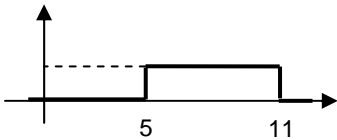


4733 Probability & Statistics 2

Penalise over-specified answers (> 6 SF) first time but only once per paper.

Use **A** or **C** to annotate “over-assertive” or “no context” respectively

1	$\hat{\mu} = \bar{x} = 15.16$ $\hat{\sigma}^2 = \frac{5}{4}s^2$ <p style="text-align: center;">= 1.363</p>	B1 M1 M1 A1	4 15.16 or 15.2 as answer only Use $\frac{\sum x^2}{5} - \bar{x}^2$ [=1.0904] Multiply by 5/4, or equiv for single formula Final answer 1.36 or 1.363 only, <i>not isw</i>
2	(i) Not all equally likely – those in range 0 to 199 more likely to be chosen	M1 A1	2 Not all equally likely stated or implied Justified by reference to numbers, no spurious reasons
	(ii) Ignore random numbers greater than 799, or 399	B1	1 Any valid resolution of this problem, no spurious reasons
3	$B(60, 0.35) \approx N(21, 13.65)$ $\Phi\left(\frac{18.5 - 21}{\sqrt{13.65}}\right) = \Phi(-0.6767)$ $= 1 - 0.7507$ <p style="text-align: center;">= 0.2493</p>	M1 M1 A1 M1 A1 A1	6 B(60, 0.35) stated or implied N(21, ...) Variance or SD = 13.65 Standardise, their np and \sqrt{npq} or npq , wrong or no cc Both \sqrt{npq} and cc correct Answer, a.r.t. 0.249
4	$H_0 : \mu = 60; H_1 : \mu < 60$ $(\alpha) \quad z = \frac{58.9 - 60}{\sqrt{5^2 / 80}} = -1.967$ < -1.645	B2 M1 A1 B1	6 Both correct, B2 B1 for one error, but not x, t, \bar{x} or \bar{t} Standardise 58.9 & $\sqrt{80}$, allow – or $\sqrt{\quad}$ errors z , art -1.97 or p in range [0.024, 0.025] Explicit comparison with -1.645 or 0.05 , or $+1.645$ or 0.95 if 1.967 or 0.976 used
	<i>or:</i> $(\beta)_c = 60 - 1.645 \times \frac{5}{\sqrt{80}} = 59.08$ $58.9 < 59.08$	M1 B1 A1 ✓	6 $60 - z \times 5 / \sqrt{80}$, any $z = \Phi^{-1}$, allow $\sqrt{\quad}$ errors or \pm , not just +; $z = 1.645$ and compare 58.9 59.1 or better, ✓ on wrong z
	Reject H_0	M1	Correct first conclusion, needs essentially correct method including $\sqrt{80}$ or 80
	Significant evidence that people underestimate time	A1 ✓	7 Contextualised, uncertainty acknowledged SR: $\mu = 58.9$: B0M1A0B1 max 2/7 SR: 2-tail: max 5/7
5	(i) $H_0 : \lambda = 11.0$ $H_1 : \lambda > 11.0$ $(\alpha) \quad P(\geq 19) = 1 - 0.9823$ $= 0.0177$ < 0.05	B2 M1 A1 B1	7 Allow μ . Both correct, B2 One error: B1, but not C, x etc Find $P(\geq 19)$ [or $P(< 19)$ if later 0.95] art 0.0177 [0.9823, ditto] Compare 0.05 [0.95 if consistent], needs M1
	(β) CR ≥ 18 , $P(\geq 18) = 0.0322$ $19 > 18$	M1 A1 B1	7 CR or CV 16/17/18/19 stated or clearly implied, but not < 18 and 0.0322 both seen, allow 0.9678 Explicit comparison with 19, needs M1
	Reject H_0	M1	Needs essentially correct method & comparison
	Significant evidence of an increase in number of customers	A1 ✓	7 Contextualised, uncertainty acknowledged SR: Normal, or $P(= 19)$ or $P(\leq 19)$ or $P(> 19)$: First B2 only.
	(ii) Can't deduce cause-and-effect, or there may be other factors	B1	1 Conclusion needed. No spurious reasons. If “DNR” in (i), “couldn't deduce even if...”

6	(i)	(a) Probabilities don't total 1	B1	1	Equivalent statement
		(b) $P(> 70)$ must be $< P(> 50)$	B1	1	Equivalent statement
		(c) $P(> 50) = 0.3 \Rightarrow \mu < 50$ $P(< 70) = 0.3 \Rightarrow \mu > 70$	B1	1	Any relevant valid statement, e.g. " $P(< 50) = 0.7$ but $P(< 50)$ must be $< P(< 70)$ "
	(ii)	$\mu = 60$ by symmetry $\frac{10}{\sigma} = \Phi^{-1}(0.7) = 0.524(4)$ $\sigma = 10/0.5243$ = 19.084	B1 M1 B1 A1	4	$\mu = 60$ obtained at any point, allow from Φ One standardisation, equate to Φ^{-1} , not 0.758 $\Phi^{-1} \in [0.524, 0.5245]$ seen σ in range [19.07, 19.1], e.g. 19.073
7	(i)		M1 A1	2	Horizontal line Evidence of truncation <i>[no need for labels]</i>
	(ii)	$\mu = 8$ $\int_5^{11} \frac{1}{6} t^2 dt = \left[\frac{1}{18} t^3 \right]_5^{11} \quad [= 67]$ $- 8^2$ = 3	B1 M1 B1 M1 A1	5	8 only, cwd Attempt $\int kt^2 dt$, limits 5 and 11 seen $k = 1/6$ stated or implied Subtract their (non-zero) mean ² Answer 3 only, <i>not</i> from MF1
	(iii)	$N(8, 3/48)$ $1 - \Phi\left(\frac{8.3 - 8}{\sqrt{3/48}}\right) = 1 - \Phi(1.2)$ $= 1 - 0.8848$ = 0.1151 Normal distribution only approx.	M1 A1 A1 M1 A1 B1	6	Normal stated or implied Mean 8 Variance their (non-zero) (ii)/48 Standardise, \sqrt{n} , ignore sign or $\sqrt{\text{errors}}$. cc: M0 Answer, art 0.115 Any equivalent comment, e.g. CLT used
8	(i)	$P(\leq 4) = 0.0473$ Therefore CR is ≤ 4 $P(\text{Type I error}) =$ 4.73%	M1 B1 A1	3	$P(\leq r)$ from $B(10, 0.7)$, $r = 3/4/5$, <i>not</i> N " ≤ 4 " stated, not just "4", nothing else Answer, art 0.0473 or 4.73%, must be stated
	(ii)	$B(10, 0.4)$ and find $P(> 4)$ $1 - P(\leq 4)$ = 0.3669	M1 M1 A1	3	Must be this, <i>not</i> isw, ✓ on (i) Allow for 0.6177 or 0.1622 Answer, art 0.367
	(iii)	0.5×0.3669 = 0.18345	M1 A1 ✓	2	$0.5 \times$ (ii) Ans correct to 3 SF, e.g. 0.184 from 0.367

9	(i)	$1 - P(\leq 7) = 1 - 0.9881$ $= \mathbf{0.0119}$	M1 A1	2	Allow for 0.0038 or 0.0335 Answer, a.r.t. 0.0119												
	(ii)	Po(12) $P(\leq 14) - P(\leq 12)$ [0.7720 - 0.5760] $= \mathbf{0.196}$	M1 M1 A1	3	Po(12) stated or implied Formula, 2 consecutive correct terms, or tables, e.g. .0905 or .3104 or .1629 Answer, art 0.196												
	(iii)	Po(60) \approx N(60, 60) $\Phi\left(\frac{69.5-60}{\sqrt{60}}\right) = \Phi(1.226)$ $= \mathbf{0.8899}$	M1 A1 M1 A1 A1	5	N(60, ...) Variance or SD 60 Standardise, λ & $\sqrt{\lambda}$, allow λ or wrong or no cc $\sqrt{\lambda}$ and cc both correct Answer 0.89 or a.r.t. 0.890												
	(iv)	(a) $1 - e^{-3m}(1 + 3m)$	M1 A1	2	M1 for one error, e.g. no "1 -", or extra term, or 0 th term missing; answer, aesf												
		(b) $m = 1.29,$ $p = 0.89842$ $m = 1.3, \quad p = 0.9008$ Straddles 0.9, therefore solution between 1.29 and 1.3	M1 A1 A1 A1	4	Substitute 1.29 or 1.3 into appropriate fn <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th>Comp</th> <th>0.9</th> <th>0.1</th> <th>0</th> </tr> </thead> <tbody> <tr> <td>1.29</td> <td>0.898</td> <td>0.10158</td> <td>-.00158</td> </tr> <tr> <td>1.3</td> <td>0.901</td> <td>0.09918</td> <td>.0008146</td> </tr> </tbody> </table> Explicit comparison with relevant value, & conclusion, needs both p s correct	Comp	0.9	0.1	0	1.29	0.898	0.10158	-.00158	1.3	0.901	0.09918	.0008146
	Comp	0.9	0.1	0													
	1.29	0.898	0.10158	-.00158													
1.3	0.901	0.09918	.0008146														
or	Method for iteration; 1.296... 1.2965 or better; conclusion stated	M1A1 A1A1		Can be implied by at least 1.296... Need at least 4 dp for M1A2													