

Mathematics

Advanced GCE

Unit **4733**: Probability and Statistics 2

Mark Scheme for January 2011

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1		$\hat{\mu} = \bar{x} = \frac{468}{9} = 52$ $\frac{24820}{9} - 52^2 [= 53.78]$ $\hat{\sigma}^2 = \frac{9}{8} \times 53.78 = \mathbf{60.5}$	B1 M1 M1 A1 4	52 stated Correct method for biased estimator Multiply by 9/8 [if single formula, allow M0 M1 if wrong but divisor 8 seen anywhere] Answer 60.5 or exact equivalent
2		$\frac{53.28 - \mu}{5/\sqrt{n}} = 1.96$ $\frac{\mu - 51.65}{5/\sqrt{n}} = 1.3$ $\sqrt{n} = 10, \quad n = \mathbf{100}$ $\mu = \mathbf{52.3}$	M1dep A1 B1 depM1 A1 B1 6	Standardise with \sqrt{n} once & equate to z , allow sign, square/ $\sqrt{\quad}$ errors twice, signs correct, z s may be wrong Both correct z values seen Solve to get \sqrt{n} or μ , needs first M1 $n = 100$, not from wrong signs a.r.t. 52.3, right arithmetic needed but \sqrt{n} can be omitted
3		B(200, 0.0228) Po(4.56) $e^{-4.56} \left(1 + 4.56 + \frac{4.56^2}{2}\right)$ $= \mathbf{0.167}$ $n \text{ large or } n > 50; p \text{ small or } np < 5$	M1 A1 M1 A1 A1 B1 6	B(200, 0.0228) stated or implied Po(4.56) stated or implied, allow 4.6 here Correct formula for $P(\leq 2) \pm 1$ term, any λ (tables: M0) Correct formula, 4.56 needed Answer, a.r.t. 0.167 [0.16694] Both, can be merely asserted. If numbers, must be these SR interpolation: clear method M1, answer A2 MR: typically $B(200, 0.228) \approx N(45.6, 3.52)$: M1A1; standardise correctly, M1; state $np, nq > 5$, B1
4	(i)	$\text{Either } z = \frac{213.4 - 230}{45/\sqrt{50}}$ $= -2.608$ $-2.608 < -2.576 \text{ or } 0.0047 < 0.005$	M1 A1 B1	Standardise z with $\sqrt{50}$, ignore sign or $\sqrt{\quad}$ or squaring errors z -value, a.r.t. -2.61 , or p in range [0.0044, 0.005) Correctly compare (-2.576) , signs consistent, <i>or</i> p explicitly with 0.005
	Or	$\text{CV is } 230 - 2.576 \times \frac{45}{\sqrt{50}} = 213.6$ $213.4 < 213.6$	M1 B1 A1	$230 - z\sigma/\sqrt{50}$, allow $\sqrt{\quad}$ or squaring errors, allow \pm but not just +; $z = 2.576$ Explicitly compare 213.4 with 213.6
		Reject H_0 . Significant evidence that population mean is not 230	M1 A1 FT 5	“Reject”, FT, needs correct method and form of comparison; interpreted, acknowledge uncertainty
5	(ii)	Yes, population distribution is not known to be normal	B2 2	<i>Not</i> , “yes, sample size is large” but ignore “can use it as ...” SR: Both right and wrong answers: B1 α “Yes as it must be assumed normal”: B1
		$H_0: \lambda = 12; \quad H_1: \lambda > 12$ $\text{Either: } P(\geq 19) = 1 - P(\leq 18)$ $= 1 - 0.9626$ $= 0.0374$ < 0.1	B2 M1 A1 B1	Both correct: B2. Allow μ . One error, B1, but <i>not</i> x, r etc. Po(12) stated or implied, e.g. 0.9787 0.0374, <i>or</i> 0.9626 if compared with 0.9 Explicitly compare $P(\geq 19)$ with 0.1, <i>or</i> $P(\leq 18)$ with 0.9
		$\text{Or: CR is } \geq 18, p = 0.063$ $19 \geq 18$	A1 B1	≥ 18 and 0.063 stated Explicit comparison of CV (right-hand CR) with 19
	Reject H_0 . Significant evidence of increase in mean number of applicants	M1 A1 FT 7	“Reject” FT, needs correct method and comparison, e.g. <i>not</i> from ≤ 19 or $= 19$, withhold if inconsistent Interpreted in context, acknowledge uncertainty	

6	(i)	If one customer arrives, it does not change the probability that another one does so; customers probably arrive in groups of at least 2	B1 B1	2	Answer that shows correct understanding of “independent”, in context; <i>not</i> just equivalent to “singly” Plausible reason, in context, nothing wrong, nothing that suggests “constant average rate”
	(ii)	0.1730	M1 A1	2	Correct use of tables or formula, e.g. .3007, or .4405 from Po(5) if Po(7) stated; answer 0.173, 0.1730 or better
	(iii)	Po(35) N(35, 35) $1 - \Phi\left(\frac{40.5 - 35}{\sqrt{35}}\right) = 1 - \Phi(0.9297)$ = 0.1763	B1 M1 A1 M1 A1 A1	6	Po(5×7) stated or implied Normal, $\mu = \text{their } \lambda$ Both parameters correct, allow 35^2 , $\sqrt{35}$ Standardise 40 with λ , $\sqrt{\lambda}$, allow $\sqrt{\quad}$, cc errors Both $\sqrt{\lambda}$ and cc correct Answer, a.r.t. 0.176 [penalise 0.1765]
7	(i)		B1 B1 B1	3	Horizontal line above axis Concave decreasing curve above axis Both correct including approx relationship, not extending beyond [1, 3], verticals and scale not needed
	(ii)	$\int_1^3 \frac{a}{x^2} dx = 1, \left[\frac{-a}{x}\right]_1^3 = 1; a = \frac{3}{2}$	M1 B1 A1	3	Attempt $\int f_X(x) dx$, limits 1, 3 at some stage, and equate to 1 Correct indefinite integral Correctly obtain 3/2 or 1.5 or exact equivalent
	(iii)	$\int_1^3 \frac{a}{x} dx = [a \ln x]_1^3$ $= \frac{3}{2} \ln 3$	M1 B1 FT A1 FT	3	Attempt $\int x f_X(x) dx$, limits 1, 3 at some stage Correct indefinite integral, FT on a Answer, any exact equivalent or a.r.t. 1.65, FT on a , or $a \ln 3$
	(iv)	T is equally likely to take any value between 1 and 3	B1	1	Must be “values taken by T ” (or “of T ”) or clear equivalent Any hint that they think T is an <i>event</i> gets B0. α “Same chance of occurring anywhere between 1 and 3”: 0 β “For values of T between 1 and 3, T is equally likely”: 0 γ “Each value of T is equally likely to occur”: 1
8	(i)	B(40, 0.225) $\approx N(9, 6.975)$ $\frac{5.5 - 9}{\sqrt{6.975}} = -1.325$ 0.9074 $np = 9 > 5$ or n large; and $nq = 31 > 5$ or p close to 0.5	M1 M1 A1 M1 A1 A1 B2	8	B(40, 0.225) stated or implied Normal, mean 9 Variance 6.975 or SD 2.641 or 6.975 Standardise with np and \sqrt{npq} , allow npq , no or wrong cc CC and \sqrt{npq} correct, allow from N(3600, 0.225) Answer, in range [0.907, 0.908] Full conditions B2; partial, B1 (assertions OK). Allow npq , allow from e.g. $n = 3600$
	(ii)	Number list sequentially and select using random numbers If # > 3600, ignore (etc)	B1 B1 B1	3	Number list, don’t need “sequentially” Mention random numbers (<i>not</i> “select numbers randomly”) Deal with issue of # > 3600, or “ignore repeats” α “Randomly pick numbers from 0 to 3599”: (B1) B0 B1
9	(i)	B(14, 0.7) CR is ≥ 13 with probability 0.0475	M1 A1 A1	3	B(14, 0.7) stated or implied, e.g. N(9.8, 2.94), can be recovered CV 13, or > 12 or {13, 14}, allow = but no other inequalities Exactly correct CR, and supporting prob .0475 or .9525 seen
	(ii)	$H_0: p = 0.7, H_1: p > 0.7$ $12 < 13$ Do not reject H_0 . Insufficient evidence that proportion who show improvement is greater than 0.7	B2 B1 M1 A1 FT	5	Both, B2. Allow π . One error, B1, but r, x etc: B0 Compare CV <i>from correct tail and inequality</i> with 12, or $P(\geq 12) = 0.1608$ and > 0.05 or $P(< 12) = 0.8392$ and < 0.95 Correct method & conclusion, requires like-with-like; CV method needs ≥ 13 or < 12 ; p method needs ≥ 12 or < 12 Withhold if inconsistent Contextualised, acknowledge uncertainty [SR: Normal or Po: (i) M1, (ii) B2 maximum] [0.9932 or 0.0068 probably B2 maximum]
	(iii)	B(14, 0.8) $P(\leq 12)$ from B(14, 0.8) 0.8021	M1 M1 A1	3	B(14, 0.8) stated or implied, allow from B(14, 0.75) Attempt prob of acceptance region, e.g. 0.8990, $\sqrt{\quad}$ on (i) Answer 0.802 or a.r.t. 0.8021

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