



Oxford Cambridge and RSA

Thursday 22 October 2020 – Afternoon

A Level Further Mathematics B (MEI)

Y435/01 Extra Pure

Time allowed: 1 hour 15 minutes



You must have:

- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator

INSTRUCTIONS

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION

- The total mark for this paper is **60**.
- The marks for each question are shown in brackets [].
- This document has **4** pages.

ADVICE

- Read each question carefully before you start your answer.

Answer **all** the questions.

1 The matrix \mathbf{A} is given by $\mathbf{A} = \begin{pmatrix} 0 & 2 \\ 3 & -1 \end{pmatrix}$.

Find

- the eigenvalues of \mathbf{A} ,
- an eigenvector associated with each eigenvalue.

[5]

2 A sequence is defined by the recurrence relation $t_{n+1} = \frac{t_n}{n+3}$ for $n \geq 1$, with $t_1 = 8$.

Verify that the particular solution to the recurrence relation is given by $t_n = \frac{a}{(n+b)!}$ where a and b are constants whose values are to be determined.

[5]

3 A sequence is defined by the recurrence relation $u_{n+2} = 4u_{n+1} - 5u_n$ for $n \geq 0$, with $u_0 = 0$ and $u_1 = 1$.

(a) Find an exact real expression for u_n in terms of n and θ , where $\tan \theta = \frac{1}{2}$.

[7]

A sequence is defined by $v_n = a^{\frac{1}{2}n}u_n$ for $n \geq 0$, where a is a positive constant.

(b) In each of the following cases, describe the behaviour of v_n as $n \rightarrow \infty$.

- $a = 0.1$
- $a = 0.2$
- $a = 1$

[5]

- 4 (a) In each of the following cases, a set G and a binary operation \circ are given. The operation \circ may be assumed to be associative on G .

Determine which, if any, of the other three group axioms are satisfied by (G, \circ) and which, if any, are not satisfied.

(i) $G = \{2n + 1 : n \in \mathbb{Z}\}$ and \circ is addition. [3]

(ii) $G = \{a + b\sqrt{2} : a, b \in \mathbb{Z}\}$ and \circ is multiplication. [3]

(iii) G is the set of all real numbers and \circ is multiplication. [3]

- (b) A group M consists of eight 2×2 matrices under the operation of matrix multiplication. Five of the eight elements of M are as follows.

$$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix} \quad \frac{1}{\sqrt{2}} \begin{pmatrix} -1 & i \\ i & -1 \end{pmatrix} \quad \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -i \\ -i & 1 \end{pmatrix} \quad \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix} \quad \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

(i) Find the other three elements of M . [3]

$(N, *)$ is another group of order 8, with identity element e . You are given that $N = \langle a, b, c \rangle$ where $a*a = b*b = c*c = e$.

(ii) State whether M and N are isomorphic to each other, giving a reason for your answer. [1]

5 In this question you must show detailed reasoning.

The matrix \mathbf{A} is given by $\begin{pmatrix} \frac{1}{3} & -\frac{2}{3} & -\frac{2}{3} \\ -\frac{2}{3} & \frac{1}{3} & -\frac{2}{3} \\ -\frac{2}{3} & -\frac{2}{3} & \frac{1}{3} \end{pmatrix}$ and the vector \mathbf{e} is given by $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$. You are given that \mathbf{e} is an

eigenvector of \mathbf{A} with an associated eigenvalue of -1 .

\mathbf{f} is any vector which is perpendicular to \mathbf{e} .

(a) Show that \mathbf{f} is also an eigenvector of \mathbf{A} . [4]

(b) State the eigenvalue associated with \mathbf{f} . [1]

You are now given that \mathbf{A} represents a reflection in 3-D space.

(c) Explain the significance of \mathbf{e} and \mathbf{f} in relation to the transformation that \mathbf{A} represents. [2]

(d) State the cartesian equation of the plane of reflection of the transformation represented by \mathbf{A} . [1]

6 A surface S is defined by $z = f(x, y) = 4x^4 + 4y^4 - 17x^2y^2$.

(a) (i) Show that there is only one stationary point on S . [5]

The value of z at the stationary point is denoted by s .

(ii) State the value of s . [1]

(iii) By factorising $f(x, y)$, sketch the contour lines of the surface for $z = s$. [3]

(iv) Hence explain whether the stationary point is a maximum point, a minimum point or a saddle point. [1]

C is a point on S with coordinates $(a, a, f(a, a))$ where a is a constant and $a \neq 0$. Π is the tangent plane to S at C .

(b) (i) Find the equation of Π in the form $\mathbf{r} \cdot \mathbf{n} = p$. [3]

(ii) The shortest distance from the origin to Π is denoted by d . Show that $\frac{d}{a} \rightarrow \frac{3\sqrt{2}}{4}$ as $a \rightarrow \infty$. [3]

(iii) Explain whether the origin lies above or below Π . [1]

END OF QUESTION PAPER

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