 13 [= 0]$ o.e</p> <p>use of $b^2 - 4ac$ with numbers subst (condone one error in substitution) (may be in quadratic formula)</p> <p>$b^2 - 4ac = 64 - 68$ or -4 cao [or $16 - 52$ or -36 if y used]</p> <p>[< 0] so no [real] roots [so line and curve do not intersect]</p>		<p>M1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>A1</p>	<p>or attempt to subst $(y + 10)/3$ for x</p> <p>condone one error; allow M1 for $x^2 - 8x = -17$ [oe for y] only if they go on to completing square method</p> <p>or $(x - 4)^2 = 16 - 17$ or $(x - 4)^2 + 1 = 0$ (condone one error)</p> <p>or $(x - 4)^2 = -1$ or $x = 4 \pm \sqrt{-1}$ [or $(y - 2)^2 = -9$ or $y = 2 \pm \sqrt{-9}$]</p> <p>or conclusion from comp. square; needs to be explicit correct conclusion and correct ft; allow '< 0 so no intersection' o.e.; allow '-4 so no roots' etc</p> <p>allow A2 for full argument from sum of two squares = 0; A1 for weaker correct conclusion</p> <p>some may use the condition $b^2 < 4ac$ for no real roots; allow equivalent marks, with first A1 for $64 < 68$ o.e.</p>
<p>10 (i)</p> <p>grad CD = $\frac{5-3}{3-(-1)} \left[= \frac{2}{4} \text{ o.e.} \right]$ isw</p> <p>grad AB = $\frac{3-(-1)}{6-(-2)}$ or $\frac{4}{8}$ isw</p> <p>same gradient so parallel www</p>		<p>M1</p> <p>M1</p> <p>A1</p>	<p>NB needs to be obtained independently of grad AB</p> <p>must be explicit conclusion mentioning 'same gradient' or 'parallel'</p> <p>if M0, allow B1 for 'parallel lines have same gradient' o.e.</p>
<p>10 (ii)</p> <p>[$BC^2 =$] $3^2 + 2^2$ [$BC^2 =$] 13 showing $AD^2 = 1^2 + 4^2 [=17]$ [$\neq BC^2$] isw</p>		<p>M1</p> <p>A1</p> <p>A1</p>	<p>accept $(6 - 3)^2 + (3 - 5)^2$ o.e. or [BC =] $\sqrt{13}$ or [AD =] $\sqrt{17}$</p> <p>or equivalent marks for finding AD or AD^2 first</p> <p>alt method: showing $AC \neq BD$ – mark equivalently</p>

10 (iii)	<p>[BD eqn is] $y = 3$</p> <p>eqn of AC is $y - 5 = 6/5 \times (x - 3)$ o.e [$y = 1.2x + 1.4$ o.e.]</p> <p>M is $(4/3, 3)$ o.e. isw</p>	<p>M1</p> <p>M2</p> <p>A1</p>	<p>eg allow for 'at M, $y = 3$' or for 3 subst in eqn of AC</p> <p>or M1 for grad AC = $6/5$ o.e. (accept unsimplified) and M1 for using their grad of AC with coords of A(-2, -1) or C (3, 5) in eqn of line or M1 for 'stepping' method to reach M</p> <p>allow : at M, $x = 16/12$ o.e. [eg $=4/3$] isw A0 for 1.3 without a fraction answer seen</p>
10 (iv)	<p>midpt of BD = $(5/2, 3)$ or equivalent simplified form cao</p> <p>midpt AC = $(1/2, 2)$ or equivalent simplified form cao or 'M is $2/3$ of way from A to C'</p> <p>conclusion 'neither diagonal bisects the other'</p>	<p>M1</p> <p>M1</p> <p>A1</p>	<p>or showing $BM \neq MD$ oe [$BM = 14/3, MD = 7/3$]</p> <p>or showing $AM \neq MC$ or $AM^2 \neq MC^2$</p> <p>in these methods A1 is dependent on coords of M having been obtained in part (iii) or in this part; the coordinates of M need not be correct; it is also dependent on midpts of both AC and BD attempted, at least one correct</p> <p>alt method: show that mid point of BD does not lie on AC (M1) and vice-versa (M1), A1 for both and conclusion</p>

11 (i)	centre $C' = (3, -2)$ radius 5	1 1	0 for ± 5 or -5
11 (ii)	showing $(6 - 3)^2 + (-6 + 2)^2 = 25$ showing that $\overrightarrow{AC'} = \overrightarrow{C'B} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$ o.e.	B1 B2	interim step needed or B1 each for two of: showing midpoint of $AB = (3, -2)$; showing $B(0, 2)$ is on circle; showing $AB = 10$ or B2 for showing midpoint of $AB = (3, -2)$ and saying this is centre of circle or B1 for finding eqn of AB as $y = -4/3x + 2$ o.e. and B1 for finding one of its intersections with the circle is $(0, 2)$ or B1 for showing $C'B = 5$ and B1 for showing $AB = 10$ or that AC' and BC' have the same gradient or B1 for showing that AC' and BC' have the same gradient and B1 for showing that $B(0, 2)$ is on the circle
11 (iii)	grad AC' or $AB = -4/3$ o.e. grad $tgt = -1/\text{their } AC'$ grad $y - (-6) = \text{their } m(x - 6)$ o.e. $y = 0.75x - 10.5$ o.e. isw	M1 M1 M1 A1	or ft from their C' , must be evaluated may be seen in eqn for tgt ; allow M2 for $\text{grad } tgt = 3/4$ oe soi as first step or M1 for $y = \text{their } m \times x + c$ then subst $(6, -6)$ eg A1 for $4y = 3x - 42$ allow B4 for correct equation www isw
11 (iv)	centre C is at $(12, -14)$ cao circle is $(x - 12)^2 + (y + 14)^2 = 100$	B2 B1	B1 for each coord ft their C if at least one coord correct

12 (i)	10	1	
12 (ii)	[x =] 5 or ft their (i) $\div 2$ ht = 5[m] cao	1 1	not necessarily ft from (i) eg they may start again with calculus to get $x = 5$
12 (iii)	$d = 7/2$ o.e. [y =] $1/5 \times 3.5 \times (10 - 3.5)$ o.e. or ft = $91/20$ o.e. cao isw	M1 M1 A1	or ft their (ii) $- 1.5$ or their (i) $\div 2 - 1.5$ o.e. or $7 - 1/5 \times 3.5^2$ or ft or showing $y - 4 = 11/20$ o.e. cao
12 (iv)	$4.5 = 1/5 \times x(10 - x)$ o.e. $22.5 = x(10 - x)$ o.e. $2x^2 - 20x + 45 [= 0]$ o.e. eg $x^2 - 10x + 22.5 [= 0]$ or $(x - 5)^2 = 2.5$ [x =] $\frac{20 \pm \sqrt{40}}{4}$ or $5 \pm \frac{1}{2}\sqrt{10}$ o.e. width = $\sqrt{10}$ o.e. eg $2\sqrt{2.5}$ cao	M1 M1 A1 M1 A1	eg $4.5 = x(2 - 0.2x)$ etc cao; accept versions with fractional coefficients of x^2 , isw or $x - 5 = [\pm]\sqrt{2.5}$ o.e.; ft their quadratic eqn provided at least M1 gained already; condone one error in formula or substitution; need not be simplified or be real accept simple equivalents only