RECOGNISING ACHIEVEMENT

Oxford Cambridge and RSA Examinations
Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS

APPLICATIONS OF ADVANCED MATHEMATICS, C4
PAPER A

4754

Specimen Paper

Additional materials: Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF 2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, Centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You **may** use a graphical or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is **72**.

Section A (36 marks)

1 Find the binomial expansion of $\sqrt{1+2x}$ up to and including the term in x^3, simplifying the coefficients.
State the values of x for which this expansion is valid. **[5]**

2 PQR is a straight line, with the points in that order.
The coordinates of P and Q are (2, 1) and (7, 1) respectively. The point S has coordinates (7,13).
The length of SR is 20 units.
Find $\tan \mathrm{P\hat{S}Q}$ and $\tan \mathrm{Q\hat{S}R}$ and hence show that $\tan \mathrm{P\hat{S}R} = \frac{63}{16}$. **[5]**

3 Write $3\sin\theta + 4\cos\theta$ in the form $R\sin(\theta+\alpha)$ where R and α are to be determined.
Solve $3\sin\theta + 4\cos\theta = 1$ for $0° \le \theta \le 360°$. **[6]**

4 **(i)** A curve, C, has parametric equations

$$x = \sin\theta - \cos\theta + 1$$

$$y = \sin 2\theta$$

Show that the cartesian equation of the curve is $y = -x^2 + 2x$. **[4]**

(ii) Sketch the curve $y = -x^2 + 2x$ and indicate which part of it corresponds to the curve C. **[2]**

5 Show that $\frac{x}{x+1} = 1 - \frac{1}{x+1}$.
The curve $y = \frac{x}{x+1}$, from $x = 0$ to 2, is rotated through $360°$ about the x-axis.
Show that the volume of revolution is $(\frac{8}{3} - 2\ln 3)\pi$. **[7]**

6 A curve has parametric equations $x = 3t,\ y = \frac{4}{t}$.
Show that the straight line joining (0, 4) to (12, 0) is a tangent to the curve and state the value of t at the point where the line touches the curve. **[7]**

Section B (36 marks)

7 The population of a city is P millions at time t years. When $t=0$, $P=1$.

(i) A simple model is given by the differential equation: $\dfrac{\mathrm{d}P}{\mathrm{d}t}=kP$ where k is a constant.

(***A***) Verify that $P=A\mathrm{e}^{kt}$ satisfies this differential equation, and show that $A=1$.
Given that $P=1.24$ when $t=1$, find k. **[5]**

(***B***) Why is this model unsatisfactory in the long term? **[1]**

(ii) An alternative model is given by the differential equation: $4\dfrac{\mathrm{d}P}{\mathrm{d}t}=P(2-P)$.

(***A***) Express $\dfrac{4}{P(2-P)}$ in partial fractions. **[3]**

(***B***) Hence, by integration, show that: $\dfrac{P}{2-P}=\mathrm{e}^{\frac{1}{2}t}$. **[5]**

(***C***) Express P in terms of t.
Verify that, when $t=1$, P is approximately 1.24. **[3]**

(***D***) According to this model, what happens to the population of the city in the long term? **[1]**

8 Fig. 8 illustrates the flight path of a helicopter H taking off from an airport.
Coordinate axes Oxyz are set up with the origin O at the base of the airport control tower. The x-axis is due east, the y-axis due north, and the z-axis vertical. The units of distance are kilometres throughout.
The helicopter takes off from the point G. The position vector r of the helicopter t minutes after take-off is given by: $\mathbf{r} = (1+t)\mathbf{i} + (0.5+2t)\mathbf{j} + 2t\mathbf{k}$.

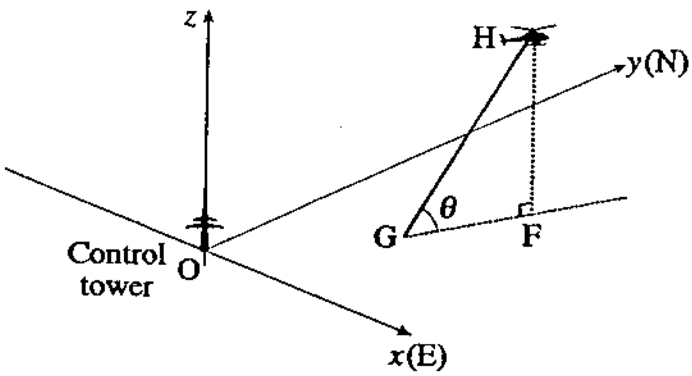

Fig. 8

(i) Write down the coordinates of G. **[1]**

(ii) Find the angle the flight path makes with the horizontal. (This angle is shown as θ in Fig. 8). **[3]**

(iii) Find the bearing of the flight path. (This is the bearing of the line GF shown in Fig. 8). **[2]**

(iv) The helicopter enters a cloud at a height of 2 km.
Find the coordinates of the point where the helicopter enters the cloud. **[3]**

(v) A mountain top is situated at M (5, 4.5, 3).
Find the value of t when HM is perpendicular to the flight path GH.
Find the distance from the helicopter to the mountain top at this time. **[5]**

(vi) Find, in vector form, the equation of the line GM.
Find also the angle between the line from G to the mountain top and the helicopter's flight path. **[4]**

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PAPER B: COMPREHENSION

4754

Specimen Insert

TIME Up to 1 hour

INSTRUCTIONS TO CANDIDATES

- This insert contains the text for use with the questions in the related Specimen Paper.

Acknowledgement

The research referred to in this article was carried out by Martyn Gorman and John Speakman of the University of Aberdeen, and Michael Mills and Jacobus Raath from the Kruger National Park in South Africa. OCR would particularly like to thank Martyn Gorman for his help in the production of this article.

Saving the African Wild Dog

Introduction

The African Wild Dog (*Lycaon pictus*) was once plentiful south of the Sahara. However, in recent years its numbers have declined sharply and it is believed that as few as 5000 individuals now remain.

This article outlines some recent work on a mathematical model for one possible cause of its decline, and considers the implications for conservation measures.

The African Wild Dog

The African Wild Dog is a completely different species from the domestic dog and it is illustrated in Fig. 1. The large rounded ears are a characteristic feature.

African Wild Dogs live in packs of up to about 40 individuals and survive by hunting. They usually prey on larger animals such as wildebeest, impala and gazelle. Their method is to approach a herd, select an individual, and then chase it until it is exhausted. At any time two dogs take the lead with the others following on behind; when those two get tired another two take over.

A pack of dogs hunts twice a day, in the morning and the evening, and spends the rest of its time eating and resting.

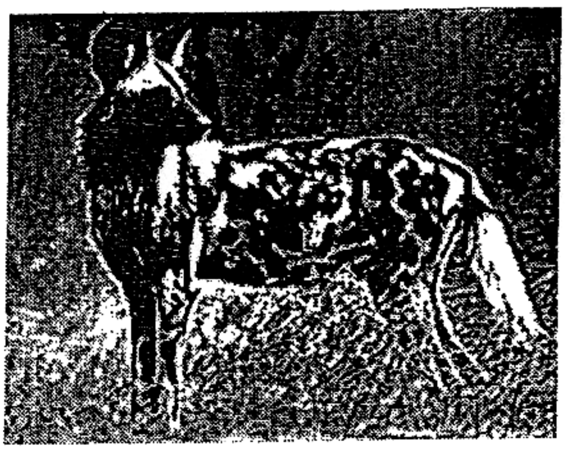

Fig. 1: An African Wild Dog

Fig. 2: A Spotted Hyena

Various reasons have been suggested for the decline in the numbers of African Wild Dogs. One of these is their relationship with the Spotted Hyena (*Crocuta crocuta*) (Fig. 2).

Data from a number of places in Africa suggest that where the density of hyenas is high, the density of wild dogs is low, and vice versa.

Hyenas are much feared by other animals and consequently are able to steal food which others, such as cheetahs and wild dogs, have just hunted and killed. (This habit is called *kleptoparasitism*.) It is believed that this may be a cause of the diminishing number of wild dogs.

Balancing Energy

Before considering the effect of food being stolen it is helpful to model the simpler situation in which the dogs eat all the meat they capture. This is done in terms of energy.

For any dog the energy output over a reasonable period of time must be the same as the energy input over the same period. A common unit for energy is the megajoule (MJ) and this is used throughout this article.

Taking a period of 24 hours gives the equation

$$E = ht + r(24 - t) \qquad \textbf{(1)}$$

where

E is the energy, in MJ, expended in a 24-hour day, the *daily energy expenditure*,
t is the number of hours hunting per day,
h is the rate of energy output when hunting, in MJ per hour,
r is the rate of energy output when not hunting, in MJ per hour.

Notice that these variables represent average values. They will vary from day to day and from one dog to another. This article is looking at a typical dog on a typical day.

The dog takes in energy by eating meat that has been captured. (One kilogram of meat coverts into about 4.4 MJ of energy.) The rate of capturing meat can thus be thought of as a rate of energy capture as a result of hunting. A further variable, c, is thus needed, where

c is the rate of energy capture while hunting, in MJ per hour.

The energy captured in a day's hunting, in MJ, is therefore ct.

Thus, assuming the dogs eat all the meat they capture, the energy balance is expressed by the equation

$$ct = ht + r(24 - t)\,. \qquad \textbf{(2)}$$

Equation **(2)** can be rearranged to make t the subject, giving

$$t = \frac{24r}{c + r - h}\,. \qquad \textbf{(3)}$$

Equation **(3)** gives the number of hours that a dog needs to hunt in a day. The sketch graph in Fig. 3 shows t plotted against c.

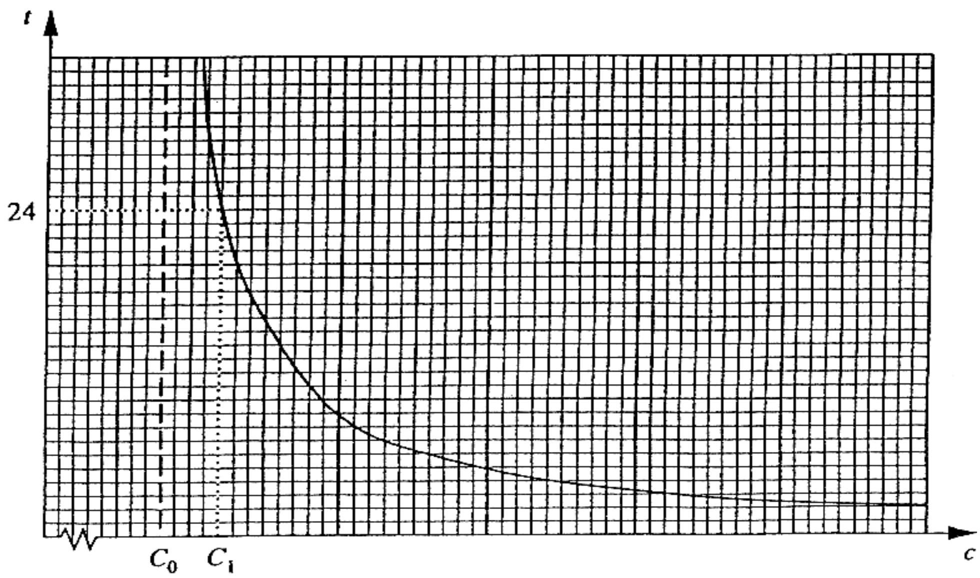

Fig. 3

There are a number of features of this graph to notice.

- The larger the value of c, the less time a dog needs to hunt.
- There is an asymptote for a certain value of c, marked C_0. The value of c must exceed C_0.
- There is however another value of c, marked as C_1 ($C_1 > C_0$), which corresponds to $t = 24$. Unless c exceeds C_1, there are not enough hours in a day for a dog to catch sufficient meat to fulfil its energy requirements.

The value C_1 represents a theoretical rather than a practical limit. No dog can hunt for anything like 24 hours a day; the value of c must be sufficiently greater than C_1 for t to have a realistic value, much less than 24.

Finding Values for the Variables

Recent research on a pack of wild dogs in the Kruger National Park has meant that, for the first time, it is possible for estimates to be made of the values of all the variables used in this article.

- The dogs' hunting times were recorded for days when their meat was not stolen and an average value calculated: $t \approx 3.45$ in hours (i.e. 3 hours 27 minutes).
- Measurements on **six** of the dogs in the pack were used to estimate the daily energy expenditure of a wild dog: $E \approx 15.3$ in MJ.
- The quantity r was estimated using an established experimental formula for domestic dogs relating the mass of a dog to its rate of energy expenditure when resting: $r \approx 0.22$ in MJ per hour.
- Substituting these figures into equation **(1)** allows an estimate to be made of the rate of energy expenditure when hunting: $h \approx 3.12$ in MJ per hour.
- Substitution also gives an estimate of the rate of energy capture when no meat is stolen: $c \approx 4.43$ in MJ per hour.

These values of h and r would suggest that the value, C_0, of c for which the asymptote occurs in Fig. 3 is 2.90. The value of c obtained above, 4.43, is quite well above this.

Food Loss

The model used so far has assumed that the dogs eat all the meat they capture. This is not the case; it is observed that hyenas often steal meat from wild dogs.

At first sight it would seem that the loss of, say, 10% of a pack's food would be made up by spending about 10% extra time hunting. Since wild dogs only hunt for a few hours a day this would represent a minor loss of their leisure time. However a suitable refinement of the model shows that this is not the case.

A further variable p is introduced to represent the proportion of the food (or energy) that is stolen: $0 \le p < 1$.

So, although the energy a dog captures in a 24-hour period is ct, its energy intake over that period is $(1-p)ct$.

Replacing ct by $(1-p)ct$ in equation **(2)**, and rearranging it to make t the subject, gives

$$t = \frac{24r}{(1-p)c + r - h} \qquad \textbf{(4)}$$

The graph in Fig. 4, in which t is plotted against p, illustrates this relationship. The values taken for r, h and c are those calculated on page 4.

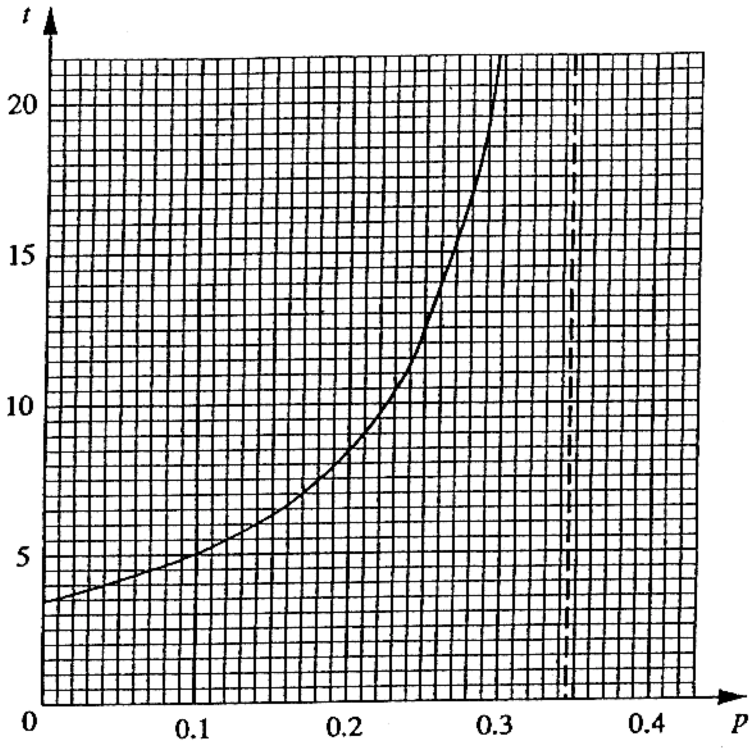

Fig. 4

Conclusions

Fig. 4 shows just how close to the limit the dogs are living. For example, if just 25% of the meat they capture is stolen, they must increase their hunting from about 3½ hours to over 12 hours a day.

Even without having any meat stolen, wild dogs work extremely hard. A wild dog is comparable in size to a collie sheep dog. A working collie sheep dog has an energy output of about 8 MJ per day, compared with the estimated 15.3 MJ for a wild dog. This is believed to be close to the limit of what the wild dogs' bodies can take. The extra energy requirements produced by having quite small quantities of food stolen may well prove fatal.

As a result of the study described in this article, some conservationists have concluded that it is pointless to try to protect the African Wild Dog in open country where there are many hyenas, and where the hyenas find it easy to detect that a kill has just taken place. The situation is different in areas of thick vegetation, both because few hyenas live there and because they are less likely to detect a kill.

Consequently efforts to save the African Wild Dog from extinction are now likely to be concentrated on those populations living in areas of thick vegetation.

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TIME Up to 1 hour

Candidate Name	Centre Number	Candidate Number

INSTRUCTIONS TO CANDIDATES

- Write your name, Centre number and candidate number in the spaces above.
- Write your answers, in blue or black ink.
- Answer **all** the questions.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- You **may** use a graphical or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The insert contains the text for use with the questions.
- You may find it helpful to make notes and do some calculations as you read the passage.
- You are **not** required to hand in these notes with your question paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **18**.

1 State the meaning of the terms ht and $r(24-t)$ in equation **(1)**.

[2]

2 In line 62, the value of h is stated to be 3.12.
Explain how this figure was obtained from the information given in lines 54 to 60.

[2]

3 Show how equation **(3)** is obtained from equation **(2)**.

[2]

4 In Fig. 3, there is an asymptote at $c = C_0$.
Find an expression, in terms of r and h, for C_0.
Justify the value given for C_0 in line 66.

[3]

5 Using figures given in the article calculate the hunting time if 20% of the meat is stolen.

[2]

6 Use equation **(4)** to calculate the value of p at the asymptote in Fig. 4.

[2]

7 The article gives estimates that were made of the values of the variables involved. While these estimates were the best that could be obtained under the circumstances, it is possible that they are not particularly accurate.

(i) State one likely source of error.

[1]

(ii) Explain briefly how you could assess the effect of any such errors on the value of p for which the asymptote in Fig. 4 occurs.

[1]

8 The article contains information that allows you to calculate the average number of kilograms of meat that a wild dog eats in a day.
Find this information and carry out the calculation.

[3]