

Mathematics

Advanced GCE

Unit **4724**: Core Mathematics 4

Mark Scheme for June 2011

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- 1 Attempt to factorise **both** numerator & denominator M1 completely or partially
 Num = e.g. $(x^2 - 1)(x^2 - 9)$ or $(x^2 - 2x - 3)(x^2 + 2x - 3)$ B1 or $(x - 3)(x + 3)(x - 1)(x + 1)$
 Denominator = e.g. $(x^2 - 2x - 3)(x + 5)(x + 3)$ B1 or $(x - 3)(x + 1)(x + 5)(x + 3)$
 $\frac{x-1}{x+5}$ or $1 - \frac{6}{x+5}$ WWW A1 4 ISW but not if any further 'cancellation'

Alternative start, attempting long division

Expand denom as quartic & attempt to divide $\frac{\text{numerator}}{\text{denominator}}$ M1 but not divide $\frac{\text{denominator}}{\text{numerator}}$

Obtain quotient = 1 & remainder = $-6x^3 - 6x^2 + 54x + 54$ B1

Final B1 A1 available as before

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- 2 $2^2 + (-3)^2 + (\sqrt{12})^2$ soi e.g. 25 or 5 M1 Allow $2^2 - 3^2 + \sqrt{12}^2$
 5 A1 May be implied by 5 or 1/5 in final answer

$\frac{1}{5} \begin{pmatrix} 2 \\ -3 \\ \sqrt{12} \end{pmatrix}$ or $\begin{pmatrix} \frac{2}{5} \\ -\frac{3}{5} \\ \frac{\sqrt{12}}{5} \end{pmatrix}$ AEF $\sqrt{A1}$ 3 FT their '5'. Accept $-\frac{1}{5} \begin{pmatrix} \\ \\ \phantom{\sqrt{12}} \end{pmatrix}$ or $\frac{1}{\pm 5} \begin{pmatrix} \\ \\ \phantom{\sqrt{12}} \end{pmatrix}$

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- 3 (i) The words quotient and remainder need not be explicit

Long division For leading term $3x$ in quotient B1

Suff evidence of div process ($3x$, mult back, attempt sub) M1

(Quotient) = $3x - 1$ A1

(Remainder) = x **AG** A1 4 No wrong working, partic on penult line

Identity $3x^3 - x^2 + 10x - 3 = Q(x^2 + 3) + R$ *M1

$Q = ax + b, R = cx + d$ & attempt at least 2 operations dep*M1 If $a = 3$, this \Rightarrow 1 operation

$a = 3, b = -1$ A1

$c = 1, d = 0$ A1

Inspection $3x^3 - x^2 + 10x - 3 = (x^2 + 3)(3x - 1) + x$ B2 or state quotient = $3x - 1$

Clear demonstration of LHS = RHS B2

- (ii) Change integrand to 'their (i) quotient' + $\frac{x}{x^2 + 3}$ M1

Correct FT integration of 'their (i) quotient' $\sqrt{A1}$

$\int \frac{x}{x^2 + 3} dx = \frac{1}{2} \ln(x^2 + 3)$ A1

Exact value of integral = $\frac{1}{2} + \frac{1}{2} \ln 4 - \frac{1}{2} \ln 3$ AEF ISW A1 4 Answer as decimal value (only) \rightarrow A0

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4	<u>Indefinite integral</u> Attempt to connect dx and $d\theta$ Denominator $(1-9x^2)^{\frac{3}{2}}$ becomes $\cos^3\theta$ Reduce original integral to $\frac{1}{3} \int \frac{1}{\cos^2\theta} d\theta$ Change $\int \frac{1}{\cos^2\theta} d\theta$ to $\tan\theta$ Use <u>appropriate</u> limits for θ (allow degrees) or x $\frac{\sqrt{3}}{9}$ AEF, exact answer required, ISW	M1 Incl $\frac{dx}{d\theta} =, \frac{d\theta}{dx} =, dx = \dots d\theta$; not $dx = d\theta$ B1 A1 May be implied, seen only as $\frac{1}{3} \int \sec^2\theta d\theta$ B1 Ignore $\frac{1}{3}$ at this stage M1 Integration need not be accurate A1 6
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5 (i)	Attempt to set up 3 equations $(s,t) = (-1,4)$ or $(-1,-3)$ or $(-\frac{10}{3}, -\frac{2}{3})$ Show clear contradiction e.g. $3 \neq -4, 4 \neq -3, -6 \neq 1$ dep* A1 3 Allow ✓ unsimpl contradictions. No ISW. <u>SC</u> If $s = -\frac{10}{3}$ found from 2 nd & 3 rd eqns and contradiction shown in 1 st eqn, all 3 marks may be awarded.	M1 of type $4 + 3s = 1, 6 + 2s = t, 4 + s = -t$ *A1 or $s = -1$ & $-\frac{10}{3}$ <u>or</u> $t =$ two of $(4, -3, -\frac{2}{3})$
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(ii)	Work with $\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$ Clear method for scalar product of any 2 vectors Clear method for modulus of any vector 79.1 ^(e) or better (79.1066..) 1.38 (rad) (1.38067..) ISW	M1 M1 M1 A1 4 (From $\frac{1}{\sqrt{14} \cdot \sqrt{2}}$)
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(iii)	Use $\begin{pmatrix} 4+3s \\ 6+2s \\ 4+s \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} = 0$ Obtain $s = -2$ A is $\begin{pmatrix} -2 \\ 2 \\ 2 \end{pmatrix}$ or $-2\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$ final answer	M1 A1 from $12 + 9s + 12 + 4s + 4 + s = 0$ B1 3 Accept $(-2, 2, 2)$
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6 $(1+ax)^{1/2} = 1 + \frac{1}{2}ax + \dots + \frac{\frac{1}{2} \cdot \frac{-1}{2}}{2} (ax)^2$ B1, B1 N.B. third term = $-\frac{1}{8}a^2x^2$

Change $(4-x)^{-1/2}$ into $k(1-\frac{x}{4})^{-1/2}$, where k is likely to be $\frac{1}{2}/2/4/-2$, & work out expansion of $(1-\frac{x}{4})^{-1/2}$

$(1-\frac{x}{4})^{-1/2} = 1 + \frac{1}{8}x + \dots + \frac{\frac{-1}{2} \cdot \frac{-3}{2}}{2} (\frac{-x}{4})^2$ B1, B1 N.B. third term = $\frac{3}{128}x^2$

OR Change $\{4-x\}^{1/2}$ into $l(1-\frac{x}{4})^{1/2}$, where l is likely to be $\frac{1}{2}/2/4/-2$, & work out expansion of $(1-\frac{x}{4})^{1/2}$

$(1-\frac{x}{4})^{1/2} = 1 - \frac{1}{8}x - \frac{1}{128}x^2$ B1 (for all 3 terms simplified)

$k = \frac{1}{2}$ (with possibility of M1 + A1 + A1 to follow) B1 $l = 2$ (with no further marks available)

Multiply $(1+ax)^{1/2}$ by $(4-x)^{-1/2}$ or $(1-\frac{x}{4})^{-1/2}$ M1 Ignore irrelevant products

The required three terms (with/without x^2) identified as

$-\frac{1}{16}a^2 + \frac{1}{32}a + \frac{3}{256}$ or $\frac{-16a^2+8a+3}{256}$ AEF ISW A1+A1 8 A1 for one correct term + A1 for other two

SC B1 for $\frac{1}{4}(1-\frac{x}{4})^{-1}$; B1 for $(1-\frac{x}{4})^{-1} = 1 + \frac{x}{4} + \frac{x^2}{16}$; M1 for multiplying $(1+ax)$ by their $(4-x)^{-1}$.

If result is $p+qx+rx^2$, then to find $(p+qx+rx^2)^{1/2}$ award B1 for $p^{1/2}(\dots)$,

B1 correct 1st & 2nd terms of expansion, B1 correct 3rd term; A1, A1 as before, for correct answers.

8

7 Attempt to sep variables in format $\int py^2 (dy) = \int \frac{q}{x+2} (dx)$ M1 where constants p and/or q may be wrong

Either y^3 & $\ln(x+2)$ or $\frac{1}{3}y^3$ & $\frac{1}{3}\ln(x+2)$ A1+A1 Accept $\frac{1}{3}\ln(3x+6)$ for $\frac{1}{3}\ln(x+2)$ & $|$ for ()

If indefinite integrals are being used (most likely scenario)

Substitute $x=1, y=2$ into an eqn containing '+const' M1

Sub $y=1.5$ and their value of 'const' & solve for x or q M1

x or $q = -1.97$ only A2

[SC x or $q = -1.970$ or -1.971 or -1.9705 or -1.9706 A1] 7

If definite integrals are used (less likely scenario)

Use $\int_{1.5}^2 \dots dy = \int_q^1 \dots dx$ where 2 corresponds with 1..... M2 & 1.5 corresp with q (at top/bottom or v.v.)

Then A2 or SC A1 as above

Use $\int_{1.5}^2 \dots dy = \int_1^q \dots dx$ where 2 corresponds with q M1 & 1.5 corresp with 1 (at top/bottom or v.v.)

Then A1 for 1.97 only

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8 Cartesian equation may be used in parts (i) - (iii) and corresponding marks awarded

(i) Sub parametric eqns into $y = 3x$ & produce $t = -2$

OR sub $t = -2$ into para eqs, obtain $(-1, -3)$ & state $y = 3x$

OR other similar methods producing (or verifying) $t = -2$ B1

Value of t at other point is 2 B1 2 $t = \pm 2$ is sufficient for B1+B1

(ii) Use (not just quote) $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$ M1

$= -(t+1)^2$ A1 or $\frac{-1}{x^2}$ or $\frac{-(2+y)}{x}$

Attempt to use $-\frac{1}{\frac{dy}{dx}}$ for gradient of normal M1

Gradient normal = 1 cao A1

Subst $t = -2$ into the parametric eqns. M1 to find pt at which normal is drawn

Produce $y = x - 2$ as equation of the normal WWW A1 6 'A' marks in (ii) are dep on prev 'A'

(iii) Substitute the parametric values into their eqn of normal M1

Produce $t = 0$ as final answer cao A1 2 This is dep on final A1 in (ii)

N.B. If $y = x - 2$ is found fortuitously in (ii) (& \therefore given A0 in (ii)), you must award A0 here in (iii).

(iv) Attempt to eliminate t from the parametric equations M1

Produce any correct equation A1 e.g. $x = \frac{1}{y+2}$

Produce $y = \frac{1}{x} - 2$ or $y = \frac{1-2x}{x}$ ISW A1 3 Must be seen in (iv)

{N.B. Candidate producing only $y = \frac{1}{x} - 2$ is awarded both A1 marks.}

- 9 (i) Treat $x \ln x$ as a product M1 If $\int \ln x$, use parts $u = \ln x$, $dv = 1$
- Obtain $x \cdot \frac{1}{x} + \ln x$ A1 $x \ln x - \int 1 dx = x \ln x - x$
- Show $x \cdot \frac{1}{x} + \ln x - 1 = \ln x$ WWW AG A1 3 And state given result

(ii)(a) Part (a) is mainly based on the indef integral $\int (\ln x)^2 dx$

[A candidate stating e.g. $\int (\ln x)^2 dx = \int 2 \ln x dx$ or $= \int (\ln x - x)^2 dx$ is awarded 0 for (ii)(a)]

Correct use of $\int \ln x dx = x \ln x - x$ anywhere in this part B1 Quoted from (i) or derived

Use integ by parts on $\int (\ln x)^2 dx$ with $u = \ln x$, $dv = \ln x$ M1 or $u = (\ln x)^2$, $dv = 1$

[For 'integration by parts, candidates must get to a 1st stage with format $f(x) + / - \int g(x) dx$]

1st stage = $\ln x(x \ln x - x) - \int \frac{1}{x}(x \ln x - x) dx$ soi A1 $x(\ln x)^2 - \int x \cdot \frac{2}{x} \ln x dx$

2nd stage = $x(\ln x)^2 - 2x \ln x + 2x$ AEF (unsimplified) A1

\therefore Value of definite integral between 1 & e = $e - 2$ cao A1 Use limits on 2nd stage & produce cao

Volume = $\pi(e - 2)$ ISW A1 6 Answer as decimal value (only) \rightarrow A0

Alternative method when subst. $u = \ln x$ used

Attempt to connect dx and du M1

Becomes $\int u^2 e^u du$ A1

First stage $u^2 e^u - \int 2u e^u du$ A1

Third stage $(u^2 - 2u + 2)e^u$ A1

Final A1 A1 available as before

(b) Indication that reqd vol = vol cylinder – vol inner solid M1

Clear demonstration of either vol of cylinder being πe^2
(including reason for height = $\ln e$) or rotation of $x = e$

about the y-axis (including upper limit of $y = \ln e$) A1 Could appear as $\pi \int_0^1 e^2 dy$

$(\pi) \int x^2 dy = (\pi) \int e^{2y} dy$ B1

$\frac{\pi(e^2 + 1)}{2}$ or 13.2 or 13.18 or better B1 4 May be from graphical calculator

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Possible helpful points

1. M is Method; does the candidate know what he/she should be doing? It does not ask how accurate it is..
e.g. in Qu.4, a candidate saying $\frac{dx}{d\theta} = -\frac{1}{3} \cos \theta$ is awarded M1.
2. When checking if decimal places are acceptable, accept both rounding & truncation.
3. In general we ISW unless otherwise stated.
4. The symbol \surd is sometimes used to indicate 'follow-through' in this scheme.

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