



Oxford Cambridge and RSA

Tuesday 7 June 2022 – Afternoon

A Level Mathematics A

H240/01 Pure Mathematics

Time allowed: 2 hours



You must have:

- the Printed Answer Booklet
- a scientific or graphical calculator

INSTRUCTIONS

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION

- The total mark for this paper is **100**.
- The marks for each question are shown in brackets [].
- This document has **8** pages.

ADVICE

- Read each question carefully before you start your answer.

Formulae
A Level Mathematics A (H240)

Arithmetic series

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a+(n-1)d\}$$

Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \quad \text{for } |r| < 1$$

Binomial series

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \dots + {}^nC_r a^{n-r}b^r + \dots + b^n \quad (n \in \mathbb{N})$$

$$\text{where } {}^nC_r = {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \dots \quad (|x| < 1, n \in \mathbb{R})$$

Differentiation

$f(x)$	$f'(x)$
$\tan kx$	$k \sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

$$\text{Quotient rule } y = \frac{u}{v}, \quad \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

Differentiation from first principles

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Integration

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + c$$

$$\int f'(x)(f(x))^n dx = \frac{1}{n+1}(f(x))^{n+1} + c$$

$$\text{Integration by parts } \int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

Small angle approximations

$$\sin \theta \approx \theta, \quad \cos \theta \approx 1 - \frac{1}{2}\theta^2, \quad \tan \theta \approx \theta \quad \text{where } \theta \text{ is measured in radians}$$

Trigonometric identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad \left(A \pm B \neq \left(k + \frac{1}{2}\right)\pi\right)$$

Numerical methods

Trapezium rule: $\int_a^b y \, dx \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1})\}$, where $h = \frac{b-a}{n}$

The Newton-Raphson iteration for solving $f(x) = 0$: $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A)P(B|A) = P(B)P(A|B) \quad \text{or} \quad P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Standard deviation

$$\sqrt{\frac{\sum(x-\bar{x})^2}{n}} = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2} \quad \text{or} \quad \sqrt{\frac{\sum f(x-\bar{x})^2}{\sum f}} = \sqrt{\frac{\sum fx^2}{\sum f} - \bar{x}^2}$$

The binomial distribution

If $X \sim B(n, p)$ then $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$, mean of X is np , variance of X is $np(1-p)$

Hypothesis test for the mean of a normal distribution

If $X \sim N(\mu, \sigma^2)$ then $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$ and $\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1)$

Percentage points of the normal distribution

If Z has a normal distribution with mean 0 and variance 1 then, for each value of p , the table gives the value of z such that $P(Z \leq z) = p$.

p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

Kinematics

Motion in a straight line

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}(u + v)t$$

$$v^2 = u^2 + 2as$$

$$s = vt - \frac{1}{2}at^2$$

Motion in two dimensions

$$\mathbf{v} = \mathbf{u} + \mathbf{a}t$$

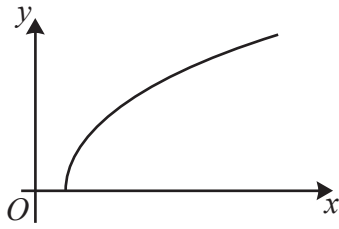
$$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$$

$$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$$

$$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$$

Answer **all** the questions.

1



The diagram shows part of the curve $y = \sqrt{x^2 - 1}$.

(a) Use the trapezium rule with 4 intervals to find an estimate for $\int_1^3 \sqrt{x^2 - 1} \, dx$.

Give your answer correct to **3** significant figures. [4]

(b) State whether the value from part (a) is an under-estimate or an over-estimate, giving a reason for your answer. [1]

(c) Explain how the trapezium rule could be used to obtain a more accurate estimate. [1]

2 (a) Given that a and b are real numbers, find a counterexample to disprove the statement that, if $a > b$, then $a^2 > b^2$. [1]

(b) A student writes the statement that $\sin x^\circ = 0.5 \iff x^\circ = 30^\circ$.

(i) Explain why this statement is incorrect. [1]

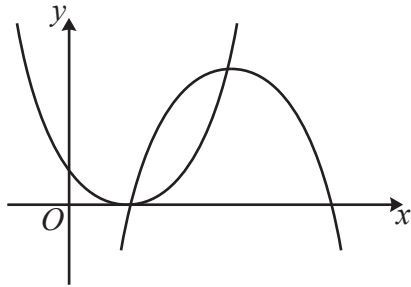
(ii) Write a corrected version of this statement. [1]

(c) Prove that the sum of four consecutive multiples of 4 is always a multiple of 8. [3]

3 (a) **In this question you must show detailed reasoning.**

Find the coordinates of the points of intersection of the curves with equations $y = x^2 - 2x + 1$ and $y = -x^2 + 6x - 5$. [4]

- (b) The diagram shows the curves $y = x^2 - 2x + 1$ and $y = -x^2 + 6x - 5$. This diagram is repeated in the Printed Answer Booklet.



On the diagram in the Printed Answer Booklet, draw the line $y = 2x - 2$. [2]

- (c) Show on your diagram in the Printed Answer Booklet the region of the x - y plane within which all three of the following inequalities are satisfied.

$$y \geq x^2 - 2x + 1 \quad y \leq -x^2 + 6x - 5 \quad y \leq 2x - 2$$

You should indicate the region for which all the inequalities hold by labelling the region R . [1]

- 4 (a) Write $2x^2 + 6x + 7$ in the form $p(x+q)^2 + r$, where p , q and r are constants. [3]

- (b) State the coordinates of the minimum point on the graph of $y = 2x^2 + 6x + 7$. [2]

- (c) Hence deduce

- the minimum value of $2 \tan^2 \theta + 6 \tan \theta + 7$,
- the smallest positive value of θ , in degrees, for which the minimum value occurs. [3]

- 5 (a) The graph of $y = 2^x$ can be transformed to the graph of $y = 2^{x+4}$ **either** by a translation **or** by a stretch.

- (i) Give full details of the translation. [2]

- (ii) Give full details of the stretch. [2]

- (b) **In this question you must show detailed reasoning.**

Solve the equation $\log_2(8x) = 1 - \log_2(1-x)$. [4]

- 6 (a) Find the first four terms in the expansion of $(3 + 2x)^5$ in ascending powers of x . [4]
- (b) Hence determine the coefficient of y^3 in the expansion of $(3 + 2y + 4y^2)^5$. [4]

7 A curve has equation $2x^3 + 6xy - 3y^2 = 2$.

Show that there are no points on this curve where the tangent is parallel to $y = x$. [8]

- 8 (a) Substance A is decaying exponentially such that its mass is m grams at time t minutes. Find the missing values of m and t in the following table.

t	0	10		50
m	1250	750	450	

[2]

- (b) Substance B is also decaying exponentially, according to the model $m = 160e^{-0.055t}$, where m grams is its mass after t minutes.

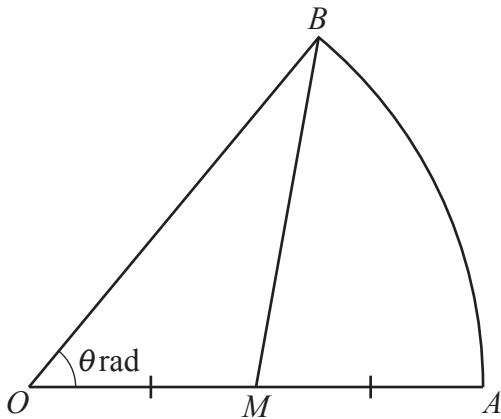
(i) Determine the value of t for which the mass of substance B is half of its original mass. [3]

(ii) Determine the rate of decay of substance B when $t = 15$. [3]

- (c) State whether substance A or substance B is decaying at a faster rate, giving a reason for your answer. [1]

- 9 Use the substitution $x = 2 \sin \theta$ to show that $\int_1^{\sqrt{3}} \sqrt{4-x^2} dx = \frac{1}{3}\pi$. [7]

10



The diagram shows a sector OAB of a circle with centre O and radius OA . The angle AOB is θ radians. M is the mid-point of OA . The ratio of areas $OMB : MAB$ is 2:3.

- (a) Show that $\theta = 1.25 \sin \theta$. [4]

The equation $\theta = 1.25 \sin \theta$ has only one root for $\theta > 0$.

- (b) This root can be found by using the iterative formula $\theta_{n+1} = 1.25 \sin \theta_n$ with a starting value of $\theta_1 = 0.5$.
- Write down the values of θ_2 , θ_3 and θ_4 .
 - Hence find the value of this root correct to **3** significant figures. [3]
- (c) The diagram in the Printed Answer Booklet shows the graph of $y = 1.25 \sin \theta$, for $0 \leq \theta \leq \pi$.
- Use this diagram to show how the iterative process used in (b) converges to this root.
 - State the type of convergence. [3]
- (d) Draw a suitable diagram to show why using an iterative process with the formula $\theta_{n+1} = \sin^{-1}(0.8\theta_n)$ does not converge to the root found in (b). [2]

- 11 The gradient function of a curve is given by $\frac{dy}{dx} = \frac{3x^2 \ln x}{e^{3y}}$.

The curve passes through the point $(e, 1)$.

- (a) Find the equation of this curve, giving your answer in the form $e^{3y} = f(x)$. [6]
- (b) Show that, when $x = e^2$, the y -coordinate of this curve can be written as $y = a + \frac{1}{3} \ln(b e^3 + c)$, where a , b and c are constants to be determined. [3]

12 A curve has parametric equations $x = \frac{1}{t}$, $y = 2t$. The point P is $\left(\frac{1}{p}, 2p\right)$.

(a) Show that the equation of the tangent at P can be written as $y = -2p^2x + 4p$. [4]

The tangent to this curve at P crosses the x -axis at the point A and the normal to this curve at P crosses the x -axis at the point B .

(b) Show that the ratio $PA:PB$ is $1:2p^2$. [8]

END OF QUESTION PAPER

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