4724 Core Mathematics 4

1 (a) $2x^2 - 7x - 4 = (2x+1)(x-4)$ or

$3x^2 + x - 2 = (3x-2)(x+1)$ **B1**

$\frac{2x+1}{3x-2}$ as final answer; this answer only **B1** Do not ISW

2

(b) For correct leading term x in quotient **B1** Identity method

For evidence of correct division process **M1** M1: $x^3 + 2x^2 - 6x - 5 = Q(x^2 + 4x + 1) + R$

Quotient = $x - 2$ **A1** M1: $Q = ax + b$ or $x + b, R = cx + d$ & ≥ 2 ops

[N.B. If $Q = x + b$, this $\Rightarrow$ 1 of the 2 ops]

Remainder = $x - 3$ **A1** A2: $a = 1, b = -2, c = 1, d = -3$ SR: B1 for two

4

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2 Parts with correct split of $u = \ln x, \frac{dv}{dx} = x^4$ ***M1** obtaining result $f(x) + / - \int g(x)\,dx$

$\frac{x^5}{5}\ln x - \int \frac{x^5}{5}.\frac{1}{x}(dx)$ **A1**

$\frac{x^5}{5}\ln x - \frac{x^5}{25}$ **A1**

Correct method with the limits dep***M1** Decimals acceptable here

$\frac{4e^5}{25} + \frac{1}{25}$ ISW (Not '+c') **A1** Accept equiv fracts; like terms amalgamated

5

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3 (i) $\frac{d}{dx}(x^2 y) = x^2\frac{dy}{dx} + 2xy$ or $\frac{d}{dx}(xy^2) = 2xy\frac{dy}{dx} + y^2$ ***B1**

Attempt to solve their differentiated equation for $\frac{dy}{dx}$ dep***M1**

$\frac{dy}{dx} = \frac{y^2 - 2xy}{x^2 - 2xy}$ only **A1** WWW **AG** Must have intermediate line &…

…could imply "=0" on 1st line

3

(ii)(a) Attempt to solve **only** $y^2 - 2xy = 0$ & derive $y = 2x$ **B1** **AG** Any effort at solving $x^2 - 2xy = 0 \rightarrow$ B0

Clear indication why $y = 0$ is not acceptable **B1** Substituting $y = 2x \rightarrow$ B0, B0

2

(b) Attempt to solve $y = 2x$ simult with $x^2 y - xy^2 = 2$ **M1**

Produce $-2x^3 = 2$ or $y^3 = -8$ **A1** AEF

$(-1, -2)$ or $x = -1, y = -2$ **only** **A1**

3

4	**(i)**	For (either point) + t(difference between vectors) $\mathbf{r} = (3\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ or $\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}) + t(-2\mathbf{i} + \mathbf{j} + \mathbf{k}$ or $2\mathbf{i} - \mathbf{j} - \mathbf{k})$	**M1** **A1** **2**	't' can be 's', 'λ' etc. '**r**' must be '**r**' but need not be bold Check other formats, e.g. $ta + (1-t)b$
	(ii)	State/imply that their **r** and their $-2\mathbf{i} + \mathbf{j} + \mathbf{k}$ are perpendicular Consider scalar product = 0	***M1** dep***M1**	N.B.This *M1 is dep on M1 being earned in (i)
		Obtain $t = -\frac{1}{6}$ or $\frac{1}{6}$ or $-\frac{5}{6}$ or $\frac{5}{6}$	**A1**	
		Subst their t into their equation of AB	**M1**	
		Obtain $\frac{1}{6}(16\mathbf{i} + 13\mathbf{j} + 19\mathbf{k})$ AEF	**A1** **5**	Accept decimals if clear
5	**(i)**	$(1-x)^{\frac{1}{2}} = 1 - \frac{1}{2}x - \frac{1}{8}x^2$ ignoring x^3 etc	**B2**	SR Allow B1 for $1 - \frac{1}{2}x + kx^2$, $k \neq -\frac{1}{8}$ or 0
		$(1+x)^{-\frac{1}{2}} = 1 - \frac{1}{2}x + \frac{3}{8}x^2$ ignoring x^3 etc	**B2**	SR Allow B1 for $1 - \frac{1}{2}x + kx^2$, $k \neq \frac{3}{8}$ or 0
		Product = $1 - x + \frac{1}{2}x^2$ ignoring x^3 etc	**B1** **5**	**AG**; with (at least) 1 intermediate step (cf x^2)
	(ii)	$\sqrt{\frac{5}{9}}$ or $\frac{\sqrt{5}}{3}$ seen	**B1**	
		$\frac{37}{49}$ or $1 - \frac{2}{7} + \frac{1}{2}\left(\frac{2}{7}\right)^2$ seen	**B1**	
		$\frac{\sqrt{5}}{3} \approx \frac{37}{49} \Rightarrow \sqrt{5} \approx \frac{111}{49}$	**B1** **3**	**AG**
6	**(i)**	Produce at least 2 of the 3 relevant equations in t and s Solve for t and s $(t, s) = (4, -3)$ AEF Subst $(4, -3)$ into suitable equation(s) & show consistency	**M1** **M1** ***A1** dep***A1** **4**	$1 + 2t = 12 + s$, $3t = -4s$, $-5 + 4t = 5 - 2s$ Either into "3rd" eqn or into all 3 coordinates. N.B. Intersection coords not asked for
	(ii)	Method for finding magnitude of any vector Method for finding scalar product of any 2 vectors Using $\cos\theta = \frac{\mathbf{a.b}}{\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert}$ AEF for the correct 2 vectors 137 (136.8359) or 43.2(43.164…)	***M1** ***M1** dep***M1** **A1** **4**	Expect $\sqrt{29}$ and $\sqrt{21}$ Expect -18 Should be $-\frac{18}{\sqrt{29}\sqrt{21}}$ 2.39 (2.388236..) or 0.753(0.75335…) rads

7	(i)	Correct (calc) method for dealing with $\frac{1}{\sin x}$ or $(\sin x)^{-1}$	**M1**	
		Obtain $-\frac{\cos x}{\sin^2 x}$ or $-(\sin x)^{-2}\cos x$	**A1**	
		Show manipulation to –cosec x cot x (or vice-versa)	**A1**	WWW **AG** with ≥ 1 line intermed working
			3	
	(ii)	Separate variables, $\int(-)\frac{1}{\sin x \tan x}\,dx = \int \cot t\,dt$	**M1**	or $\int \frac{1}{\sin x \tan x}\,dx = \int(-)\cot t\,dt$
		Style: For the M1 to be awarded, dx and dt must appear on correct sides or there must be $\int$ sign on both sides		
		$\int -\text{cosec } x \cot x\,dx = \text{cosec } x$ (+c)	**A1**	or $\int \text{cosec } x \cot x\,dx = -\text{cosec } x$
		$\int \cot t\,dt = \ln \sin t$ or $\ln\lvert\sin t\rvert$ (+c)	**B1**	or $\int -\cot t\,dt = -\ln \sin t$ or $-\ln\lvert\sin t\rvert$
		Subst $(t,x) = \left(\frac{1}{2}\pi, \frac{1}{6}\pi\right)$ into their equation containing 'c'	**M1**	and attempt to find 'c'
		cosec $x = \ln \sin t + 2$ or $\ln\lvert\sin t\rvert + 2$	**A1**	WWW ISW; cosec $\frac{\pi}{6}$ to be changed to 2
			5	
8	(i)	$A(t+1) + B = 2t$	**M1**	Beware: correct values for A and/or B can be ...
		$A = 2$	**A1**	… obtained from a wrong identity
		B = –2	**A1**	Alt method: subst suitable values into given… …expressions
			3	
	(ii)	Attempt to connect dx and dt	**M1**	But not just dx = dt. As **AG**, look carefully.
		dx = t dt s.o.i. AEF	**A1**	
		$x + \sqrt{2x-1} \rightarrow \frac{t^2+1}{2} + t = \frac{(t+1)^2}{2}$ s.o.i.	**B1**	Any wrong working invalidates
		$\int \frac{2t}{(t+1)^2}\,dt$	**A1**	**AG** WWW The 'dt' must be present
			4	
	(iii)	$\int \frac{1}{t+1}\,dt = \ln(t+1)$	**B1**	Or parts $u = 2t, dv = (t+1)^{-2}$ or subst $u = t + 1$
		$\int \frac{1}{(t+1)^2}\,dt = -\frac{1}{t+1}$	**B1**	
		Attempt to change limits (expect 1 & 3) and use f(t)	**M1**	or re-substitute and use 1 and 5 on g(x)
		$\ln 4 - \frac{1}{2}$	**A1**	AEF (like terms amalgamated); if A0 A0 in (i), then final A0
			4	

9	**(i)**	$A: \theta = \frac{1}{2}\pi$ (accept 90°)	**B1**	
		$B: \theta = 2\pi$ (accept 360°)	**B2**	SR If B0 awarded for point *B*, allow B1 SR for any angle s.t. $\sin\theta = 0$
			3	
	(ii)	$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$	**M1**	or $\frac{dy}{d\theta}.\frac{d\theta}{dx}$ Must be used, not just quoted
		$\frac{dx}{d\theta} = 2 + 2\cos 2\theta$	**B1**	
		$2 + 2\cos 2\theta = 4\cos^2\theta$ with $\geq$ 1 line intermed work	***B1**	
		$\frac{dy}{dx} = \frac{4\cos\theta}{2 + 2\cos 2\theta}$ s.o.i.	**A1**	This & previous line are interchangeable
		$= \sec\theta$	dep***A1**	WWW **AG**
			5	
	(iii)	Equating $\sec\theta$ to 2 and producing at least one value of θ	**M1**	degrees or radians
		$(x =) -\frac{2}{3}\pi - \frac{\sqrt{3}}{2}$	**A1**	'Exact' form required
		$(y =) -2\sqrt{3}$	**A1**	'Exact' form required
			3	