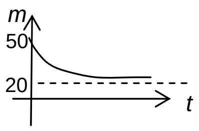
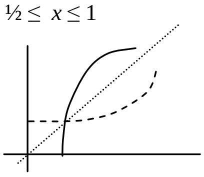
4753 (C3) Methods for Advanced Mathematics

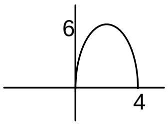
Section A

1 $\lvert 2x-1\rvert \le 3$ $\Rightarrow$ $-3 \le 2x-1 \le 3$ $\Rightarrow$ $-2 \le 2x \le 4$ $\Rightarrow$ $-1 \le x \le 2$ *or* $(2x-1)^2 \le 9$ $\Rightarrow$ $4x^2-4x-8 \le 0$ $\Rightarrow$ $(4)(x+1)(x-2) \le 0$ $\Rightarrow$ $-1 \le x \le 2$	M1 A1 M1 A1 M1 A1 A1 A1 [4]	$2x-1 \le 3$ (or =) $x \le 2$ $2x-1 \ge -3$ (or =) $x \ge -1$ squaring and forming quadratic = 0 (or $\le$) factorising or solving to get $x = -1, 2$ $x \ge -1$ $x \le 2$ (www)
2 Let $u = x$, $dv/dx = e^{3x} \Rightarrow v = e^{3x}/3$ $\Rightarrow$ $\int x e^{3x}\,dx = \frac{1}{3}xe^{3x} - \int \frac{1}{3}e^{3x}.1.dx$ $= \frac{1}{3}xe^{3x} - \frac{1}{9}e^{3x} + c$	M1 A1 A1 B1 [4]	parts with $u = x$, $dv/dx = e^{3x} \Rightarrow v$ $= \frac{1}{3}xe^{3x} - \frac{1}{9}e^{3x}$ $+c$
3 (i) $f(-x) = f(x)$ Symmetrical about Oy.	B1 B1 [2]	
(ii) (A) even (B) neither (C) odd	B1 B1 B1 [3]	
4 Let $u = x^2 + 2 \Rightarrow du = 2x\,dx$ $\int_1^4 \frac{x}{x^2+2}dx = \int_3^{18} \frac{1/2}{u}du$ $= \frac{1}{2}[\ln u]_3^{18}$ $= ½ (\ln 18 - \ln 3)$ $= ½ \ln(18/3)$ $= ½ \ln 6$*	M1 A1 M1 E1 [4]	$\int \frac{1/2}{u}du$ or $k \ln (x^2 + 1)$ ½ ln u or ½ ln(x^2 + 2) substituting correct limits (u or x) must show working for ln 6
5 $y = x^2 \ln x$ $\Rightarrow$ $\frac{dy}{dx} = x^2.\frac{1}{x} + 2x\ln x$ $= x + 2x\ln x$ $dy/dx = 0$ when $x + 2x\ln x = 0$ $\Rightarrow$ $x(1 + 2\ln x) = 0$ $\Rightarrow$ $\ln x = -½$ $\Rightarrow$ $x = e^{-½} = 1/\sqrt{e}$ *	M1 B1 A1 M1 M1 E1 [6]	product rule $d/dx (\ln x) = 1/x$ soi oe their deriv = 0 or attempt to verify $\ln x = -½ \Rightarrow x = e^{-½}$ or $\ln(1/\sqrt{e}) = -½$

6(i) Initial mass = 20 + 30 e^0 = 50 grams Long term mass = 20 grams	M1A1 B1 [3]	
(ii) 30 = 20 + 30 $e^{-0.1t}$ ⇒ $e^{-0.1t}$ = 1/3 ⇒ –0.1t = ln (1/3) = –1.0986… ⇒ t = 11.0 mins	M1 M1 A1 [3]	 anti-logging correctly 11, 11.0, 10.99, 10.986 (not more than 3 d.p)
(iii) 	B1 B1 [2]	correct shape through (0, 50) – ignore negative values of t → 20 as $t \to \infty$
7 $x^2 + xy + y^2 = 12$ ⇒ $2x + x\frac{dy}{dx} + y + 2y\frac{dy}{dx} = 0$ ⇒ $(x+2y)\frac{dy}{dx} = -2x - y$ ⇒ $\frac{dy}{dx} = -\frac{2x+y}{(x+2y)}$	M1 B1 A1 M1 A1 [5]	Implicit differentiation $x\frac{dy}{dx} + y$ correct equation collecting terms in dy/dx and factorising oe cao

Section B

8(i) $y = 1/(1+\cos\pi/3) = 2/3$.	B1 [1]	or 0.67 or better
(ii) $f'(x) = -1(1+\cos x)^{-2}.-\sin x$ $= \frac{\sin x}{(1+\cos x)^2}$ When $x = \pi/3$, $f'(x) = \frac{\sin(\pi/3)}{(1+\cos(\pi/3))^2}$ $= \frac{\sqrt{3}/2}{(1\frac{1}{2})^2} = \frac{\sqrt{3}}{2}\times\frac{4}{9} = \frac{2\sqrt{3}}{9}$	M1 B1 A1 M1 A1 [5]	chain rule or quotient rule d/dx (cos x) = –sin x soi correct expression substituting $x = \pi/3$ oe or 0.38 or better. (0.385, 0.3849)
(iii) deriv = $\frac{(1+\cos x)\cos x - \sin x.(-\sin x)}{(1+\cos x)^2}$ $= \frac{\cos x + \cos^2 x + \sin^2 x}{(1+\cos x)^2}$ $= \frac{\cos x + 1}{(1+\cos x)^2}$ $= \frac{1}{1+\cos x}$ * Area $= \int_0^{\pi/3} \frac{1}{1+\cos x}dx$ $= \left[\frac{\sin x}{1+\cos x}\right]_0^{\pi/3}$ $= \frac{\sin\pi/3}{1+\cos\pi/3}(-0)$ $= \frac{\sqrt{3}}{2}\times\frac{2}{3} = \frac{\sqrt{3}}{3}$	M1 A1 M1dep E1 B1 M1 A1 cao [7]	Quotient or product rule – condone uv' – u'v for M1 correct expression $\cos^2 x + \sin^2 x = 1$ used dep M1 www substituting limits or $1/\sqrt{3}$ - must be exact
(iv) $y = 1/(1 + \cos x)$ $x \leftrightarrow y$ $x = 1/(1 + \cos y)$ $\Rightarrow$ $1 + \cos y = 1/x$ $\Rightarrow$ $\cos y = 1/x - 1$ $\Rightarrow$ $y = \arccos(1/x - 1)$ * Domain is $\frac{1}{2} \le x \le 1$ 	M1 A1 E1 B1 B1 [5]	attempt to invert equation www reasonable reflection in $y = x$

9 (i) $y=\sqrt{4-x^2}$ $\Rightarrow$ $y^2 = 4 - x^2$ $\Rightarrow$ $x^2 + y^2 = 4$ which is equation of a circle centre O radius 2 Square root does not give negative values, so this is only a semi-circle.	M1 A1 B1 [3]	squaring $x^2 + y^2 = 4$ + comment (correct) oe, e.g. f is a function and therefore single valued
(ii) (*A*) Grad of OP = *b*/*a* $\Rightarrow$ grad of tangent = $-\frac{a}{b}$ (*B*) $f'(x)=\frac{1}{2}(4-x^2)^{-1/2}.(-2x)$ $= -\frac{x}{\sqrt{4-x^2}}$ $\Rightarrow$ $f'(a)=-\frac{a}{\sqrt{4-a^2}}$ (*C*) $b = \sqrt{(4 - a^2)}$ so $f'(a)=-\frac{a}{b}$ as before	M1 A1 M1 A1 B1 E1 [6]	 chain rule or implicit differentiation oe substituting *a* into their f'(*x*)
(iii) Translation through $\begin{pmatrix}2\\0\end{pmatrix}$ followed by stretch scale factor 3 in *y*-direction 	M1 A1 M1 A1 M1 A1 [6]	Translation in *x*-direction through $\begin{pmatrix}2\\0\end{pmatrix}$ or 2 to right ('shift', 'move' M1 A0) $\begin{pmatrix}2\\0\end{pmatrix}$ alone is SC1 stretch in *y*-direction (condone *y* 'axis') (scale) factor 3 elliptical (or circular) shape through (0, 0) and (4, 0) and (2, 6) (soi) –1 if whole ellipse shown
(iv) $y = 3f(x - 2)$ $= 3\sqrt{(4 - (x - 2)^2)}$ $= 3\sqrt{(4 - x^2 + 4x - 4)}$ $= 3\sqrt{(4x - x^2)}$ $\Rightarrow$ $y^2 = 9(4x - x^2)$ $\Rightarrow$ $9x^2 + y^2 = 36x$ *	M1 A1 E1 [3]	or substituting $3\sqrt{(4 - (x - 2)^2)}$ oe for *y* in $9x^2 + y^2$ $4x - x^2$ www