4723 Core Mathematics 3

1 (i)	Obtain integral of form ke^{-2x}	M1		any constant k different from 8
	Obtain $-4e^{-2x}$	A1		or (unsimplified) equiv
(ii)	Obtain integral of form $k(4x+5)^7$	M1		any constant k
	Obtain $\frac{1}{28}(4x+5)^7$	A1		in simplified form
	Include … + c at least once	B1		in either part
			5	
2 (i)	Form expression involving attempts at y values and addition	M1		with coeffs 1, 4 and 2 present at least once
	Obtain $k(\ln 4+4\ln 6+2\ln 8+4\ln 10+\ln 12)$	A1		any constant k
	Use value of k as $\frac{1}{3}\times 2$	A1		or unsimplified equiv
	Obtain 16.27	A1	**4**	or 16.3 or greater accuracy (16.27164…)
(ii)	State 162.7 or 163	B1√	**1**	following their answer to **(i)**, maybe rounded
			5	
3 (i)	Attempt use of identity for $\tan^2\theta$	M1		using $\pm\sec^2\theta\pm 1$; or equiv
	Replace $\frac{1}{\cos\theta}$ by $\sec\theta$	B1		
	Obtain $2(\sec^2\theta-1)-\sec\theta$	A1	**3**	or equiv
(ii)	Attempt soln of quadratic in $\sec\theta$ or $\cos\theta$	M1		as far as factorisation or substitution in correct formula
	Relate $\sec\theta$ to $\cos\theta$ and attempt at least one value of θ	M1		may be implied
	Obtain $60°, 131.8°$	A1		allow 132 or greater accuracy
	Obtain $60°, 131.8°, 228.2°, 300°$	A1	**4**	allow 132, 228 or greater accuracy; and no others between $0°$ and $360°$
			7	
4 (i)	Obtain derivative of form $kx(4x^2+1)^4$	M1		any constant k
	Obtain $40x(4x^2+1)^4$	A1		or (unsimplified) equiv
	State $x = 0$	A1√	**3**	and no other; following their derivative of form $kx(4x^2+1)^4$
(ii)	Attempt use of quotient rule	M1		or equiv
	Obtain $\frac{2x\ln x - x^2.\frac{1}{x}}{(\ln x)^2}$	A1		or equiv
	Equate to zero and attempt solution	M1		as far as solution involving e
	Obtain $e^{\frac{1}{2}}$	A1	**4**	or exact equiv; and no other; allow from $\pm$ (correct numerator of derivative)
			7	

5 (i)	State 40	B1		
	Attempt value of k using 21 and 80	M1		or equiv
	Obtain $40\mathrm{e}^{21k} = 80$ and hence 0.033	A1		or equiv such as $\frac{1}{21}\ln 2$
	Attempt value of M for $t = 63$	M1		using established formula or using exponential property
	Obtain 320	A1	**5**	or value rounding to this
(ii)	Differentiate to obtain $c\mathrm{e}^{0.033t}$ or $40k\mathrm{e}^{kt}$	M1		any constant c different from 40
	Obtain $40 \times 0.033\mathrm{e}^{0.033t}$	A1√		following their value of k
	Obtain 2.64	A1	**3**	allow 2.6 or 2.64 ± 0.01 or greater accuracy (2.64056…)
			[8]	

6 (i)	Attempt correct process for finding inverse	M1		maybe in terms of y so far
	Obtain $2x^3 - 4$	A1		or equiv; in terms of x now
	State $\sqrt[3]{2}$ or 1.26	B1	**3**	
(ii)	State reflection in $y = x$	B1		or clear equiv
	Refer to intersection of $y = x$ and $y = \mathrm{f}(x)$ and hence confirm $x = \sqrt[3]{\frac{1}{2}x + 2}$	B1	**2**	AG; or equiv
(iii)	Obtain correct first iterate	B1		
	Show correct process for iteration	M1		with at least one more step
	Obtain at least 3 correct iterates in all	A1		allowing recovery after error
	Obtain 1.39	A1	**4**	following at least 3 steps; answer required to exactly 2 d.p.

[0 → 1.259921 → 1.380330 → 1.390784 → 1.391684
1 → 1.357209 → 1.388789 → 1.391512 → 1.391747
1.26 → 1.380337 → 1.390784 → 1.391684 → 1.391761
1.5 → 1.401020 → 1.392564 → 1.391837 → 1.391775
2 → 1.442250 → 1.396099 → 1.392141 → 1.391801]

[9]

7 (i)	Refer to stretch and translation	M1		in either order; allow here informal terms
	State stretch, factor $\frac{1}{k}$, in x direction	A1		or equiv; now with correct terminology
	State translation in negative y direction by a	A1	**3**	or equiv; now with correct terminology
	[SC: If M0 but one transformation completely correct – B1]			
(ii)	Show attempt to reflect negative part in x-axis	M1		ignoring curvature
	Show correct sketch	A1	**2**	with correct curvature, no pronounced 'rounding' at x-axis and no obvious maximum point
(iii)	Attempt method with $x = 0$ to find value of a	M1		... other than (or in addition to) value –12
	Obtain $a = 14$	A1		and nothing else
	Attempt to solve for k	M1		using any numerical a with sound process
	Obtain $k = 3$	A1	**4**	
			[9]	

8	**(i)**	Attempt to express x or x^2 in terms of y	M1		
		Obtain $x^2 = \frac{1296}{(y+3)^4}$	A1		or (unsimplified) equiv
		Obtain integral of form $k(y+3)^{-3}$	M1		any constant k
		Obtain $-432\pi(y+3)^{-3}$ or $-432(y+3)^{-3}$	A1		or (unsimplified) equiv
		Attempt evaluation using limits 0 and p	M1		for expression of form $k(y+3)^{-n}$ obtained from integration attempt; subtraction correct way round
		Confirm $16\pi(1-\frac{27}{(p+3)^3})$	A1	**6**	AG; necessary detail required, including appearance of π prior to final line
	(ii)	State or obtain $\frac{dV}{dp} = 1296\pi(p+3)^{-4}$	B1		or equiv; perhaps involving y
		Multiply $\frac{dp}{dt}$ and attempt at $\frac{dV}{dp}$	*M1		algebraic or numerical
		Substitute $p = 9$ and attempt evaluation	M1		dep *M
		Obtain $\frac{1}{4}\pi$ or 0.785	A1	**4**	or greater accuracy
				10	
9	**(i)**	State $\cos 2\theta \cos\theta - \sin 2\theta \sin\theta$	B1		
		Use at least one of $\cos 2\theta = 2\cos^2\theta - 1$ and $\sin 2\theta = 2\sin\theta\cos\theta$	B1		
		Attempt to express in terms of $\cos\theta$ only	M1		using correct identities for $\cos 2\theta$, $\sin 2\theta$ and $\sin^2\theta$
		Obtain $4\cos^3\theta - 3\cos\theta$	A1	**4**	AG; necessary detail required
	(ii)	Either: State or imply $\cos 6\theta = 2\cos^2 3\theta - 1$	B1		
		Use expression for $\cos 3\theta$ and attempt expansion	M1		for expression of form $\pm 2\cos^2 3\theta \pm 1$
		Obtain $32c^6 - 48c^4 + 18c^2 - 1$	A1	**3**	AG; necessary detail required
		Or: State $\cos 6\theta = 4\cos^3 2\theta - 3\cos 2\theta$	B1		maybe implied
		Express $\cos 2\theta$ in terms of $\cos\theta$ and attempt expansion	M1		for expression of form $\pm 2\cos^2\theta \pm 1$
		Obtain $32c^6 - 48c^4 + 18c^2 - 1$	A1	**(3)**	AG; necessary detail required
	(iii)	Substitute for $\cos 6\theta$	*M1		with simplification attempted
		Obtain $32c^6 - 48c^4 = 0$	A1		or equiv
		Attempt solution for c of equation	M1		dep *M
		Obtain $c^2 = \frac{3}{2}$ and observe no solutions	A1		or equiv; correct work only
		Obtain $c = 0$, give at least three specific angles and conclude odd multiples of 90	A1	**5**	AG; or equiv; necessary detail required; correct work only
				12	