GCE

Further Mathematics B (MEI)

Y436/01: Further pure with technology

Advanced GCE

Mark Scheme for June 2019

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

Text Instructions

Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This indicates that the instruction **In this question you must show detailed reasoning** appears in the question.

Subject-specific Marking Instructions for A Level Mathematics B (MEI)

a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and applied in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.) We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so. When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination. There is no penalty for using a wrong value for g. E marks will be lost except when results agree to the accuracy required in the question.

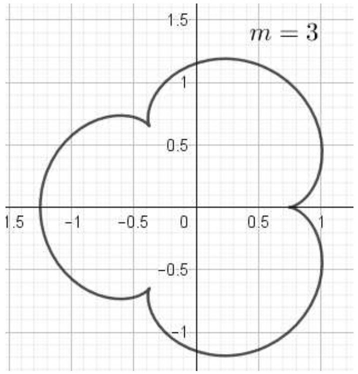
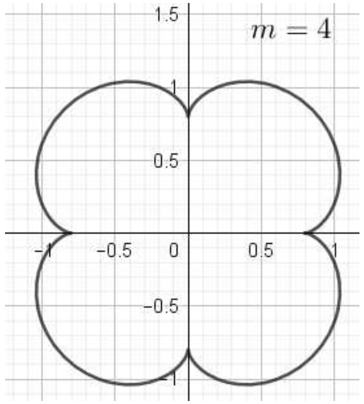
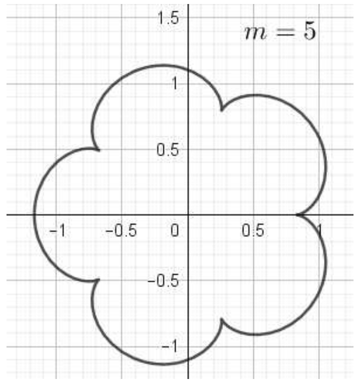
g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. 'Fresh starts' will not affect an earlier decision about a misread. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a graphical calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question	Answer	Mark	AOs		Guidance

Question			Answer	Mark	AOs		Guidance
1	(a)	(i)		B1	1.1b	Shape, position of cusps in correct quadrants or on axes, points on axes.	
				B1	1.1b	Shape, position of cusps on axes.	
				B1 [3]	1.1b	Shape, position of cusps in correct quadrants or on axes, points on axes.	

Question			Answer	Mark	AOs		Guidance
		(ii)	e.g. bounded e.g. closed e.g. reflective symmetry in the x-axis. e.g. rotational symmetry e.g. cusps	**B1** **[1]**	**1.2**	B1 for one common feature FT provided at least 1 graph correct in 1(a)(i)	
		(iii)	Reflective symmetry in the y axis OE	**B1** **[1]**	**1.2**	B1 for one feature unique to m = 4	

Question			Answer	Mark	AOs		Guidance
	(b)	(i)	$\frac{dx}{dt}=\sin((m+1)t)-\sin(t)$ $\frac{dy}{dt}=\cos(t)-\cos((m+1)t)$ $\frac{dx}{dt}=0$ $\Rightarrow \sin((m+1)t)-\sin(t)=0$	M1	1.1b	For evidence of solving $\frac{dx}{dt}=0$	
			$\Rightarrow t=\frac{2k\pi}{m}$ or $t=\frac{(2l+1)\pi}{m+2}$ where k,l are integers. $\Rightarrow t=0,\frac{2\pi}{m},\frac{4\pi}{m},\ldots\frac{2(m-1)\pi}{m}$ or $\frac{\pi}{m+2},\frac{3\pi}{m+2},\frac{5\pi}{m+2},\ldots,\frac{(2m+3)\pi}{m+2}$	M1	1.1b	At least one from each group.	
			$\frac{dy}{dt}=0$ $\Rightarrow \cos(t)-\cos((m+1)t)=0$ $\Rightarrow t=\frac{2k\pi}{m}$ or $t=\frac{2l\pi}{m+2}$ where k,l are integers. $\Rightarrow t=0,\frac{2\pi}{m},\frac{4\pi}{m},\ldots\frac{2(m-1)\pi}{m}$ or $\frac{2\pi}{m+2},\frac{4\pi}{m+2},\frac{6\pi}{m+2},\ldots,\frac{2(m+1)\pi}{m+2}$	M1	2.2b	At least one from each group	
			Values of t for which $\frac{dx}{dt}=0$ and $\frac{dy}{dt}\neq 0$ are $\frac{\pi}{m+2},\frac{3\pi}{m+2},\frac{5\pi}{m+2},\ldots,\frac{(2m+3)\pi}{m+2}$	A1 **[4]**		CAO, must include all values in correct range.	

Question			Answer	Mark	AOs		Guidance
		(ii)	The tangent to the curve is vertical at points corresponding to such values of t.	**B1** **[1]**	**1.2**		

Question			Answer	Mark	AOs		Guidance
	(c)	(i)	$\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} \cos(t) \\ \sin(t) \end{pmatrix} - \frac{1}{m+1}\begin{pmatrix} \cos((m+1)t) \\ \sin((m+1)t) \end{pmatrix}$ $\left\lvert\begin{pmatrix} \cos(t) \\ \sin(t) \end{pmatrix}\right\rvert = 1, \left\lvert -\frac{1}{m+1}\begin{pmatrix} \cos((m+1)t) \\ \sin((m+1)t) \end{pmatrix}\right\rvert = \frac{1}{m+1}$.	**M1**	**1.1b**	Consideration of the magnitude of the vector(s) involved or equivalent words	
			Considering vector addition and these magnitudes, $\begin{pmatrix} x(t) \\ y(t) \end{pmatrix}$ is at most a distance of $1+\frac{1}{m+1}$ from the origin and at least a distance of $1-\frac{1}{m+1}$ from the origin.	**E1**	**2.4**	Any reasonable argument regarding the specific distance bounds.	
			Alternative version				
			$[r =]\ \sqrt{x(t)^2 + y(t)^2}$ $= \sqrt{\left(\cos(t) - \frac{\cos((m+1)t)}{m+1}\right)^2 + \left(\sin(t) - \frac{\sin((m+1)t)}{m+1}\right)^2}$ $= \sqrt{\frac{m^2 - 2m\cos(mt) + 2m - 2\cos(mt) + 2}{m^2 + 2m + 1}}$	**M1**	**1.1b**	Or $\sqrt{1 + \frac{1}{(m+1)^2} - \frac{2\cos(mt)}{m+1}}$ Find expression for distance to the orgin and simplify to a suitable form. Square roots may be omitted.	
			$= \sqrt{\frac{m^2}{(m+1)^2}} = 1 - \frac{1}{m+1}$ when $\cos(mt) = 1$ and $= \sqrt{\frac{(m+2)^2}{(m+1)^2}} = 1 + \frac{1}{m+1}$ when $\cos(mt) = -1$	**A1**	**2.4**	Considering range of cos or use of suitable values of t (e.g. 0 and $\frac{\pi}{m}$). SC1 for max or min correctly found.	

Question			Answer	Mark	AOs		Guidance
		(ii)	Area of circles with radius r_1 where $r_1 = 1 - \frac{1}{m+1} = \frac{m}{m+1}$ and radius r_2 where $r_2 = 1 + \frac{1}{m+1} = \frac{m+2}{m+1}$.	**B1** **[1]**	**1.1b**		
		(iii)	Both $\frac{m^2\pi}{(m+1)^2}$ and $\frac{(m+2)^2\pi}{(m+1)^2}$ tend to π as m tends to infinity. Hence the result by a sandwich of limits.	**B1** **[1]**	**1.1b**	No need for a formal argument	

Question			Answer	Mark	AOs		Guidance
	(d)		$\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}$ $=\sqrt{(\sin((m+1)t)-\sin(t))^2+(\cos(t)-\cos((m+1)t)^2}$	M1	1.1a	Evidence of correct application of formula involving expressions for $\frac{dx}{dt}$ and $\frac{dy}{dt}$	
			$=\sqrt{2-2\cos(mt)}$	M1	1.1b	Suitable intermediate step	
			$=2\left\|\sin\left(\frac{mt}{2}\right)\right\|$ The length of the curve is therefore $\int_0^{2\pi} 2\left\|\sin\left(\frac{mt}{2}\right)\right\|dt$ $=2\int_0^{2\pi}\left\|\sin\left(\frac{mt}{2}\right)\right\|dt$	M1	3.1a	For obtaining an expression suitable for integration (previous M1 implied), condone lack of modulus symbol	
			$=2m\int_0^{2\pi/m}\sin\left(\frac{mt}{2}\right)dt$ (by looking at the graph of the function on previous line) $=2m\left[\frac{2}{m}\cos\left(\frac{mt}{2}\right)\right]_0^{2\pi/m}$	A1	3.1a	Setting out and splitting the integral into m equal parts	
			$=8$	A1	1.1b	Correct value for integral.	
			This is independent of m.	B1 [6]	3.2a	Conclusion about this being Independent of m.	

Question			Answer	Mark	AOs		Guidance
2	(a)		Suppose $x^2 - 2y^2 = 1$ where x and y are integers.				
			Then $x^2 = 2y^2 + 1$ and so x^2 is odd and therefore x is odd.	B1	2.1	For x^2 is odd, conclusion (allow suitable mod 2 argument)	
			So $x = 2t + 1$ for some integer t and $2y^2 = (2t + 1)^2 - 1 = 4t^2 + 4t$.	M1	2.1	For $2y^2$ is a multiple of 4	
			Thefore $y^2 = 2t^2 + 2t$ is even and so y is even.	E1 [3]	2.1	For y^2 is even and conclusion.	
			Alternative version				
			Suppose $x^2 - 2y^2 = 1$ where x and y are integers.				
			Then $x^2 = 2y^2 + 1$ and so x^2 is odd and therefore x is odd.	B1	2.1	For x^2 is odd, conclusion (allow suitable mod 2 argument)	
			So $x \equiv 1 \pmod 4$ or $x \equiv 3 \pmod 4$ $\Rightarrow x^2 \equiv 1 \pmod 4$ $\Rightarrow x^2 - 1 \equiv 0 \pmod 4$ $\Rightarrow 2y^2 \equiv 0 \pmod 4$	M1	2.1	For $2y^2 \equiv 0 \pmod 4$.	
			$\Rightarrow y^2$ is even $\Rightarrow$ y is even	E1 [3]	2.1	For y^2 is even and conclusion.	

Question			Answer	Mark	AOs		Guidance
	(b)		Example code for Python **for x in range(1,s+1):** **for y in range(1,s+1):** **if x*x-2*y*y==1:** **print(x,y)**	**M1** **M1** **A1** **[3]**	**3.3** **2.1** **2.5**	Pseudo code accepted, condone lack of syntax, give reasonable BOD on possible transcription errors and consider a correct answer to 2(c) as evidence of a correct prog. At least one correct loop Appropriate conditional statement Complete prog	
	(c)		x = 3 y = 2 x = 17 y = 12 x = 99 y = 70 x = 577 y = 408 cao	**B1** **[1]**	**1.1b**	Set s to 600 in the above. Ignore x = 1, y = 0 if seen.	

Question			Answer	Mark	AOs		Guidance
	(d)		For the smallest solution $3^2 - 2\times 2^2$ [=1] $\Rightarrow \left(3+2\sqrt{2}\right)\left(3-2\sqrt{2}\right)$ [=1]	**M1**	**2.1a**	Substituting x = 3, y = 2 (or square and simplify the given algebraic expression)	
			$\Rightarrow \left(3+2\sqrt{2}\right)^2\left(3-2\sqrt{2}\right)^2 = 1$ $\Rightarrow \left(17+12\sqrt{2}\right)\left(17-12\sqrt{2}\right) = 1$	**M1**	**2.1a**	Squaring and expanding/simplifying (or substitute x = 3, y = 2 if previous step done algebraically)	
			So the next solution is x = 17, y = 12	**A1**	**2.1a**	Stating that this means x = 17, y = 12 is a solution or rearranging to $17^2 - 2 \times 12^2 = 1$	
			By cubing and taking the fourth power of the second line above the other solutions are found.	**E1** **[4]**	**2.4**		
	(e)		$\left(3+2\sqrt{2}\right)^n\left(3-2\sqrt{2}\right)^n = 1$ simplified gives a solution for any positive integer n. So the equations has infinitely many positive integer solutions x,y	**B1** **[1]**	**2.2a**	Infinitely many solutions.	

Question			Answer	Mark	AOs		Guidance

	(f)		**Example code for Python**
			
			for m in range(1,s+1):
			    for n in range(1,s+1):
			        if m*(m+1)==2*n*n: (oe)
			            print(m,n)
				M1
				
				A1
				
				[2]
					2.1
					
					2.5
						Pseudo code accepted, condone lack of syntax, give reasonable BOD on possible transcription errors and consider a correct answer to 2(g) as evidence of a correct prog.
						
						At least one correct loop and conditional statement
						
						Complete program	

	(g)		m = 1 n = 1 m = 8 n = 6 m = 49 n = 35 m = 288 n = 204 cao	**B1** **[1]**	**1.1b**	Set s = 300 in the above. Ignore m = 0, n = 0 if seen.	

Question			Answer	Mark	AOs	Guidance
	(h)		Comparing answers to (c) and (g) suggests that, if x, y is a solution of $x^2-2y^2=1$ in which x, y are positive integers, then $m=\frac{x-1}{2}, n=\frac{y}{2}$ satisfy $T_m = n^2$.	**M1***	**3.1a**	Relationship between pairs in (c) and pairs in (g). Allow x = 2m + 1, y = 2n
			Since x is odd, m is an integer. Since y is even, n is an integer.	**M1**	**2.1**	Seen in part (a).
			$T_m = \frac{\left(\frac{x-1}{2}\right)\left(\frac{x+1}{2}\right)}{2} = \frac{x^2-1}{8}$ $n^2 = \frac{y^2}{4}$	**M1**	**2.4**	Substitutions into T_m and n^2. Allow substitution of x = 2m + 1, y = 2n into $x^2-2y^2=1$.
			Since $x^2-2y^2=1$, we have $8T_m = 8n^2$ and so $T_m = n^2$ as required.	**M1**	**2.1**	Verification that $x^2-2y^2=1$ leads to $T_m = n^2$.
			Since there are infinitely many positive integer solutions to $x^2-2y^2=1$ (part (v)) there are infinitely many triangular numbers which are square.	**B1(dep*)** **[5]**	**3.2a**	Reference to part (e). Do not allow if only converse argument seen (i.e. a solution of $T_m = n^2$ gives a solution of $x^2 - 2y^2 = 1$).

3	**(a)**	i	If a = -1 the isoclines satisfy $1-\frac{y}{x}=m \Rightarrow y=(1-m)x$. These are straight lines through the origin.	**M1** **B1** **[2]**	**1.1b** **1.2**	Substituting a = -1 and rearranging into a suitable form.	
		ii	If a = 0 the isoclines satisfy $1-y=m \Rightarrow y=1-m$. These are horizontal lines.	**M1** **B1** **[2]**	**1.1b** **1.2**	Substituting a = 0 and rearranging into a suitable form.	
		iii	If a = 1 the isoclines satisfy $1-xy=m \Rightarrow y=\frac{1-m}{x}$. This is an stretch/enlargement of $y=\frac{1}{x}$ except when m = 1 when it is the line y = 0) OR Hyperbolae with the x and y axes as asymptotes (except when m = 1 when it is the line y = 0).	**M1** **B1** **[2]**	**1.1b** **1.2**	Substituting a = 1 and rearranging into a suitable form. Any reasonable geometrical description allowed. Condone lack of consideration of m = 1.	
	(b)	i	Solution is $y=1+(b-1)e^{-x}$.	**B1** **[1]**	**1.1a**		

		ii	Asymptote is y = 1.	**B1** **[1]**	**1.2**		
	(c)	i	Solution is $y=\frac{x^2+2cd-c^2}{2x}$	**B1** **[1]**	**1.1a**		
		ii	$\frac{dy}{dx}=\frac{x^2+c^2-2cd}{2x^2}$	**M1**	**1.1b**	$[x \neq 0]$ Alternatively, using the differential equation. $\frac{dy}{dx}=0 \Rightarrow 1-\frac{y}{x}=0 \Rightarrow y=x$	
			So $\frac{dy}{dx}=0 \Leftrightarrow x^2=c(2d-c)$ o.e	**M1**	**1.1b**	Then substituting into y equation gives $x^2=c(2d-c)$	Need not be factorised.
			$\frac{dy}{dx}=0$ has a solution x if and only if $c(2d-c)>0$.	**M1**	**3.1a**	Condone use of $\geq$.	
			This is if and only if either ($c>0$ and $2d>c$) or ($c<0$ and $2d<c$). cao	**A1** **[4]**	**3.2a**	Or equivalent description of region in (c, d) plane. Must be strict inequalities since $x \neq 0$. SC 1 for correct answer unsupported. Solution possible involving identifying $y=\frac{x}{2}$ as common asymptote to solutions in (c) (i). Conclusion then needs to be fully explained. Identifying the common asymptote with correct conclusion only awarded SC 2.	

	(d)	i	**A2 = 0, B2 = 1.5, H1 = 0.05** **C2 = $H$1*(1-A2*B2)** **D2 =$H$1*(1-(A2+$H$1/2)*(B2+C2/2))** **E2 =$H$1*(1-(A2+$H$1/2)*(B2+D2/2))** **F2=$H$1*(1-(A2+$H$1)*(B2+E2))** **A3 =A2+$H$1** **B3 = B2 + (1/6)(C2 + 2*D2+2*E2+F2)**	**M1** **M1** **M1** **M1** **[4]**	**1.1** **3.1a** **3.1a** **2.5**	Give reasonable BOD on possible transcription errors and consider a correct answer to 3(d)(ii) as evidence of correct formulae in the spreadsheet. Columns for x and y or equivalent Columns for k_1 and k_2 or equivalent Columns for k_3 and k_4 or equivalent Formulae for x_{n+1} and y_{n+1}	

ii

xn	yn	k1	k2	k3	k4
0	1.5	0.05	0.04809375	0.04809494	0.046129763
0.05	1.54808452	0.04612979	0.04410819	0.04411198	0.042039017
0.1	1.59218605	0.04203907	0.03991747	0.0399241	0.037759174
0.15	1.63209961	0.03775925	0.03555393	0.03556358	0.033123368
0.2	1.66765255	0.03332347	0.03105146	0.03106424	0.02876604
0.25	1.69870604	0.02876617	0.02644502	0.02646098	0.024122495
0.3	1.72515615	0.02412266	0.02177022	0.02178933	0.019428454
0.35	1.74693452	0.01942865	0.01706283	0.01708501	0.014719609
0.4	1.76400851	0.01471983	0.01235842	0.01238351	0.010031179
0.45	1.77638099	0.01003143	0.00769183	0.00771961	0.005397485
0.5	1.78408962	0.00539776	0.0030968	0.003127	0.000851543
0.55	1.78720577	0.00085184	-0.0013944	-0.0013621	-0.00357531
0.6	1.78583302	-0.003575	-0.0057514	-0.0057174	-0.007853757
0.65	1.78010528	-0.0078534	-0.009946	-0.0099107	-0.01195681
0.7	1.77018466	-0.0119565	-0.0139525	-0.0139163	-0.015860063
0.75	1.75625898	-0.0158597	-0.0177478	-0.0177112	-0.019541912
0.8	1.73853907	-0.0195416	-0.0213117	-0.0212752	-0.022983715
0.85	1.71725589	-0.0229834	-0.0246272	-0.0245912	-0.02616991
0.9	1.69265754	-0.0261696	-0.0276802	-0.0276453	-0.029088081
0.95	1.66500608	-0.0290878	-0.03046	-0.0304266	-0.031728975
1	1.63457442	-0.0317287	-0.0329589	-0.0329274	-0.03408647

		ii	Spreadsheet gives 1.63457442 (to 8 d.p.)	**B1** **[1]**	**1.1**	Correct answer to at least 3 s.f. Must for correct for the number of significant figures given.	

<table>
<tr><td></td><td></td><td>iii</td><td>Setting h = 0.005 in the spreadsheet gives these estimates, x coordinates on the left, corresponding y coordinates on the right

(extract from spreadsheet):

<table><tr><td>0.55</td><td>1.78720579</td></tr><tr><td>0.555</td><td>1.78726854</td></tr><tr><td>0.56</td><td>1.78728649</td></tr><tr><td>0.565</td><td>1.78725977</td></tr><tr><td>0.57</td><td>1.78718852</td></tr><tr><td>0.575</td><td>1.78707284</td></tr></table></td><td>M1</td><td>3.1a</td><td>Must see at least three suitable (x, y) pairs. Or argument based sign change in gradient sufficiently close to 0.56.</td><td></td></tr>
<tr><td></td><td></td><td></td><td>This gives an estimate of 0.56 (to 2 d.p.).</td><td>A1
[2]</td><td>3.2a</td><td>SC 1 for 0.56 unsupported.</td><td></td></tr>
</table>

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