GCE

Mathematics (MEI)

Advanced GCE **4754A**

Applications of Advanced Mathematics (C4) Paper A

Mark Scheme for June 2010

Section A

1	$\dfrac{x}{x^2-1}+\dfrac{2}{x+1}=\dfrac{x}{(x-1)(x+1)}+\dfrac{2}{x+1}$	B1	$x^2-1=(x+1)(x-1)$
	$=\dfrac{x+2(x-1)}{(x-1)(x+1)}$	M1	correct method for addition of fractions
	$=\dfrac{(3x-2)}{(x-1)(x+1)}$	A1	or $\dfrac{(3x-2)}{x^2-1}$ do not isw for incorrect subsequent cancelling
or	$\dfrac{x}{x^2-1}+\dfrac{2}{x+1}=\dfrac{x(x+1)+2(x^2-1)}{(x^2-1)(x+1)}$	M1	correct method for addition of fractions
	$=\dfrac{3x^2+x-2}{(x^2-1)(x+1)}$		
	$=\dfrac{(3x-2)(x+1)}{(x^2-1)(x+1)}$	B1	$(3x-2)(x+1)$
	$=\dfrac{(3x-2)}{(x^2-1)}$	A1 [3]	accept denominator as x^2-1 or $(x-1)(x+1)$ do not isw for incorrect subsequent cancelling
2(i)	When $x=0.5$, $y=1.1180$	B1	4dp
$\Rightarrow$	$A\approx 0.25/2\{1+1.4142+2(1.0308+1.1180+1.25)\}$	M1	
	$=0.25\times 4.6059=1.151475$		(0.125×9.2118)
	$=1.151\ (3\ \text{d.p.})^*$	E1 [3]	need evidence
(ii)	Explain that the area is an over-estimate. *or* The curve is below the trapezia, so the area is an over- estimate.	B1	or use a diagram to show why
	This becomes less with more strips. *or* Greater number of strips improves accuracy so becomes less	B1 [2]	
(iii)	$V=\displaystyle\int_0^1 \pi y^2\,dx$		
	$=\displaystyle\int_0^1 \pi(1+x^2)\,dx$	M1	allow limits later
	$=\pi\left[(x+x^3/3)\right]_0^1$	B1	$x+x^3/3$
	$=1\tfrac{1}{3}\pi$	A1 [3]	exact

3 $y = \sin\theta\cos\theta = \frac{1}{2}\sin 2\theta$ $x = \cos 2\theta$ $\sin^2 2\theta + \cos^2 2\theta = 1$ $\Rightarrow$ $x^2 + (2y)^2 = 1$ $\Rightarrow$ $x^2 + 4y^2 = 1$ *	M1 M1 E1	use of $\sin 2\theta$

or $x^2 + 4y^2 = (\cos 2\theta)^2 + 4\,(\sin\theta\cos\theta)^2$
        $= \cos^2 2\theta + \sin^2 2\theta$
        $= 1$ *
	M1
	 M1
	 E1
		substitution
		 use of $\sin 2\theta$

or $\cos 2\theta = 2\cos^2\theta - 1$
     $\cos^2\theta = (x+1)/2$
     $\cos 2\theta = 1 - 2\sin^2\theta$
     $\sin^2\theta = (1-x)/2$
     $y^2 = \sin^2\theta\cos^2\theta = \left(\dfrac{1-x}{2}\right)\left(\dfrac{x+1}{2}\right)$
     $y^2 = (1 - x^2)/4$
     $x^2 + 4y^2 = 1$*
	M1
	
	 M1
	
	 E1
		for both

or $x = \cos 2\theta = \cos^2\theta - \sin^2\theta$
    $x^2 = \cos^4\theta - 2\sin^2\theta\cos^2\theta + \sin^4\theta$
    $y^2 = \sin^2\theta\cos^2\theta$
 $x^2 + 4y^2 = \cos^4\theta - 2\sin^2\theta\cos^2\theta + \sin^4\theta + 4\sin^2\theta\cos^2\theta$
      $= (\cos^2\theta + \sin^2\theta)^2$
      $= 1$*
	M1
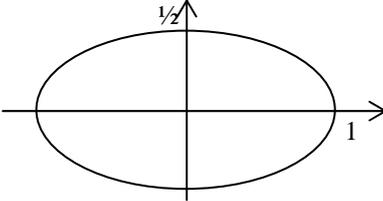	
	 M1
	 E1
		correct use of double angle formulae
		
		 correct squaring and use of $\sin^2\theta + \cos^2\theta = 1$

	M1 A1	ellipse correct intercepts
	[5]	

4	$\sqrt{4+x} = 2(1+\frac{x}{4})^{\frac{1}{2}}$	M1	dealing with √4 (or terms in $4^{\frac{1}{2}}$, $4^{\frac{-1}{2}}$,…etc)
	$= 2(1+\frac{1}{2}.\frac{x}{4}+\frac{\frac{1}{2}.-\frac{1}{2}}{2}(\frac{x}{4})^2+...)$	M1 A1	correct binomial coefficients correct **unsimplified** expression for
	$= 2(1+\frac{1}{8}x-\frac{1}{128}x^2+...)$		$(1+x/4)^{\frac{1}{2}}$ or $(4+x)^{\frac{1}{2}}$
	$= 2+\frac{1}{4}x-\frac{1}{64}x^2+...$	A1	cao
⇒	Valid for $-1 < x/4 < 1$ $-4 < x < 4$	B1 [5]	

5(i)	$\dfrac{3}{(y-2)(y+1)} = \dfrac{A}{y-2}+\dfrac{B}{y+1}$		
	$= \dfrac{A(y+1)+B(y-2)}{(y-2)(y+1)}$		
⇒	$3 = A(y+1)+B(y-2)$ $y=2 \Rightarrow 3 = 3A \Rightarrow A = 1$ $y=-1 \Rightarrow 3 = -3B \Rightarrow B = -1$	M1 A1 A1 [3]	substituting, equating coeffs or cover up
(ii)	$\dfrac{dy}{dx} = x^2(y-2)(y+1)$		
⇒	$\int \dfrac{3\,dy}{(y-2)(y+1)} = \int 3x^2\,dx$	M1	separating variables
⇒	$\int (\dfrac{1}{(y-2)}-\dfrac{1}{y+1})\,dy = \int 3x^2\,dx$		
⇒	$\ln(y-2)-\ln(y+1) = x^3+c$	B1ft B1	$\ln(y-2)-\ln(y+1)$ ft their A,B x^3+c
⇒	$\ln\left(\dfrac{y-2}{y+1}\right) = x^3+c$		
⇒	$\dfrac{y-2}{y+1} = e^{x^3+c} = e^{x^3}.e^c = Ae^{x^3}$ *	M1 E1 [5]	anti-logging including c www

6	$\tan(\theta+45) = \dfrac{\tan\theta + \tan 45}{1-\tan\theta\tan 45}$	M1	oe using sin/cos
	$= \dfrac{\tan\theta + 1}{1-\tan\theta}$	A1	
⇒	$\dfrac{\tan\theta + 1}{1-\tan\theta} = 1-2\tan\theta$		
⇒	$1 + \tan\theta = (1-2\tan\theta)(1-\tan\theta)$ $= 1 - 3\tan\theta + 2\tan^2\theta$	M1 A1 M1	multiplying up and expanding any correct one line equation solving quadratic for $\tan\theta$ oe
⇒	$0 = 2\tan^2\theta - 4\tan\theta = 2\tan\theta(\tan\theta - 2)$		
⇒	$\tan\theta = 0$ or 2		
⇒	$\theta = 0$ or 63.43	A1A1 [7]	www -1 extra solutions in the range

Section B

7(i) $\overline{AB} = \begin{pmatrix} 100-(-200) \\ 200-100 \\ 100-0 \end{pmatrix} = \begin{pmatrix} 300 \\ 100 \\ 100 \end{pmatrix}$ * $AB = \sqrt{(300^2 + 100^2 + 100^2)} = 332$ m	E1 M1 A1 [3]	 accept surds
(ii) $\mathbf{r} = \begin{pmatrix} -200 \\ 100 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 300 \\ 100 \\ 100 \end{pmatrix}$ Angle is between $\begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ $\Rightarrow \quad \cos\theta = \dfrac{3\times 0+1\times 0+1\times 1}{\sqrt{11}\sqrt{1}} = \dfrac{1}{\sqrt{11}}$ $\Rightarrow \quad \theta = 72.45°$	B1B1 M1 M1 A1 A1 [6]	oe …and $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ complete scalar product method(including cosine) for correct vectors 72.5° or better, accept 1.26 radians
(iii) Meets plane of layer when $(-200+300\lambda) + 2(100+100\lambda) + 3\times 100\lambda = 320$ $\Rightarrow \quad 800\lambda = 320$ $\Rightarrow \quad \lambda = 2/5$ $\mathbf{r} = \begin{pmatrix} -200 \\ 100 \\ 0 \end{pmatrix} + \dfrac{2}{5}\begin{pmatrix} 300 \\ 100 \\ 100 \end{pmatrix} = \begin{pmatrix} -80 \\ 140 \\ 40 \end{pmatrix}$ so meets layer at (–80, 140, 40)	M1 A1 M1 A1 [4]	
(iv) Normal to plane is $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ Angle is between $\begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ $\Rightarrow \quad \cos\theta = \dfrac{3\times 1+1\times 2+1\times 3}{\sqrt{11}\sqrt{14}} = \dfrac{8}{\sqrt{11}\sqrt{14}} = 0.6446..$ $\Rightarrow \quad \theta = 49.86°$ $\Rightarrow \quad$ angle with layer $= 40.1°$	B1 M1A1 A1 A1 [5]	 complete method ft 90–theirθ accept radians

8(i)	At A, $y = 0 \Rightarrow 4\cos\theta = 0$, $\theta = \pi/2$	B1	
	At B, $\cos\theta = -1$, $\Rightarrow \theta = \pi$	B1	
	x-coord of A = $2\times\pi/2 - \sin\pi/2 = \pi - 1$	M1	for either A or B/C
	x-coord of B = $2\times\pi - \sin\pi = 2\pi$	A1	for both A and B/C
$\Rightarrow$	OA = $\pi - 1$, AC = $2\pi - \pi + 1 = \pi + 1$		
$\Rightarrow$	ratio is $(\pi - 1):(\pi + 1)$ *	E1	
		[5]	

(ii)	$\dfrac{dy}{d\theta} = -4\sin\theta$	B1	either $dx/d\theta$ or $dy/d\theta$
	$\dfrac{dx}{d\theta} = 2 - \cos\theta$		
$\Rightarrow$	$\dfrac{dy}{dx} = \dfrac{dy/d\theta}{dx/d\theta}$		
	$= -\dfrac{4\sin\theta}{2 - \cos\theta}$	M1 A1	
	At A, gradient $= -\dfrac{4\sin(\pi/2)}{2-\cos(\pi/2)} = -2$	A1	www
		[4]	

(iii)	$\dfrac{dy}{dx} = 1 \Rightarrow -\dfrac{4\sin\theta}{2-\cos\theta} = 1$	M1	their $dy/dx = 1$
$\Rightarrow$	$-4\sin\theta = 2 - \cos\theta$		
$\Rightarrow$	$\cos\theta - 4\sin\theta = 2$ *	E1	
		[2]	

(iv)	$\cos\theta - 4\sin\theta = R\cos(\theta + \alpha)$		
	$= R(\cos\theta\cos\alpha - \sin\theta\sin\alpha)$		
$\Rightarrow$	$R\cos\alpha = 1$, $R\sin\alpha = 4$	M1	corr pairs
$\Rightarrow$	$R^2 = 1^2 + 4^2 = 17$, $R = \sqrt{17}$	B1	
	$\tan\alpha = 4$, $\alpha = 1.326$	M1 A1	accept 76.0º , 1.33 radians
$\Rightarrow$	$\sqrt{17}\cos(\theta + 1.326) = 2$		
$\Rightarrow$	$\cos(\theta + 1.326) = 2/\sqrt{17}$		
$\Rightarrow$	$\theta + 1.326 = 1.064, 5.219, 7.348$	M1	inv cos $(2/\sqrt{17})$ ft their R for method
$\Rightarrow$	$\theta = (-0.262), 3.89, 6.02$	A1 A1	-1 extra solutions in the range
		[7]	

GCE

Mathematics (MEI)

Advanced GCE **4754B**

Applications of Advanced Mathematics (C4) Paper B: Comprehension Greener Travel

Mark Scheme for June 2010

1.		Rail: $307 \times 0.0602 = 18.4814 = 18.5$ kg (3 sf) Road: $300 \times 0.2095 \div 1.58 = 39.77... = 39.8$ kg (3 sf) Reduction = 21.3 kg	**B1** for either **B1**
2.		$$y = \frac{1}{10^4}(x^3 - 100x^2 - 10000x + 2100100)..$$ $$\Rightarrow \frac{dy}{dx} = \frac{1}{10^4}(3x^2 - 200x - 10\ 000)$$ $$\frac{dy}{dx} = 0 \Rightarrow 3x^2 - 200x - 10\ 000 = 0$$ $$(3x + 100)(x - 100) = 0$$ $$x = 100 \ \ (\text{or } x = -\frac{100}{3})$$ The graph shows the minimum emission occurs at speed of 100 km hour^{-1} Substituting $x = 100$ gives $y = 110.01$ Minimum rate of emission is 110 grams per km.	**M1** **A1** **M1** solving quadratic **A1** **A1** or $\frac{d^2 y}{dx^2}$ justify min **A1**
3.	(i)	Substituting $p = 250, d = 279, s = 4$ in $E = (10 + 0.0015p)d + 200s$ $\Rightarrow E = 3694.625$ (in kg) So emissions are 3.7 tonnes to 2 s.f. *	**M1** subst **E1**
	(ii)	Emission rate = 1.5 g km^{-1} Distance = 279 km Emissions = $1.5 \times 279 = 418.5$ g $\qquad\qquad = 0.42$ kg (2 s.f.), and so is less than ½ kg. *or* p=251 in formula gives E=3695.0435, difference=0.4185kg<0.5kg	**E1**
4.	(i)	 **Approximate Step graph** **Correct with scales shown**	**M1** **A1**
	(ii)	There is a basic service of 10 trains a day for up to 1 million passengers per year. For every half million extra passengers above 1 million, an extra daily train is provided.	**B1** **B1**
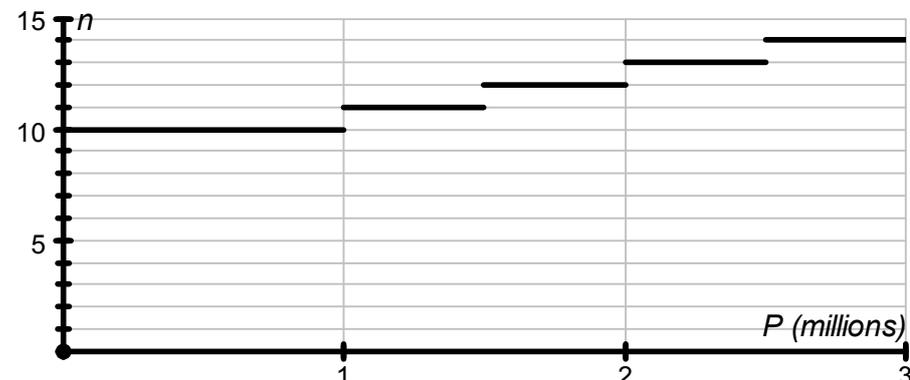
5.		100 miles = 1.609344 × 100 km	**M1**
		= 160 km 934 m 40 cm	**A1**
		So it appears to give the answer to the nearest 10 cm (option B).	**A1**
			[18]

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