General Certificate of Education June 2009 Advanced Level Examination

# MATHEMATICS Unit Pure Core 3

AQA

MPC3

Friday 5 June 2009 1.30 pm to 3.00 pm

# For this paper you must have:

• an 8-page answer book

• the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

# Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC3.
- Answer all questions.
- Show all necessary working; otherwise marks for method may be lost.

# Information

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

### Advice

• Unless stated otherwise, you may quote formulae, without proof, from the booklet.

#### Answer all questions.

1 (a) The curve with equation

$$y = \frac{\cos x}{2x+1}, \qquad x > -\frac{1}{2}$$

intersects the line  $y = \frac{1}{2}$  at the point where  $x = \alpha$ .

- (i) Show that  $\alpha$  lies between 0 and  $\frac{\pi}{2}$ . (2 marks)
- (ii) Show that the equation  $\frac{\cos x}{2x+1} = \frac{1}{2}$  can be rearranged into the form

$$x = \cos x - \frac{1}{2} \tag{1 mark}$$

(iii) Use the iteration  $x_{n+1} = \cos x_n - \frac{1}{2}$  with  $x_1 = 0$  to find  $x_3$ , giving your answer to three decimal places. (2 marks)

(b) (i) Given that 
$$y = \frac{\cos x}{2x+1}$$
, use the quotient rule to find an expression for  $\frac{dy}{dx}$ .  
(3 marks)

(ii) Hence find the gradient of the normal to the curve  $y = \frac{\cos x}{2x+1}$  at the point on the curve where x = 0. (2 marks)

2 The functions f and g are defined with their respective domains by

$$f(x) = \sqrt{2x+5}, \quad \text{for real values of } x, \ x \ge -2.5$$
$$g(x) = \frac{1}{4x+1}, \quad \text{for real values of } x, \ x \ne -0.25$$

(a) Find the range of f. (2 marks)

- (b) The inverse of f is  $f^{-1}$ .
  - (i) Find  $f^{-1}(x)$ . (3 marks)
  - (ii) State the domain of  $f^{-1}$ . (1 mark)
- (c) The composite function fg is denoted by h.
  - (i) Find an expression for h(x). (1 mark)
  - (ii) Solve the equation h(x) = 3. (3 marks)
- 3 (a) Solve the equation  $\tan x = -\frac{1}{3}$ , giving all the values of x in the interval  $0 < x < 2\pi$  in radians to two decimal places. (3 marks)
  - (b) Show that the equation

$$3 \sec^2 x = 5(\tan x + 1)$$
  
can be written in the form  $3 \tan^2 x - 5 \tan x - 2 = 0$ . (1 mark)

(c) Hence, or otherwise, solve the equation

$$3\sec^2 x = 5(\tan x + 1)$$

giving all the values of x in the interval  $0 < x < 2\pi$  in radians to two decimal places. (4 marks)

- 4 (a) Sketch the graph of  $y = |50 x^2|$ , indicating the coordinates of the point where the graph crosses the y-axis. (3 marks)
  - (b) Solve the equation  $|50 x^2| = 14$ . (3 marks)
  - (c) Hence, or otherwise, solve the inequality  $|50 x^2| > 14$ . (2 marks)
  - (d) Describe a sequence of two geometrical transformations that maps the graph of  $y = x^2$  onto the graph of  $y = 50 x^2$ . (4 marks)
- 5 (a) Given that  $2 \ln x = 5$ , find the exact value of x.
  - (b) Solve the equation

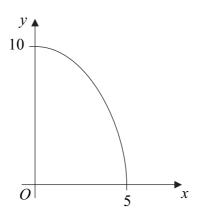
$$2\ln x + \frac{15}{\ln x} = 11$$

giving your answers as exact values of x.

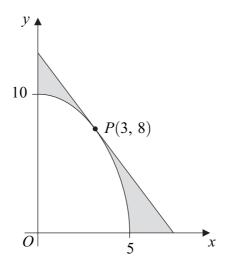
(5 marks)

(1 mark)

6 The diagram shows the curve with equation  $y = \sqrt{100 - 4x^2}$ , where  $x \ge 0$ .



- (a) Calculate the volume of the solid generated when the region bounded by the curve shown above and the coordinate axes is rotated through 360° about the *y*-axis, giving your answer in terms of  $\pi$ . (5 marks)
- (b) Use the mid-ordinate rule with five strips of equal width to find an estimate for  $\int_0^5 \sqrt{100 4x^2} \, dx$ , giving your answer to three significant figures. (4 marks)
- (c) The point P on the curve has coordinates (3, 8).
  - (i) Find the gradient of the curve  $y = \sqrt{100 4x^2}$  at the point *P*. (3 marks)
  - (ii) Hence show that the equation of the tangent to the curve at the point *P* can be written as 2y + 3x = 25. (2 marks)
- (d) The shaded regions on the diagram below are bounded by the curve, the tangent at P and the coordinate axes.



Use your answers to part (b) and part (c)(ii) to find an approximate value for the **total** area of the shaded regions. Give your answer to three significant figures. (5 marks)

- 7 (a) Use integration by parts to find  $\int (t-1) \ln t \, dt$ . (4 marks)
  - (b) Use the substitution t = 2x + 1 to show that  $\int 4x \ln(2x + 1) dx$  can be written as  $\int (t 1) \ln t dt$ . (3 marks)
  - (c) Hence find the exact value of  $\int_0^1 4x \ln(2x+1) dx$ . (3 marks)

# END OF QUESTIONS

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