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Mathematics MPC4

(Specification 6360)

Pure Core 4

Final

Mark Scheme

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
$-x$ EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q	Solution	Marks	Total	Comments
1(a)	$(\mathrm{f}(-2)=)0$	B1	1	ISW (0 seen is B1)
(b)	$\mathrm{f}\left(\frac{3}{2}\right)=4\left(\frac{3}{2}\right)^3-13\left(\frac{3}{2}\right)+6$	M1		Clear attempt at $\mathrm{f}\left(\frac{3}{2}\right)$ with 3 terms Factor theorem required; **NOT** long division
	$4\times\frac{27}{8}-13\times\frac{3}{2}+6$ or $13.5-19.5+6$			Must see this, or equivalent
	$=0\Rightarrow(2x-3)$ is a factor	A1	2	Shown = 0 and statement.
(c)	Any appropriate method to find third factor	M1		Full long division Compare coefficients Factor Theorem $\mathrm{f}\left(\frac{1}{2}\right)$
	$(x+2)(2x-3)(2x-1)$	A1		Or $\left(2x^2+x-6\right)(2x-1)$ NMS M1A1 **SC1** $(2x+1)$ or $(1-2x)$ or $\left(x-\frac{1}{2}\right)$ or $\left(\frac{1}{2}-x\right)$ for third factor
	$2x^2+x-6=(x+2)(2x-3)$	M1		Factorise **numerator** correctly or cancel $2x^2+x-6$
	$\dfrac{2x^2+x-6}{\mathrm{f}(x)}=\dfrac{1}{2x-1}$	A1	4	No ISW
			7	

Q	Solution	Marks	Total	Comments
2(a)(i)	$(A=)80$	B1	1	Ignore units
(ii)	$2000 = A \times k^{25}$	M1		A or their value from (a)(i)
	$k = \sqrt[25]{25}$ or $25^{\frac{1}{25}}$ or $k = 10^{0.04\log 25}$ or $e^{0.04\ln 25}$ $\Rightarrow k = 1.137411$ **AG**	A1	2	Correct expression for k, or 1.13741146….seen, **and** correct answer to 6 d.p.
(b)	$\ln\left(\dfrac{100000}{their\ A}\right) = t\ln k$	M1		Take logs correctly. Condone miscopied k $\ln 1250 = t\ln k$ or t = $\log_k 1250$
	$t = 55.38$	A1		Condone 55.3 or 55.4 PI
	$\Rightarrow 2016$	A1	3	
			6	
2(b)	**Alternative** By trial and improvement $1250 = k^t$	M1		Attempt to calculate k^{55} and k^{56}.
	$t = 56$ or $55 < t < 56$	A1		
	$\Rightarrow$ 2016	A1	3	

Q	Solution	Marks	Total	Comments
3 (a)(i)	$(1-x)^{\frac{1}{3}} = 1-\frac{1}{3}x$	M1		Condone $1^{\frac{1}{3}} + -\frac{1}{3}x$ for M1
	$= 1-\frac{1}{3}x-\frac{1}{9}x^2$	A1	2	Must simplify coefficients including signs
(ii)	$(125-27x)^{\frac{1}{3}} = 125^{\frac{1}{3}}\left(1-\frac{27}{125}x\right)^{\frac{1}{3}}$	B1		May have 5 instead of $125^{\frac{1}{3}}$
	$\left(1-\frac{27}{125}x\right)^{\frac{1}{3}} = \left(1-\frac{1}{3}\times\frac{27}{125}x-\frac{1}{9}\left(\frac{27}{125}x\right)^2\right)$	M1		Attempt to replace x by $\pm\frac{27}{125}x$ condone missing brackets, or start binomial again.
	$= 5-\frac{9}{25}x-\frac{81}{3125}x^2$	A1	3	Condone $5+\frac{-9}{25}x+\frac{-81}{3125}x^2$
(b)	$x=\frac{2}{9}$ **used** in answer to (a)(ii)	M1		Condone $x=\frac{6}{27}$ or x = 0.222 or better
	$\sqrt[3]{119} \approx 5-\frac{9}{25}\times\frac{2}{9}-\frac{81}{3125}\left(\frac{2}{9}\right)^2$			
	$= 4.91872$	A1	2	This answer only and must follow from correct expansion
			7	
3(a) (ii)	**Alternative** using $(a+bx)^n$			
	$(125-27x)^{\frac{1}{3}} = 125^{\frac{1}{3}} + \frac{1}{3}\times125^{-\frac{2}{3}}\times(-27x)$ $+\frac{1}{3}\left(-\frac{2}{3}\right)\frac{1}{2}\times125^{-\frac{5}{3}}(-27x)^2$	M1		Allow one error; condone missing brackets
	$= 5-\frac{9}{25}x-\frac{81}{3125}x^2$	A2	3	

Q	Solution	Marks	Total	Comments
4 (a)(i)	$\left(\frac{\mathrm{d}x}{\mathrm{d}\theta}=\right)-6\sin 2\theta$, $\left(\frac{\mathrm{d}y}{\mathrm{d}\theta}=\right)-2\sin\theta$	M1		$\left(\frac{\mathrm{d}x}{\mathrm{d}\theta}=\right)p\sin 2\theta$ or $r\sin\theta\cos\theta$ $\left(\frac{\mathrm{d}y}{\mathrm{d}\theta}=\right)q\sin\theta$
		A1		Both correct.
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{-2\sin\theta}{-6\sin 2\theta}$	M1		Use chain rule $\frac{\frac{\mathrm{d}y}{\mathrm{d}\theta}}{\frac{\mathrm{d}x}{\mathrm{d}\theta}}$; condone one slip
	$=\frac{2\sin\theta}{6\times 2\sin\theta\cos\theta}=\frac{1}{6\cos\theta}$	A1	4	$k=6$ must come from correct working seen **AG**
(ii)	$\theta=\frac{\pi}{3}$ $m_{\mathrm{T}}=\frac{1}{3}$	B1ft		ft on k $\left(\frac{1}{k\times\frac{1}{2}}\right)$ k need not be numerical
	$m_{\mathrm{N}}=-3$	B1ft		ft on m_{T}
	$(x,y)=\left(-\frac{3}{2},1\right)$	B1		
	Normal $y-1=-3\left(x+\frac{3}{2}\right)$	B1	4	CAO; any correct form, ISW. $2y+6x+7=0$
(b)	$\sin^2 x=\frac{1}{2}(1-\cos 2x)$	M1 A1		$p+q\cos 2x$; Allow different letters for x or mixture eg θ even for A1and the following A1ft
	$\int p\ \mathrm{d}x=px$ $\int q\cos 2x=\frac{1}{2}q\sin 2x$	A1ft		Both integrals correct; ft on p and q
	$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\sin^2 x\ \mathrm{d}x=\left[\frac{x}{2}-\frac{1}{4}\sin 2x\right]$			
	$=\left(\frac{\pi}{8}-\frac{1}{4}\right)-\left(-\frac{\pi}{8}-\left(-\frac{1}{4}\right)\right)$	m1		Correct use of limits; $\mathrm{F}\left(\frac{\pi}{4}\right)-\mathrm{F}\left(-\frac{\pi}{4}\right)$ or $2\mathrm{F}\left(\frac{\pi}{4}\right)$ $\mathrm{F}(x)=px+r\sin 2x$ and $\sin\frac{\pi}{2}$, $\sin\left(-\frac{\pi}{2}\right)$ must be evaluated correctly for m1
	$=\frac{\pi}{4}-\frac{1}{2}$	A1	5	CSO OE ISW
			13	

4 (b)	**Alternative**			
	$\int \sin^2 x \ \mathrm{d}x = -\sin x \cos x - \int -\cos x \cos x \ \mathrm{d}x$	M1		Use parts; condone sign slips
	$= -\sin x \cos x + \int 1 - \sin^2 x \ \mathrm{d}x$	m1		Use $\cos^2 x = 1 - \sin^2 x$
	$2\int \sin^2 x \ \mathrm{d}x = -\sin x \cos x + x$	A1		
	$2\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sin^2 x \ \mathrm{d}x = \mathrm{G}\left(\frac{\pi}{4}\right) - \mathrm{G}\left(-\frac{\pi}{4}\right)$	m1		Correct use of limits
	$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sin^2 x \ \mathrm{d}x = \frac{\pi}{4} - \frac{1}{2}$	A1	5	

Q	Solution	Marks	Total	Comments
5 (a)	$\overrightarrow{AB}=\begin{bmatrix}4\\-1\\3\end{bmatrix}-\begin{bmatrix}5\\1\\-2\end{bmatrix}=\begin{bmatrix}-1\\-2\\5\end{bmatrix}$	B1		$\pm\left(\overrightarrow{OA}-\overrightarrow{OB}\right)$ Co-ordinate form only is B0 Condone one component incorrect
	Line through A and B	M1		$\overrightarrow{OA}+\lambda\mathbf{d}$ or $\overrightarrow{OB}+\lambda\mathbf{d}$ where $\mathbf{d}=\overrightarrow{AB}$ or $\overrightarrow{BA}$ all in components and identified.
	$\mathbf{r}=\begin{bmatrix}5\\1\\-2\end{bmatrix}+\lambda\begin{bmatrix}-1\\-2\\5\end{bmatrix}$ or $\mathbf{r}=\begin{bmatrix}4\\-1\\3\end{bmatrix}+\lambda\begin{bmatrix}-1\\-2\\5\end{bmatrix}$	A1	3	OE $\mathbf{r}$ or $\begin{bmatrix}x\\y\\z\end{bmatrix}$ required Condone missing brackets on $\overrightarrow{OA}$ or $\overrightarrow{OB}$
(b)(i)	$5-\lambda=-8+5\mu$ $1-2\lambda=5$ $-2+5\lambda=-6-2\mu$	M1		Clear attempt to set up and solve at least two simultaneous equations in μ and a different parameter. Allow in column vector form.
	$\lambda=-2 \quad \mu=3$	A1		One of λ **or** μ correct OE
	$-2+5\times-2=-12 \quad -6-2\times3=-12$ Both equal -12 so intersect	E1		Verify intersect, λ **and** μ correct or verify $(7,5,-12)$ is on both lines; **statement required**
	P is $(7,5,-12)$	B1	4	CAO condone $P=\begin{bmatrix}7\\5\\-12\end{bmatrix}$ OE and missing brackets
(ii)	$\overrightarrow{BC}=\begin{bmatrix}-8+5\mu\\5\\-6-2\mu\end{bmatrix}-\begin{bmatrix}4\\-1\\3\end{bmatrix}$	B1		$\overrightarrow{BC}=\overrightarrow{OC}-\overrightarrow{OB}$ or $\overrightarrow{CB}=\overrightarrow{OB}-\overrightarrow{OC}$
	$\begin{bmatrix}3\\6\\-15\end{bmatrix}\bullet\overrightarrow{BC}=0$	M1		Clear attempt at $\pm\overrightarrow{BP}$ or $\pm\overrightarrow{AB}$ or $\pm\overrightarrow{AP}$ in components sp with $\overrightarrow{BC}=0$
	$-36+15\mu+36+135+30\mu=0$	m1		Linear equation in μ using *their* $\overrightarrow{BC}$ and solved for μ. Condone one arithmetical or sign slip
	$\mu=-3$	A1		
	C is $(-23,5,0)$	A1	5	CSO Condone column vector.
			12	

Q	Solution	Marks	Total	Comments
6 **(a)**	$(C=)\frac{2}{e}$ or $2e^{-1}$ or $2\left(\frac{1}{e}\right)$ or $2\left(e^{-1}\right)$	B1	1	One of these answers only. Not 0.736 but allow ISW.
(b)	$\frac{d}{dx}(2y)=2\frac{dy}{dx}$	B1		
	$\frac{d}{dx}\left(e^{2x}y^2\right)=2e^{2x}y^2+e^{2x}2y\frac{dy}{dx}$	M1		Product; 2 terms added, one with $\frac{dy}{dx}$;
		A1		A1 for each term
		A1		
	$\frac{d}{dx}\left(x^2+C\right)=2x$	B1		
	$\frac{dy}{dx}=$	M1		Solve *their* equation correctly for $\frac{dy}{dx}$
	$\frac{x-e^{2x}y^2}{e^{2x}y+1}$	A1	7	Condone factor of 2 in both numerator and denominator. ISW
(c)	Evaluate $\frac{dy}{dx}$ at $\left(1,\frac{1}{e}\right)$	M1		Substitute $x=1$ and $y=\frac{1}{e}$ into numerator of $\frac{dy}{dx}$; allow one slip
	numerator $=1-e^2e^{-2}=0 \Rightarrow$ stationary point	A1	2	Conclusion required; must score full marks in part (b) Allow $1-1=0$ or $2-2=0$
			10	

Q	Solution	Marks	Total	Comments
Q7 (a)	$\dfrac{dA}{dt}$	B1		
	$= -k$	B1	2	
(b)(i)	$A = -kt(+\ C)$	M1		Integrate
	$C = 4\pi \times 60^2$	A1		C correct from $A = \pm kt + C$
	$4\pi \times 30^2 = -9k + 4\pi \times 60^2$	m1		Use $r = 30$ $t = 9$ and attempt to find k, as far as $k =$ … $k = 1200\pi$
	$A = -1200\pi t + 14400\pi$ $= 1200\pi(12 - t)$	A1	4	**AG** CSO
(ii)	$t = 12$ (days)	B1	1	
			7	

Q	Solution	Marks	Total	Comments
Q8 (a)	$1 = A(1-x)^2 + B(1-x)(3-2x) + C(3-2x)$	M1		Attempt to clear fractions
	$x=1 \quad x=\frac{3}{2} \quad x=0$ $C=1 \quad 1=A\left(-\frac{1}{2}\right)^2 \quad 1=A+3B+3C$	m1		Use any two (or three) values of x to set up two (or three) equations
	$A=4 \quad B=-2 \quad C=1$	A1 A1	4	Two values correct All values correct
(b)	$\int \frac{1}{2\sqrt{y}}\ \mathrm{d}y = \int \frac{4}{3-2x} - \frac{2}{1-x} + \frac{1}{(1-x)^2}\ \mathrm{d}x$	B1ft		Separate using partial fractions; correct notation; condone missing integral signs but d*y* and d*x* must be in correct place. ft on their *A, B, C* and on each integral.
	$\int \frac{1}{2\sqrt{y}}\ \mathrm{d}y = \sqrt{y} =$	B1		OE $\int \frac{k}{\sqrt{y}}\ \mathrm{d}y = 2k\sqrt{y}$ is B1
	$-2\ln(3-2x)$	B1ft		Condone missing brackets on one ln integral.
	$+2\ln(1-x)$	B1ft		
	$+\frac{1}{1-x} \quad (+C)$	B1ft		Condone omission of +*C*
	$x=0 \quad y=0 \quad \Rightarrow 0=-2\ln 3+0+1+C$	M1		Use $(0,0)$ to find *C*. Must get to *C* = …..
	$C = 2\ln 3 - 1$	A1		Correct *C* found from correct equation. *C* must be exact, in any form but not decimal.
	$\sqrt{y} = 2\ln\left(\frac{3-3x}{3-2x}\right) + \frac{1}{1-x} - 1$	m1		Correct use of rules of logs to progress towards requested form of answer . *C* must be of the form *r* ln*s* + *t*
	$y^{\frac{1}{2}} = 2\ln\left(\frac{3-3x}{3-2x}\right) + \frac{x}{1-x}$	A1	9	OE CSO condone B0 for separation
			13	
	TOTAL		**75**	

Q8 (a)	**Alternative** $1 = A(1-x)^2 + B(1-x)(3-2x) + C(3-2x)$ $1 = A + 3B + 3C$ $0 = -2A - 5B - 2C$ $0 = A + 2B$ $A = 4 \quad B = -2 \quad C = 1$	M1 m1 A1 A1	 4	 Set up three simultaneous equations Two values correct All values correct