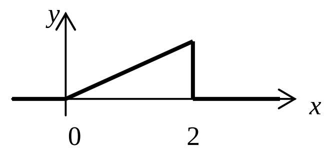
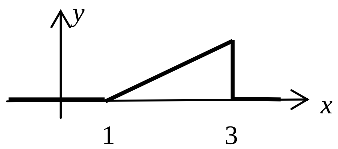
4733 Probability & Statistics 2

1		$\frac{80-\mu}{\sigma} = \Phi^{-1}(0.95) = 1.645$ $\frac{\mu-50}{\sigma} = \Phi^{-1}(0.75) = 0.674(5)$ Solve simultaneously $\mu = 58.7$, $\sigma = 12.9$	M1 B1 A1 M1 A1 A1 **6**	Standardise once with Φ^{-1}, allow σ^2, cc Both 1.645 (1.64, 1.65) and [0.674, 0.675], ignore signs Both equations correct apart from wrong *z*, *not* 1–1.645 Solve two standardised equations μ, a.r.t 58.7 σ, a.r.t. 12.9 [*not* σ^2] [σ^2: M1B1A0M1A1A0]
2	(i)	Let *R* denote the number of choices which are 500 or less. $R \sim B(12, \frac{5}{6})$ $P(R = 12) = (\frac{5}{6})^{12}$ [=0.11216] = **0.112**	M1 M1 A1 **3**	$B(12, \frac{5}{6})$ stated or implied, allow 501/600 etc p^{12} or q^{12} or equivalent Answer, a.r.t. 0.112 [SR: $\frac{500}{600} \times \frac{499}{599} \times \frac{498}{598} \times \ldots$; 0.110: M1A1] [M1 for 0.910 or 0.1321 or vague number of terms]
	(ii)	Method unbiased; unrepresentative by chance	B1 B1 **2**	State that method is unbiased Appropriate comment (e.g. "not unlikely") [SR: partial answer, e.g. not necessarily biased: B1]
3	(i)	$P(\le 1) = 0.0611$ $P(\ge 9) = 1 - P(\le 8) = 1 - 0.9597$ $= 0.0403$ $0.0611 + 0.0403$ [= 0.1014] = **10.1**%	B1 M1 A1 M1 A1 **5**	0.0611 seen Find $P(\ge 9)$, allow 8 or 10 [0.0866, 0.0171] 0.0403 correct Add probabilities of tails, *or* 1 tail × 2 Answer [10.1, 10.2]% or probability
	(ii)	$P(2 \le G \le 8)$ $= 0.8944 - 0.0266$ [= 0.8678] = **0.868**	M1 M1 A1 **3**	Attempt at $P(2 \le G \le 8)$, *not* isw, allow $1 \le G \le 9$ etc Po(5.5) tables, P(≤ top end) – P(≤ bottom end) Answer, a.r.t. 0.868, allow %
4	(i)	$\hat{\mu} = \bar{y} = \frac{3296.0}{40} = 82.4$ $\frac{286800.4}{40} - 82.4^2$ [= 380.25] $S^2 \times \frac{40}{39}$; = 390	B1 M1 M1 A1 **4**	Mean 82.4, c.a.o. Use correct formula for biased estimate Multiply by $n/(n-1)$ [SR: all in one, M2 or M0] Variance 390, c.a.o.
	(ii)	$\Phi\left(\frac{60-82.4}{\sqrt{390}}\right) = \Phi(-1.134)$ = 1 – 0.8716 = **0.128**	M1 A1 **2**	Standardise, allow 390, cc or biased estimate, +/–, do not allow $\sqrt{n}$ Answer in range [0.128, 0.129]
	(iii)	No, distribution irrelevant	B1 **1**	"No" stated or implied, any valid comment
5	(i)	$H_0 : \mu = 500$ where μ denotes $H_1 : \mu < 500$ the population mean *α*: $z = \frac{435-500}{100/\sqrt{4}} = -1.3$ Compare –1.282	B2 M1 A1 B1	Both hypotheses stated correctly [SR: 1 error, B1, but $\bar{x}$ etc: B0] Standardise, use $\sqrt{4}$, can be + $z = -1.3$ (allow –1.29 from cc) *or* $\Phi(z) = 0.0968$ (.0985) Compare *z* & –1.282 *or* *p* (< 0.5) & 0.1 or equivalent
		β: $500 - 1.282 \times 100/\sqrt{4}$ = 435.9; compare 435	M1 A1√;B1	$500 - z \times 100/\sqrt{4}$, allow √ errors, any Φ^{-1}, must be – CV correct, √ on their *z*; 1.282 correct and compare
		Reject H_0 Significant evidence that number of visitors has decreased	M1√ A1√ **7**	Correct deduction, needs $\sqrt{4}$, $\mu = 500$, like-with-like Correct conclusion interpreted in context
	(ii)	CLT doesn't apply as *n* is small So need to know distribution	M1 B1 **2**	Correct reason ["*n* is small" is sufficient] Refer to distribution, e.g. "if not normal, can't do it"

6	(i)	(a) $1 - 0.8153$ $= 0.1847$ (b) $0.8153 - 0.6472$ $=$ **0.168**	M1 A1 M1 A1	 **2** **2**	Po(3) tables, "1 –" used, e.g. 0.3528 or 0.0839 Answer 0.1847 or 0.185 Subtract 2 tabular values, or formula [$e^{-3}\,3^4/4!$] Answer, a.r.t. 0.168
	(ii)	N(150, 150) $1-\Phi\left(\frac{165.5-150}{\sqrt{150}}\right)$ $= 1 - \Phi(1.266) =$ **0.103**	B1 B1 M1 A1 A1	 **5**	Normal, mean 3×50 stated or implied Variance or SD = 3 × 50, or same as μ Standardise 165 with λ, √λ or λ, any or no cc √λ and 165.5 Answer in range [0.102, 0.103]
	(iii)	(a) The sale of one house does not affect the sale of any others (b) The average number of houses sold in a given time interval is constant	B1 B1	 **2**	Relevant answer that shows evidence of correct understanding [but *not* just examples] Different reason, in context [Allow "constant rate" or "uniform" but not "number constant", "random", "singly", "events".]
7	(i)	$\int_0^2 kx\,dx = \left[\frac{kx^2}{2}\right]_0^2 = 2k$ $= 1$ so $k = \frac{1}{2}$	M1 A1	 **2**	Use $\int_0^2 kx\,dx = 1$, or area of triangle Correctly obtain $k = \frac{1}{2}$ **AG**
	(ii)		B1 B1	 **2**	Straight line, positive gradient, through origin Correct, some evidence of truncation, no need for vertical
	(iii)	$\int_0^2 \frac{1}{2}x^2\,dx = \left[\frac{1}{6}x^3\right]_0^2 = \frac{4}{3}$ $\int_0^2 \frac{1}{2}x^3\,dx = \left[\frac{1}{8}x^4\right]_0^2$ [= 2] $2 - \left(\frac{4}{3}\right)^2 = \frac{2}{9}$	M1 A1 M1 M1 A1	 **5**	Use $\int_0^2 kx^2\,dx$; $\frac{4}{3}$ seen or implied Use $\int_0^2 kx^3\,dx$; subtract their mean^2 Answer $\frac{2}{9}$ or a.r.t. 0.222, c.a.o.
	(iv)		M1 A1√	 **2**	Translate horizontally, allow stated, or "1, 2" on axis One unit to right, 1 and 3 indicated, nothing wrong seen, no need for vertical or emphasised zero bits [If in doubt as to → or ↓, M0 in this part]
	(v)	$\frac{7}{3}$ $\frac{2}{9}$	B1√ B1√	 **2**	Previous mean + 1 Previous variance [If in doubt as to → or ↓, B1B1 in this part]

8	(i)	$H_0 : p = 0.65$ OR $p \geq 0.65$ $H_1 : p < 0.65$ B(12, 0.65)	B2 M1	Both hypotheses correctly stated, in this form [One error (but not r, x or $\bar{x}$): B1] B(12, 0.65) stated or implied
		α: P(≤ 6) = 0.2127 Compare 0.10	A1 B1	Correct probability from tables, *not* P(= 6) Explicit comparison with 0.10
		β: Critical region ≤ 5; 6 > 5 Probability 0.0846	B1 A1	Critical region ≤5 or ≤6 or {≤4} ∩ {≥11} & compare 6 Correct probability
		Do not reject H_0 Insufficient evidence that proportion of population in favour is not at least 65%	M1√ A1√ **7**	Correct comparison and conclusion, needs correct distribution, correct tail, like-with-like Interpret in context, e.g. "consistent with claim" [SR: N(7.8, 2.73): can get B2M1A0B1M0: 4 ex 7]
	(ii)	Insufficient evidence to reject claim; test and p/q symmetric	B1√ B1 **2**	Same conclusion as for part (i), don't need context Valid relevant reason, e.g. "same as (i)"
	(iii)	$R \sim B(2n, 0.65)$, $P(R \leq n) > 0.15$ B(18, 0.65), $p = 0.1391$ Therefore $n = 9$	M1 A1 A1 A1 **4**	B(2n, 0.65), P($R \leq n$) > 0.15 stated or implied Any probability in list below seen $p = 0.1391$ picked out (i.e., not just in a list of > 2) Final answer $n = 9$ only [SR <n: M1A0, $n = 4$, 0.1061 A1A0] [SR 2-tail: M1A1A0A1 for 15 or 14] [SR: 9 only, no working: M1A1] [MR B(12, 0.35): M1A0, $n = 4$, 0.1061 A1A0] 3 0.3529 7 0.1836 12 0.0942 4 0.2936 8 0.1594 13 0.0832 5 0.2485 9 0.1391 14 0.0736 6 0.2127 10 0.1218 15 0.0652