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Mark Scheme (Results)

Summer 2009

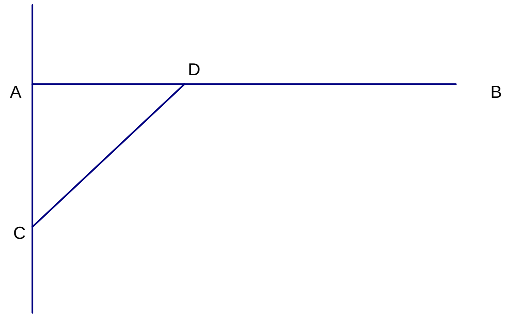
GCE

GCE Mathematics (6678/01)

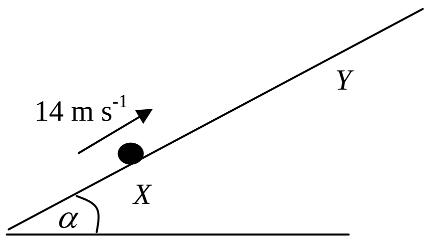
A PEARSON COMPANY

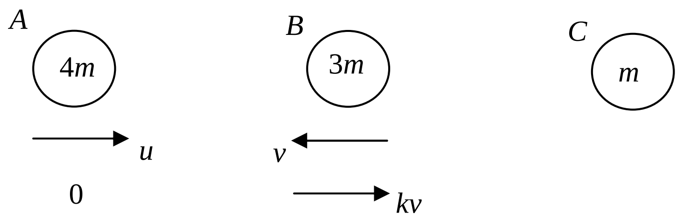
June 2009
6678 Mechanics M2
Mark Scheme

Question Number	Scheme	Marks
Q1	$\mathbf{I} = m\mathbf{v} - m\mathbf{u}$ $5\mathbf{i} - 3\mathbf{j} = \frac{1}{4}\mathbf{v} - \frac{1}{4}(3\mathbf{i} + 7\mathbf{j})$ $\mathbf{v} = 23\mathbf{i} - 5\mathbf{j}$ $\lvert\mathbf{v}\rvert = \sqrt{23^2 + 5^2} = 23.5$	 M1A1 A1 M1A1 **[5]**
Q2 (a)	$\frac{dv}{dt} = 8 - 2t$ $8 - 2t = 0$ Max $v = 8 \times 4 - 4^2 = 16\,(\text{ms}^{-1})$	M1 M1 M1A1 (4)
(b)	$\int 8t - t^2 dt = 4t^2 - \frac{1}{3}t^3 (+c)$ (t=0, displacement = 0 $\Rightarrow$ c=0) $4T^2 - \frac{1}{3}T^3 = 0$ $T^2(4 - \frac{T}{3}) = 0 \Rightarrow T = 0, 12$ T = 12 (seconds)	M1A1 DM1 DM1 A1 (5) **[9]**
Q3 (a)	Constant v $\Rightarrow$ driving force = resistance $\Rightarrow$ F=120 (N) $\Rightarrow$ P=120 x 10 = 1200W	 M1 M1 (2)
(b)	Resolving parallel to the slope, zero acceleration: $\frac{P}{v} = 120 + 300g\sin\theta (= 330)$ $\Rightarrow$ v = $\frac{1200}{330} = 3.6\,(\text{ms}^{-1})$	 M1A1A1 A1 (4) **[6]**

Question Number	Scheme	Marks
Q4 (a)	 Taking moments about A: $3g \times 0.75 = \frac{T}{\sqrt{2}} \times 0.5$ $T = 3\sqrt{2}g \times \frac{7.5}{5} = \frac{9\sqrt{2}g}{2} (= 62.4N)$	M1A1A1 A1 (4)
(b)	$\leftarrow \pm H = \frac{T}{\sqrt{2}} (= \frac{9g}{2} \approx 44.1N)$	B1
	$\uparrow \ \pm V + \frac{T}{\sqrt{2}} = 3g \quad (\Rightarrow V = 3g - \frac{9g}{2} = \frac{-3g}{2} \approx -14.7\text{ N})$	M1A1
	$\Rightarrow \ \lvert R \rvert = \sqrt{81+9} \times \frac{g}{2} \approx 46.5(N)$	M1A1
	at angle $\tan^{-1}\frac{1}{3} = 18.4^{\circ}$ (0.322 radians) below the line of BA 161.6° (2.82 radians) below the line of AB (108.4° or 1.89 radians to upward vertical)	M1A1 (7) **[11]**
Q5 (a)	Ratio of areas triangle:sign:rectangle = 1 : 5 : 6 (1800:9000:10800) Centre of mass of the triangle is 20cm down from *AD* (seen or implied)	B1 B1
	$\Rightarrow 6 \times 45 - 1 \times 20 = 5 \times \bar{y}$ $\bar{y} = 50cm$	M1A1 A1 (5)
(b)	Distance of centre of mass from *AB* is 60cm	B1
	Required angle is $\tan^{-1}\frac{60}{50}$ (their values) $= 50.2^{\circ}$ (0.876 rads)	M1A1ft A1 (4) **[9]**

Question Number	Scheme	Marks
Q6 (a)	$\rightarrow \quad x = u\cos\alpha t = 10$	M1A1
	$\uparrow \quad y = u\sin\alpha t - \frac{1}{2}gt^2 = 2$	M1A1
	$\Rightarrow t = \frac{10}{u\cos\alpha}$	
	$2 = u\sin\alpha \times \frac{10}{u\cos\alpha} - \frac{g}{2} \times \frac{100}{u^2\cos^2\alpha}$	M1
	$= 10\tan\alpha - \frac{50g}{u^2\cos^2\alpha}$ (given answer)	A1 (6)
(b)	$2 = 10 \times 1 - \frac{100g \times 2}{2u^2 \times 1}$	M1A1
	$u^2 = \frac{100g}{8}$, $u = \sqrt{\frac{100g}{8}} = 11.1$ (m s^{-1})	A1
	$\frac{1}{2}mu^2 = m \times 9.8 \times 2 + \frac{1}{2}mv^2$	M1A1
	$v = 9.1ms^{-1}$	A1 (6)
		[12]

Question Number	Scheme	Marks
Q7 (a)	 	
	KE at $X = \frac{1}{2}mv^2 = \frac{1}{2}\times 2\times 14^2$	B1
	GPE at $Y =$ $mgd\sin\alpha \left(= 2\times g\times d\times \frac{7}{25}\right)$	B1 B1
	Normal reaction $R = mg\cos\alpha$	M1
	Friction = $\mu\times R = \frac{1}{8}\times 2g\times\frac{24}{25}$	M1A1
	Work Energy: $\frac{1}{2}mv^2 - mgd\sin\alpha = \mu\times R\times d$ or equivalent	
	$196 = \frac{14gd}{25} + \frac{6gd}{25} = \frac{20gd}{25}$	A1
	$d = 25$ m	(7)
(b)	Work Energy	
	First time at X: $\frac{1}{2}mv^2 = \frac{1}{2}m14^2$	
	Work done = $\mu\times R\times 2d = \frac{1}{8}\times 2g\times\frac{24}{25}\times 2d$	
	Return to X: $\frac{1}{2}mv^2 = \frac{1}{2}m14^2 - \frac{1}{8}\times 2g\times\frac{24}{25}\times 50$	M1A1
	$v = 8.9$ ms^{-1} (accept 8.85 ms^{-1})	DM1A1
		(4)
	OR: Resolve parallel to XY to find the acceleration and use of $v^2 = u^2 + 2as$	
	$2a = 2g\sin\alpha - F_{\max} = 2g\times\frac{7}{25} - \frac{6g}{25} = \frac{8g}{25}$	M1A1
	$v^2 = (0+)2\times a\times s = 8g$; $v = 8.9$ (accept 8.85 ms^{-1})	DM1;A1
		[11]

Question Number	Scheme	Marks
Q8 (a)		
	Conservation of momentum: $4mu - 3mv = 3mkv$	M1A1
	Impact law: $kv = \frac{3}{4}(u + v)$	M1A1
	Eliminate k: $4mu - 3mv = 3m \times \frac{3}{4}(u + v)$	DM1
	$u = 3v$ (Answer given)	A1
		(6)
(b)	$kv = \frac{3}{4}(3v + v)$, $k = 3$	M1,A1
		(2)
(c)	Impact law: $(kv + 2v)e = v_C - v_B$ $\quad (5ve = v_C - v_B)$	B1
	Conservation of momentum : $3 \times kv - 1 \times 2v = 3v_B + v_c$ $\quad (7v = 3v_B + v_c)$	B1
	Eliminate v_C : $v_B = \frac{v}{4}(7 - 5e) > 0$ hence no further collision with A.	M1 A1
		(4)
		[12]