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ASSESSMENT and QUALIFICATIONS ALLIANCE

## General Certificate of Education

# Mathematics 6360

*MPC2 Pure Core 2*

# Mark Scheme

*2006 examination – June series*

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

# Key To Mark Scheme And Abbreviations Used In Marking

| | |
|---|---|
| M | mark is for method |
| m or dM | mark is dependent on one or more M marks and is for method |
| A | mark is dependent on M or m marks and is for accuracy |
| B | mark is independent of M or m marks and is for method and accuracy |
| E | mark is for explanation |

| | | | |
|---|---|---|---|
| $\surd$ or ft or F | follow through from previous incorrect result | MC | mis-copy |
| CAO | correct answer only | MR | mis-read |
| CSO | correct solution only | RA | required accuracy |
| AWFW | anything which falls within | FW | further work |
| AWRT | anything which rounds to | ISW | ignore subsequent work |
| ACF | any correct form | FIW | from incorrect work |
| AG | answer given | BOD | given benefit of doubt |
| SC | special case | WR | work replaced by candidate |
| OE | or equivalent | FB | formulae book |
| A2,1 | 2 or 1 (or 0) accuracy marks | NOS | not on scheme |
| $-x$ EE | deduct $x$ marks for each error | G | graph |
| NMS | no method shown | c | candidate |
| PI | possibly implied | sf | significant figure(s) |
| SCA | substantially correct approach | dp | decimal place(s) |

**No Method Shown**

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

**Otherwise we require evidence of a correct method for any marks to be awarded.**

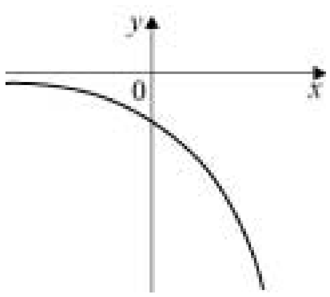
**MPC2**

| Question | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **1(a)** | Area of sector = $\frac{1}{2}r^2\theta = \frac{1}{2}\times 5^2\times\theta$ | M1 | | $\frac{1}{2}r^2\theta$ seen or used |
| | $12.5\theta = 8.1 \Rightarrow \theta = 0.648$ | A1 | 2 | AG Condone $\theta = 0.648$ used to show that area = 8.1 |
| **(b)** | Arc = $5\theta$; | M1 | | $5\theta$ |
| | ….. = 3.24 cm | A1 | | PI by a correct perimeter |
| | $\Rightarrow$ Perimeter = 10 + arc = 13.24 cm | A1$\sqrt{}$ | 3 | CSO Condone missing/wrong units; condone 3sf i.e. 13.2 if no obvious error NMS 3/3 |
| | **Total** | | **5** | |
| **2(a)** | $\frac{\sin B}{4.8} = \frac{\sin 100}{12}$ | M1 | | Use of the sine rule |
| | $\sin B = \frac{4.8\sin 100}{12}$ [= 0.39(392…)] | m1 | | Rearrangement |
| | ( angle $ABC$) = 23.19(8…) {= 23.2°.} | A1 | 3 | AG Need >1dp eg 23.19 or 23.20 |
| **(b)** | Angle $C$ = 80° – 23.2° = 56.8° | M1 | | Valid method to find a relevant angle eg $C$ (PI eg by correct sin $C$ ) or 23.2°+10° |
| | Area of triangle = 0.5×12×4.8×sin $C$ | M1 | | OE eg 0.5×4.8×12×cos ($B$+10) |
| | ….. = 24.09…. = 24.1 cm$^2$. (to 3sf) | A1 | 3 | Condone missing/wrong units |
| | **Total** | | **6** | |
| **3(a)** | (Tenth term) = $a + (10-1)\,d$ | M1 | | |
| | ………….. = 1 + 9(6) = 55 | A1 | 2 | NMS or rep. addn. B2 CAO<br>SC if M0 award B1 for $6n-5$ OE |
| **(b)(i)** | $S_n = \frac{n}{2}[2+(n-1)6]$ | M1 | | Formula for $\{S_n\}$ with either $a$ = 1 or $d$ = 6 substituted |
| | $\frac{n}{2}[2+6n-6]$ = 7400 | A1 | | Eqn formed with some expansion of brackets |
| | $3n^2 - 2n = 7400 \Rightarrow 3n^2 - 2n - 7400 = 0$ | A1 | 3 | CSO AG |
| **(ii)** | $(3n+148)(n-50) = 0$ | M1 | | Formula/factorisation OE |
| | $\Rightarrow$ $n$ = 50 | A1 | 2 | NMS single ans. 50.. B2 CAO<br>NMS 50 and –49.3(3…) B1 CAO |
| | **Total** | | **7** | |

**MPC2 (cont)**

| Question | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **4(a)** | $(1-2x)^4 = (1)^4 + 4(1)^3(-2x) + 6(1^2)(-2x)^2 + [4(1)(-2x)^3 + (-2x)^4]$ | M1 | | Any valid method as far as term(s) in $x$ and term(s) in $x^2$. |
| | $= [1] - 8x + 24x^2 + [-32x^3 + 16x^4]$ | A1 | | $p = -8$ Accept $-8x$ even within a series. |
| | | A1 | 3 | $q = 24$ Accept $24x^2$ even within a series. |
| **(b)** | $x$ term is $\binom{9}{1} 2^8 x$ | M1 | | OE |
| | Coefficient of $x$ term is $= 9\times 2^8 = 2304$ $(=k)$ | A1 | 2 | Condone $2304x$ |
| **(c)** | $(1-2x)^4(2+x)^9 = (1+px+..)(2^9+kx...)$ | M1 | | Uses (**a**) and (**b**) oe (PI) |
| | $= ...$ | | | |
| | $= .....+ kx + px(2^9) + .....$ | M1 | | Multiply the two expansions to get $x$ terms |
| | Coefficient of $x$ is $k + 512p$ | | | |
| | $= 2304 - 4096 = -1792$ | A1ft | 3 | ft on candidate's values of $k$ and $p$. Condone $-1792x$<br>SC If 0/3 award B1ft for $p+k$ evaluated |
| | **Total** | | **8** | |
| **5(a)** | $\log_a x = \log_a 6^2 - \log_a 3$ | M1 | | One law of logs used correctly |
| | $\log_a x = \log_a \left(\frac{6^2}{3}\right)$ | M1 | | A second law of logs used correctly |
| | $\log_a x = \log_a \frac{36}{3} \Rightarrow x = 12$ | A1 | 3 | CSO AG |
| **(b)** | $\log_a y + \log_a 5 = 7 \Rightarrow \log_a 5y = 7$ | M1 | | |
| | $\Rightarrow 5y = a^7$ or $y = \frac{1}{5}a^7$ or $a = (5y)^{1/7}$ | m1<br>A1 | 3 | Eliminates logs<br>Accept these forms |
| | **Total** | | **6** | |

**MPC2 (cont)**

| Question | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **6(a)(i)** | $y$-coordinate of $A$ is $27-3^0$; $=26$ | M1A1 | 2 | |
| **(ii)** | When $x=3$, $y=27-3^3=0 \Rightarrow B(3,0)$ | B1 | 1 | AG; be convinced |
| **(b)** | $h=1$ | B1 | | PI |
| | Area $\approx h/2\{\ldots\}$<br>$\{\ldots\}=$ f(0)+f(3)+2[f(1)+f(2)] | M1 | | OE summing of areas of the 'trapezia'.. |
| | $\{\ldots\}=$ "26" + 0 + 2(24 + 18) | A1$\sqrt{}$ | | on (a)(i) ($\sum$trap= "25"+21+9) |
| | (Area $\approx$) 55 | A1$\sqrt{}$ | 4 | on [42 + 0.5× "(a)(i)"] |
| **(c)(i)** | $\log_{10} 3^x = \log_{10} 13$ | M1 | | Takes ln or $\log_{10}$ on both<br>or $x = \log_3 13$ |
| | $x\log_{10} 3 = \log_{10} 13$ | m1 | | Use of $\log 3^x = x\log 3$ or<br>$\log_3 13 = \dfrac{\lg 13}{\lg 3}$ OE (PI by $\log_3 13 = 2.335$ or better) |
| | $x = \dfrac{\lg 13}{\lg 3} = 2.334717\ldots$<br>= 2.3347 to 4dp | A1 | 3 | Must show that logarithms have been used |
| **(ii)** | $\{k=\}$ 14 | B1 | 1 | Condone $y = 14$; Accept final answer 14 with only zeros after decimal point eg 14.000 |
| **(d)(i)** | Translation; | B1; | | 'Translation'/'translate(d)'<br>B0 if more than one transformation |
| | $\begin{bmatrix} 0 \\ -27 \end{bmatrix}$ | B1 | 2 | Accept full equivalent in words provided linked to 'translation/move/shift' and negative $y$-direction<br>(Note: B0 B1 is possible) |
| **(ii)** |  | B1 | | Correct shape (translation of given curve vertically downwards) |
| | | B1 | 2 | Only point of intersection with coord axes is on negative $y$-axis and curve is asymptotic to the negative $x$-axis |
| | **Total** | | **15** | |

**MPC2 (cont)**

| Question | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **7(a)(i)** | When $x = 4$, $\frac{dy}{dx} = 3(2) + \frac{16}{16} - 7 = 0$ | B1 | 1 | AG Be convinced |
| **(ii)** | $\frac{16}{x^2} = 16x^{-2}$ | B1 | 1 | Accept $k = -2$ |
| **(iii)** | $\frac{d^2y}{dx^2} = 3 \times \frac{1}{2}x^{-\frac{1}{2}} + 16 \times (-2)x^{-3} - 0$ | M1 | | A power decreased by 1 |
| | $\frac{d^2y}{dx^2} = \frac{3}{2}x^{-\frac{1}{2}}; \quad -32x^{-3}$ | A1; A1√ | 3 | candidate's negative integer $k$ [−1 for >2 term(s)] |
| **(iv)** | When $x = 4$, $\frac{d^2y}{dx^2} = \frac{3}{4} - \frac{32}{64} = \frac{1}{4}$ | M1 | | Attempt to find $y''(4)$ reaching as far as two simplified terms |
| | Minimum since $y''(4) > 0$ | E1√ | 2 | candidate's sign of $y''(4)$ |
| | **[Alternative:** Finds the sign of $y'(x)$ either side of the point where $x$=4, need evidence rather than just a statement: (M1) Correct ft conclusion with valid reason E1√**]** [In both, condone absent statement $y'(4)$=0] | | | |
| **(b)(i)** | At $P(1,8)$, $\frac{dy}{dx} = 3(1)^{\frac{1}{2}} + \frac{16}{1^2} - 7$ =12 | B1 | 1 | AG Be convinced |
| **(ii)** | Gradient of normal = $-\frac{1}{12}$ | M1 | | Use of or stating $m \times m' = -1$ |
| | Equation of normal is $y - 8 = m[x - 1]$ | M1 | | Can be awarded even if m=12 |
| | $y - 8 = -\frac{1}{12}(x - 1) \Rightarrow 12y - 96 = -x + 1$ $\Rightarrow 12y + x = 97$ | A1 | 3 | Any correct form of the equation |
| **(c)(i)** | $\int 3x^{\frac{1}{2}} + \frac{16}{x^2} - 7\, dx =$ | | | |
| | …….. $= 3\frac{x^{\frac{3}{2}}}{\frac{3}{2}} + 16\frac{x^{-1}}{-1} - 7x + c$ | M1 A2,1,0 √ | 3 | One power correct. A1 if 2 of 3 terms correct candidate's negative integer $k$ Condone absence of "+ $c$" |
| **(ii)** | $y = 2x^{\frac{3}{2}} - 16x^{-1} - 7x + c$ (*) | B1√ | | $y$ = candidate's answer to (c)(i) with tidied coefficients and with '+c'. ('$y$ =' PI by next line) |
| | When $x = 1, y = 8 \Rightarrow 8 = 2–16–7+c$ | M1 | | Substitute. (1,8) in attempt to find constant of integration |
| | $y = 2x^{\frac{3}{2}} - 16x^{-1} - 7x + 29$ | A1 | 3 | Accept $c$ = 29 after (*), including $y$ =, stated |
| | **Total** | | **17** | |

**MPC2 (cont)**

| Question | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| 8(a) | Stretch **(I)** in $x$-direction **(II)** scale factor 2 **(III)** | M1<br>A1 | 2 | Need**(I)** and one of **(II)**,**(III)**<br>M0 if more than one transformation |
| (b) | $\tan^{-1} 3 = 1.2(49\ldots.)\ (=\alpha)$ | M1 | | $\tan^{-1} 3$ [PI by 71.(56..)°] |
| | $\{\frac{1}{2}x=\}\quad \pi+\alpha$ ; | m1 | | Correct quadrant; condone degrees or mix |
| | $\frac{1}{2}x = 1.249\ldots;\ 4.3906\ldots$ | | | |
| | $x = 2.498... = 2.50$ to 3 sf | A1 | | Condone 2.5 otherwise deduct **max** of 1 mark throughout Q8 from A marks if 'correct' rads. but to 2sf or final answers in degrees. (143°, 503°) |
| | $x = 8.781... = 8.78$ to 3 sf | A1 | 4 | As usual, accept greater accuracy answers. Ignore extra values outside the given interval (0 to12.6). If $> 2$ values inside interval lose an A mark for each one.<br>NB M1m0A1A0 is possible |
| | SC after M0 for error $\tan x = 6$;<br>Either $x = 1.40(5),\ 4.54(7),\ 7.68(8),\ 10.8(3)$ or $x = 80.5°, 260.5°, 440.5°, 620.5°$ SC B1<br>(accept each rounded or truncated to 3 sf) | | | |
| (c) | $\cos\theta = 0,\quad \sin\theta - 3\cos\theta = 0$ | M1 | | Need both |
| | $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ or $\tan\theta = 3$ | M1 | | $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ seen/used |
| | $\cos\theta = 0 \Rightarrow\ \theta = \dfrac{\pi}{2} = 1.57(07\ldots)$ | B1 | | Accept $\dfrac{\pi}{2}$ |
| | or $\theta = \dfrac{3\pi}{2} = 4.71(23..)$ | B1 | | Accept $\dfrac{3\pi}{2}$ |
| | $\tan\theta = 3 \Rightarrow$<br>$\theta = 1.249..;\ 4.3906\ldots = 1.25,\ 4.39$ to 3sf | A1$\surd$ | 5 | If not correct, ft on **(b)**<br>NB M0M1(B0B0)A1ft is possible<br>90°; 270°;<br>71.5(6)°; 251.5(6)° |
| | **Total** | | **11** | |
| | **TOTAL** | | **75** | |