Mark Scheme (Results)

Summer 2019

Pearson Edexcel GCE
In Mathematics (9MA0_31) Statistics

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Summer 2019
Publications Code 9MAO_03_1906_MS
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## General Marking Guidance

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- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- $\quad$ All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification/indicative content will not be exhaustive.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, a senior examiner must be consulted before a mark is awarded.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.


## PEARSON EDEXCEL GCE MATHEMATICS

## General Instructions for Marking

1. The total number of marks for the paper is 50 .
2. These mark schemes use the following types of marks:

- M marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- A marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- B marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod - benefit of doubt
- ft - follow through
- the symbol $\sqrt{ }$ will be used for correct ft
- cao - correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw - ignore subsequent working
- awrt - answers which round to
- SC: special case
- o.e. - or equivalent (and appropriate)
- d or dep - dependent
- indep - independent
- dp decimal places
- sf significant figures
-     * The answer is printed on the paper or ag- answer given

4. All M marks are follow through.

A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but answers that don't logically make sense e.g. if an answer given for a probability is $>1$ or $<0$, should never be awarded A marks.
5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.
6. Where a candidate has made multiple responses and indicates which response they wish to submit, examiners should mark this response.
If there are several attempts at a question which have not been crossed out, examiners should mark the final answer which is the answer that is the most complete.
7. Ignore wrong working or incorrect statements following a correct answer.
8. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used. If no such alternative answer is provided but the response is deemed to be valid, examiners must escalate the response for a senior examiner to review.

| Question | Scheme | Marks | AOs |
| :---: | :---: | :---: | :---: |
| 1(a) |  | B1 <br> dB1 | 1.1 b 1.1 b |
|  |  | (2) |  |
| (b) | $\frac{9}{10} \times \frac{4}{5} \times \frac{2}{3}$ | M1 | 1.1b |
|  | $=\frac{12}{25}(=0.48)$ | A1 | 1.1b |
|  |  | (2) |  |
| (c) | $\frac{9}{10} \times \frac{1}{5}+\frac{9}{10} \times \frac{4}{5} \times \frac{1}{3} \quad$ or $\quad 1-\left(\frac{1}{10}+\frac{9}{10} \times \frac{4}{5} \times \frac{2}{3}\right)$ | M1 | 3.1b |
|  | $=\frac{21}{50}(=0.42)$ | A1 | 1.1b |
|  |  | (2) |  |
| (d) | $[\mathrm{P}($ Red from $B \mid$ Red selected $)]=\frac{\frac{9}{10} \times \frac{1}{5}}{\frac{1}{10}+\frac{9}{10} \times \frac{1}{5}+\frac{9}{10} \times \frac{4}{5} \times \frac{1}{3}}\left[\frac{\frac{9}{50}}{\frac{10}{13}}\right.$ | M1 | 3.1b |
|  | $=\frac{9}{26}$ | A1 | 1.1b |
|  |  | (2) |  |
| (8 marks) |  |  |  |
| Notes |  |  |  |
|  | Allow decimals or percentages throughout this question. |  |  |
| (a) | B1: for correct shape (3 pairs) and at least one label on at least two pairs G (reen) and R (ed) <br> allow $G$ and $G^{\prime}$ or $R$ and R' as labels, etc. condone 'extra' pairs if they are labelled with a probability of 0 <br> dB1: (dep on previous B1) all correct i.e. for all 6 correct probabilities on the correct branches with at least one label on each pair |  |  |
| (b) | M1: Multiplication of 3 correct probabilities (allow ft from their tree diagram) <br> A1: $\frac{12}{25}$ oe |  |  |
| (c) | M1: Either addition of only two correct products (product of two probs + product of three probs) which may ft from their tree diagram or for $1-\left({ }^{1} \frac{1}{10}{ }^{\prime}+'(b)\right.$ ') <br> A1: $\quad \frac{21}{50}$ oe |  |  |
| (d) | M1: Correct ratio of probabilities <br>  <br> A1: $\quad \frac{9}{26}$ (allow awrt 0.346) |  |  |


| Question |  | Scheme | Marks | AOs |
| :---: | :---: | :---: | :---: | :---: |
| 2(a) | $\mathrm{IQR}=26.6-19.4[=7.2]$ |  | B1 | 2.1 |
|  | $19.4-1.5 \times$ ' 7.2 ' [= 8.6] or $26.6+1.5 \times$ ' 7.2 ' [= 37.4] |  | M1 | 1.1b |
|  | Plotting one upper whisker to 32.5 and one lower whisker to 8.6 or 9.1 |  | A1 | 1.1b |
|  | Plotting 7.6 and 8.1 as the only two outliers |  | A1 | 1.1b |
|  |  |  | (4) |  |
| (b) | October (since it is the month with the coldest temperatures between May and October in Beijing) |  | B1 | 2.4 |
|  |  |  | (1) |  |
| (c) | $\left[\sigma=\sqrt{\frac{4952.906}{184}} \quad\right.$ or e.g. $[\sigma=] \sqrt{\frac{S_{x x}}{n}}=5.188 \ldots \quad\left[=5.19^{*}\right]$ |  | B1cso* | 1.1b |
|  | $z=( \pm) 1.28(16) \quad\left[P_{90}=\right] 29.251 \ldots$ or $\left[P_{10}=\right] 15.948 \ldots$ |  | (1) |  |
| (d) |  |  | B1 | 3.1b |
|  | $2 \times 1.2816 \times 5.19$ | '29.251...' - '15. | M1 | 1.1b |
|  | $\square=$ awrt 13.3 |  | A1 | 1.1b |
|  |  |  | (3) |  |
| (e) | Daily mean wind speed/Beaufort conversion since it is qualitative Rainfall since it is not symmetric/lots of days with 0 rainfall |  | $\begin{aligned} & \hline \text { B1 } \\ & \text { B1 } \\ & \hline \end{aligned}$ | $\begin{aligned} & 2.4 \\ & 2.4 \\ & \hline \end{aligned}$ |
|  | (12) ${ }^{\text {(11 marks }}$ ( ${ }^{\text {(1) }}$ |  |  |  |
|  |  |  |  |  |
| Notes |  |  |  |  |
| (a) | $\begin{array}{ll}\text { B1: } & \text { for a correct calculation for the IQR (implied by } 10.8 \text { or } 8.6 \text { or } 37.4 \text { seen) } \\ \text { M1: } & \text { for a complete method for either lower outlier limit or upper outlier limit } \\ & \text { (allow ft on their IQR) (may be implied by the } 1^{\text {st }} \mathrm{A} 1 \text { or a lower whisker at } 8.6 \text { ) } \\ \text { A1: } & \text { both whiskers plotted correctly (allow } 1 / 2 \text { square tolerance) } \\ \text { A1: } & \text { only two outliers plotted, } 7.6 \text { and } 8.1 \text { (must be disconnected from whisker) } \\ \text { NOTE: A fully correct box plot with no incorrect working scores } 4 / 4\end{array}$ |  |  |  |
| (c) | B1cso*: Correct expression with square root or correct formula and 5.188 or better Allow a complete correct method finding $\sum x^{2}=$ awrt 98720 and $\sigma=\sqrt{\frac{98715.9 \ldots}{184}-\left(\frac{4153.6}{184}\right)^{2}}$ |  |  |  |
| (d) | B1: $\quad$ Identifying $z$-value for 10th or 90th percentile (allow awrt ( $\pm$ ) 1.28) <br> or for identifying $\left[P_{90}=\right] 29.251 \ldots$ (awrt 29.3) or $\left[P_{10}=\right] 15.948 \ldots$ (awrt 15.9) <br> (This may be implied by a correct answer awrt 13.3) <br> M1: for $2 \times z \times 5.19$ where $1<z<2$ <br> or for their $P_{90}-P_{10}$ where $25<P_{90}<35$ and $10<P_{10}<20$ <br> A1: awrt 13.3 |  |  |  |
| (e) | B1: for one variable identified and a correct supporting reason <br> B1: for two variables identified and a correct supporting reason for each <br> Allow any two of the following: <br> - Wind speed/Beaufort since the data is non-numeric (o.e.). They need not mention Beaufort provided there is a description of the data as non-numeric (Do not allow wind direction/wind gust) <br> - Rainfall as not symmetric/is skewed/is not bell shaped/lots of $0 \mathrm{~s} /$ many days with no rain/mean $\neq$ mode or median <br> - Date since each data value appears once/it is uniformly distributed <br> - Daily mean pressure since it is not symmetric/is skewed/not bell shaped <br> - Daily mean wind speed since it is not symmetric/is skewed/not bell shaped <br> Do not allow 'not continuous' or 'discrete' as a supporting reason. <br> Ignore extraneous non-contradicting statements |  |  |  |


| Question | Scheme |  | Marks | AOs |
| :---: | :---: | :---: | :---: | :---: |
| 3(a) | $\mathrm{H}_{0}: \rho=0 \quad \mathrm{H}_{1}: \rho>0$ |  | B1 | 2.5 |
|  | Critical value 0.3438 |  | M1 | 1.1a |
|  | ( $0.446>0.3438$ ) so there is evidence that the product moment correlation coefficient (pmcc) is greater than $0 /$ there is positive correlation |  | A1 | 2.2b |
|  |  |  | (3) |  |
| (b) | The value is close(r) to 1 or there is strong(er) (positive) correlation |  | B1 | 2.4 |
|  |  |  | (1) |  |
| (c) | $\log _{10} y=-1.82+0.89\left(\log _{10} x\right)$ | $\begin{aligned} & y=a x^{n} \rightarrow \\ & \log _{10} y=\log _{10}\left(a x^{n}\right) \end{aligned}$ | M1 | 1.1b |
|  | $y=10^{-1.82+0.89\left(\log _{10} x\right)}$ | $\log _{10} y=\log _{10} a+\log _{10} x^{n}$ | M1 | 2.1 |
|  | $\begin{aligned} & y=10^{-1.82} \times 10^{0.89\left(\log _{10} x\right)} \\ & {\left[=10^{-1.82} \times 10^{\left(\log _{10} x\right)^{0.89}}\right]} \end{aligned}$ | $\begin{aligned} & \log _{10} y=\log _{10} a+n \log _{10} x \\ & {\left[\log _{10} a=-1.82, n=0.89\right]} \end{aligned}$ | M1 | 1.1b |
|  | $y=0.015 x^{0.89}$ | $y=0.015 x^{0.89}$ | A1A1 | 1.1b 1.1b |
|  |  |  | (5) |  |
| marks) |  |  |  |  |
| Notes |  |  |  |  |
| (a) | B1: for both hypotheses correct in terms of $\rho$ <br> M1: for the critical value: sight of 0.3438 or any cv such that $0.25<\|\mathrm{cv}\|<0.45$ <br> A1: a comment suggesting a significant result/ $\mathrm{H}_{0}$ is rejected on the basis of seeing +0.3438 and which mentions "pmcc/correlation/relationship" and "greater than 0/positive" (not just $\rho>0$ ) <br> or an answer in context e.g. 'as "income"(o.e.) increases, " $\mathrm{CO}_{2} /$ emissions"(o.e.) increases' A contradictory statement scores A0 e.g. 'Accept $\mathrm{H}_{0}$, therefore positive correlation' |  |  |  |
| (b) | B1: for suitable reason e.g. $r$ is close(r) to 1 or "strong(er)"/"near perfect" "correlation" Do not allow 'association' |  |  |  |
| (c) | Method 2: (working from the model) <br> M1: Taking the $\log$ of both sides (may be implied by $2^{\text {nd }}$ M1 mark) <br> M1: Correct use of addition rule (may be implied by $3^{\text {rd }}$ M1 mark) <br> M1: Correct multiplication of power (this line implies M1M1M1 provided no previous incorrect working seen) <br> A1: $n=0.89$ or $a=\operatorname{awrt} 0.015$ or $y=a x^{0.89}$ or $y=\operatorname{awrt} 0.015 x^{n}$ (dep on M3) <br> A1: $n=0.89$ and $a=\operatorname{awrt} 0.015 / y=\operatorname{awrt} 0.015 x^{0.89}$ (dep on M3) <br> do not award the final A1 if answer is given in an incorrect form e.g $y=0.015+x^{0.89}$ |  |  |  |


| Question | Scheme | Marks | AOs |
| :---: | :---: | :---: | :---: |
| 4 (a) | $\frac{132}{184}=0.71739 \ldots \quad$ awrt $\underline{\mathbf{0 . 7 1 7}}$ | B1 | 1.1b |
|  |  | (1) |  |
| (b)(i) | $\mathrm{P}(X \geq 6)=1-\mathrm{P}(X \leq 5)$ or $\mathrm{P}([X=] 6)+\mathrm{P}([X=] 7)+\mathrm{P}([X=] 8)$ | M1 | 3.4 |
|  | $=1-0.296722 \ldots$ awrt $\underline{\mathbf{0 . 7 0 3}}$ | A1 | 1.1b |
|  |  | (2) |  |
| (b)(ii) | $184 \times \mathrm{P}(X=7) \quad[=184 \times 0.2811 \ldots]$ | M1 | 1.1b |
|  | $=51.7385 \ldots$ awrt $5 \underline{1.7}$ | A1 | 1.1b |
|  |  | (2) |  |
| (c) | Part (a) and part (b)(i) are similar and the expected number of 7 s ( 51.7 or 0.281 ) matches with the number of 7 s found in the data set ( 52 or 0.283 ) so Magali's model is supported. | B1ft | 3.5a |
|  |  | (1) |  |
| (d) | $\frac{23}{28}=0.82142 \ldots$ <br> awrt $\mathbf{0 . 8 2 1}$ | B1 | 1.1b |
|  |  | (1) |  |
| (e) | Any one of... <br> - Part (d)/‘0.821' differs from part (a)/(b)(i)/(0.7...) <br> - there is a greater/different probability of high cloud cover/more likely to have high cloud cover if the previous day had high cloud cover <br> - independence(o.e.) does not hold | B1 | 2.4 |
|  | ...therefore Magali's (binomial) model may not be suitable. | dB1 | 3.5a |
|  |  | (2) |  |
| (9 marks) |  |  |  |
| Notes |  |  |  |
|  | Allow fractions, decimals or percentages throughout this question. |  |  |
| (a) | Allow equivalent fraction, e.g. $\frac{33}{46}$ |  |  |
| (b)(i) | M1: for writing or using $1-\mathrm{P}(X \leq 5)$ or $\mathrm{P}(X=6)+\mathrm{P}(X=7)+\mathrm{P}(X=8)$ <br> A1: awrt 0.703 (correct answer scores 2 out of 2) |  |  |
| (b)(ii) | M1: for $184 \times \mathrm{P}(X=7)$ o.e. e.g., $184 \times[\mathrm{P}(X \leq 7)-\mathrm{P}(X \leq 6)]$ <br> A1: awrt 51.7 |  |  |
| (c) | B1ft: comparing ' 0.717 ' with ' 0.703 ' and ' 51.7 or ' 0.281 ' with 52 or 0.283 and concluding that Magali's model is supported (must be comparing prob. with prob. and days with days). Allow not supported or mixed conclusions if consistent with their f.t. answers in (a) and (b) |  |  |
| (e) | B1: Any bullet point <br> dB1: (dep on previous B1) for Magali's model may not be suitable (o.e.) Condone not accurate for not suitable <br> SC: part (d) is similar to part (a)/(b)(i) and a compatible conclusion (i.e. Magali's model is supported) to score B1B1. |  |  |


| Question | Scheme | Marks | AOs |
| :---: | :---: | :---: | :---: |
| 5(a) | $\frac{24.63-25}{\prime} \sigma^{\prime}{ }^{\text {a }}=-1.0364$ | M1 | 3.1b |
|  | [ $\sigma=10.357$ (must come from compatible signs) | A1 | 1.1b |
|  | $\mathrm{P}(D>k)=0.4$ or $\mathrm{P}(D<k)=0.6$ | B1 | 1.1b |
|  | $\frac{k-25}{{ }^{0.357}}=0.2533$ | M1 | 3.4 |
|  | $k=$ awrt 25.09 | A1 | 1.1b |
|  |  | (5) |  |
| (b) | $[Y \sim \mathrm{~B}(200,0.45) \rightarrow] W \sim \mathrm{~N}(90,49.5)$ | B1 | 3.3 |
|  | $\mathrm{P}(Y<100) \approx \mathrm{P}(W<99.5)\left[=\mathrm{P}\left(Z<\frac{99.5-90}{\sqrt{49.5}}\right)\right]$ | M1 | 3.4 |
|  | $=0.9115 \ldots$ awrt $\underline{\mathbf{0 . 9 1 2}}$ | A1 | 1.1b |
|  |  | (3) |  |
| (c) | $\mathrm{H}_{0}: \mu=25 \quad \mathrm{H}_{1}: \mu<25$ | B1 | 2.5 |
|  | [ $\bar{D} \sim] \mathrm{N}\left(25, \frac{0.16^{2}}{20}\right)$ | M1 | 3.3 |
|  | $\mathrm{P}(\bar{D}<24.94)[=\mathrm{P}(Z<-1.677 \ldots)]=0.046766 \ldots$ | A1 | 3.4 |
|  | $\begin{aligned} & p=0.047<0.05 \text { or } z=-1.677 \ldots<-1.6449 \\ & \underline{\text { or }} 24.94<24.94115 \ldots \\ & \text { or reject } \mathrm{H}_{0} / \text { in the critical region/significant } \end{aligned}$ | M1 | 1.1b |
|  | There is sufficient evidence to support Hannah's belief. | A1 | 2.2b |
|  |  | (5) |  |
| (13 marks) |  |  |  |
| Notes |  |  |  |
| (a) | M1: for standardising 24.63, 25 and ' $\sigma$ ' (ignore label) and setting $=$ to $z$ where $1<\|z\|<2$ <br> A1: $[\sigma=$ ] awrt 0.36. Do not award this mark if signs are not compatible. <br> B1: for either correct probability statement (may be implied by correct answer) this mark may be scored for a correct region shown on a diagram <br> M1: for a correct expression with $z=$ awrt 0.253 (may be implied by correct answer) <br> A1: awrt 25.09 (Correct answer with no incorrect working scores 5 out of 5) |  |  |
| (b) | B1: setting up normal distribution approximation of binomial $\mathrm{N}(90,49.5)$ (may be implied by a correct answer) Look out for e.g. $\sigma=\frac{3 \sqrt{22}}{2}$ or $\sigma=$ awrt 7.04 <br> M1: attempting a probability using a continuity correction i.e. $\mathrm{P}(W<100.5), \mathrm{P}(W<99.5)$ or $\mathrm{P}(W<98.5)$ condone $\leq$ (The continuity correction may be seen in a standardisation). <br> A1: awrt 0.912 [Note: $0.911299 \ldots$ from binomial scores 0 out of 3] |  |  |
| (c) | B1: for both hypotheses in terms of $\mu$ <br> M1: selecting suitable model must see N (ormal), mean 25 , $\mathrm{sd}=\frac{0.16}{\sqrt{20}}$ (o.e.) or var $=\frac{4}{3125}$ <br> (o.e.) <br> Condone $N\left(25, \frac{0.16}{\sqrt{20}}\right)$ if $\frac{0.16}{\sqrt{20}}$ then used as s.d. <br> A1: $p$ value $=$ awrt $0.047 \quad$ or test statistic awrt -1.68 or $\mathbf{C V}$ awrt 24.941 <br> (any of these values imply the M1 provided they do not come from Normal mean $=24.94$ ) <br> M1: a correct comparison (including compatible signs) or correct non-contextual conclusion (f.t. their $p$ value, test statistic or critical value in the comparison) M1 may be implied by a correct contextual statement <br> NB Any contradictory non contextual statements/comparisons score M0A0 e.g. 'p<0.05, not significant' <br> A1: correct conclusion in context mentioning Hannah's belief or the mean amount/liquid in each bottle is now less than 25 ml (dep on M1A1M1) |  |  |

## P Pearson Edexcel

## Mark Scheme (Results)

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Summer 2019
Publications Code 9MA0_03_1906_MS
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- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
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- ft - follow through
- the symbol will be used for correct ft
- cao - correct answer only
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- dep - dependent
- indep - independent
- dp decimal places
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- $\boldsymbol{*}$ The answer is printed on the paper
- The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

## General Principles for Mechanics Marking

(But note that specific mark schemes may sometimes override these general principles)

- Rules for M marks: correct no. of terms; dimensionally correct; all terms that need resolving (i.e. multiplied by cos or sin) are resolved.
- Omission or extra g in a resolution is an accuracy error not method error.
- Omission of mass from a resolution is a method error.
- Omission of a length from a moments equation is a method error.
- Omission of units or incorrect units is not (usually) counted as an accuracy error.
- DM indicates a dependent method mark i.e. one that can only be awarded if a previous specified method mark has been awarded.
- Any numerical answer which comes from use of $g=9.8$ should be given to 2 or 3 SF .
- Use of $\mathrm{g}=9.81$ should be penalised once per (complete) question.
N.B. Over-accuracy or under-accuracy of correct answers should only be penalised once per complete question. However, premature approximation should be penalised every time it occurs.
- Marks must be entered in the same order as they appear on the mark scheme.
- In all cases, if the candidate clearly labels their working under a particular part of a question i.e. (a) or (b) or (c),.....then that working can only score marks for that part of the question.
- Accept column vectors in all cases.
- Misreads - if a misread does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, bearing in mind that after a misread, the subsequent A marks affected are treated as A ft
- Mechanics Abbreviations

M(A) Taking moments about A.
N2L Newton's Second Law (Equation of Motion)
NEL Newton's Experimental Law (Newton's Law of Impact)
HL Hooke's Law
SHM Simple harmonic motion
PCLM Principle of conservation of linear momentum
RHS, LHS Right hand side, left hand side.

9MA0-32: Mechanics 1906
Mark scheme

| Question |  | Scheme | Marks | AO |
| :---: | :---: | :---: | :---: | :---: |
| 1(a) |  | Differentiate $\mathbf{v}$ | M1 | 1.1a |
|  |  | $(\mathbf{a}=) 6 \mathbf{i}-\frac{15}{2} t^{\frac{1}{2}} \mathbf{j}$ | A1 | 1.1b |
|  |  | $=6 \mathbf{i}-15 \mathbf{j}\left(\mathrm{~m} \mathrm{~s}^{-2}\right)$ | A1 | 1.1b |
|  |  |  | (3) |  |
| 1(b) |  | Integrate $\mathbf{v}$ | M1 | 1.1a |
|  |  | $(\mathbf{r}=)\left(\mathbf{r}_{0}\right)+3 t^{2} \mathbf{i}-2 t^{\frac{5}{2}} \mathbf{j}$ | A1 | 1.1b |
|  |  | $=(-20 \mathbf{i}+20 \mathbf{j})+(48 \mathbf{i}-64 \mathbf{j})=28 \mathbf{i}-44 \mathbf{j}(\mathrm{~m})$ | A1 | 2.2a |
|  |  |  | (3) |  |
|  |  |  | (6) |  |
| Marks |  | Notes |  |  |
|  |  | N.B. Accept column vectors throughout and condone missing brackets in working but they must be there in final answers |  |  |
| 1a | M1 | Use of $\mathbf{a}=\frac{\mathrm{d} \mathbf{v}}{\mathrm{d} t}$ with attempt to differentiate (both powers decreasing by 1 ) M0 if i's and $\mathbf{j}$ 's omitted and they don't recover |  |  |
|  | A1 | Correct differentiation in any form |  |  |
|  | A1 | Correct and simplified. <br> Ignore subsequent working (ISW) if they go on and find the magnitude. |  |  |
| 1b | M1 | Use of $\mathbf{r}=\int \mathbf{v} \mathrm{d} t$ with attempt to integrate (both powers increasing by 1 ) M0 if i's and $\mathbf{j}$ 's omitted and they don't recover |  |  |
|  | A1 | Correct integration in any form. Condone $\mathbf{r}_{0}$ not present |  |  |
|  | A1 | Correct and simplified. |  |  |




| Question | Scheme | Marks | AO |
| :---: | :---: | :---: | :---: |
| 3(a) |  |  |  |
|  | $R=2 m g \cos \alpha$ | B1 | 3.4 |
|  | $F=\frac{2}{3} R$ | B1 | 1.2 |
|  | Equation of motion for $A$ : | M1 | 3.3 |
|  | $T-F-2 m g \sin \alpha=2 m a$ | A1 | 1.1b |
|  | Equation of motion for $B$ : | M1 | 3.3 |
|  | $3 m g-T=3 m a$ | A1 | 1.1b |
|  | Complete strategy to find an equation in $T, m$ and $g$ only. | M1 | 3.1b |
|  | $T=\frac{12 m g}{5}$ * | A1* | 2.2a |
|  |  | (8) |  |
| (b) | $\left(F_{\text {max }}=\right) \frac{16 m g}{13}>\frac{10 m g}{13}$ | M1 | 2.1 |
|  | ...... so $A$ will not move. | A1 | 2.2a |
|  |  | (2) |  |
| (c) | - Extensible string <br> - Weight of string <br> - Friction at pulley e.g. rough pulley <br> - Allow for the dimensions of the blocks e.g. "Do not model blocks as particles"; "(include) air resistance","include rotational effects of forces on blocks i.e. spin" | $\begin{aligned} & \text { B1 } \\ & \text { B1 } \end{aligned}$ | $\begin{aligned} & 3.5 \mathrm{c} \\ & 3.5 \mathrm{c} \end{aligned}$ |
|  |  | (2) |  |
|  |  | (12) |  |
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| Marks |  | Notes |
| :---: | :---: | :---: |
| 3a | B1 | Normal reaction between $A$ and the plane seen or implied, $\cos \alpha$ does not need to be substituted. |
|  | B1 | $F=\frac{2}{3} R$ seen or implied anywhere, including part (b) |
|  | M1 | Form an equation of motion for $A$. Must include all relevant terms. Must be the correct mass but condone consistent missing $m$ 's. Condone sign errors and $\sin / \mathrm{cos}$ confusion |
|  | A1 | Correct unsimplified equation ( $F$ does not need to be substituted). Allow consistent use of $(-a)$ <br> N.B. If $T-2 m \mathrm{~g}=2 m \mathrm{a}$ is seen with no working, M0A0 unless both B1 marks have been scored. |
|  | M1 | Form an equation of motion for $B$. Must be the correct mass on RHS but condone consistent missing $m$ 's. Condone sign errors and sin/cos confusion. |
|  | A1 | Correct unsimplified equation ( $F$ does not need to be substituted). Allow consistent use of $(-a)$ |
|  |  | N.B. Allow the 'whole system' equation to replace the equation for $A$ or $B$. $3 m g-F-2 m g \sin \alpha=5 m a$ <br> Must be the correct mass on RHS but condone consistent missing $m$ 's. Condone sign errors and $\sin /$ cos confusion. |
|  | M1 | Complete method to give an equation in $T, m$ and $g$ only. N.B. Allow $\theta$ in the equation if they have defined what $\theta$ is: e.g. $\theta=\tan ^{-1}\left(\frac{5}{12}\right)$ <br> This is an independent mark but they must have two simultaneous equations in $T$ and $a$ unless one of the equations is the whole system equation in which case one equation will be in $T$ and $a$ and the other equation will be in $a$ only. |
|  | A1* | Obtain the given answer from correct working using EXACT trig ratios. (not available if using a decimal angle) |
| 3b | M1 | Comparison of their $F_{\max }\left(\frac{2}{3} R\right)$ and their component of weight down the slope, must be comparing numerical values. oe e.g. if they consider the difference <br> N.B. Allow comparison of $\mu$ and $\tan \alpha$ with numerical values |
|  | A1 | Correctly justified conclusion and no errors seen <br> N.B. If they equate their difference to an ' $m a$ ' term then A0 |
| 3c | $\begin{aligned} & \text { B1 } \\ & \text { B1 } \end{aligned}$ | Deduct 1 mark for each extra (more than 2 ) incorrect answer up to a maximum of 2 incorrect answers. Ignore extra correct answers. <br> e.g. two correct, one incorrect B1 B0 <br> one correct, one incorrect B1 B0 <br> one correct, two incorrect B0 B0 <br> Ignore incorrect reasons or consequences. <br> Ignore any mention of wind or a general reference to friction. |


| Question | Scheme | Marks | AO |
| :---: | :---: | :---: | :---: |
| 4(a) | Drum smooth, or no friction, (therefore reaction is perpendicular to the ramp) | B1 | 2.4 |
|  |  | (1) |  |
| (b) | N.B. In (b), for a moments equation, if there is an extra $\sin \theta$ or $\cos \theta$ on a length, give M 0 for the equation <br> e.g. $\mathrm{M}(A): 20 g \times 4 \cos \theta=5 N \sin \theta$ would be given M0A0 |  |  |
|  |  |  |  |
|  | Possible equns$\begin{aligned} & (\nearrow): F \cos \theta+R \sin \theta=20 g \sin \theta \\ & (\nwarrow): N+R \cos \theta=20 g \cos \theta+F \sin \theta \\ & (\uparrow) R+N \cos \theta=20 g \\ & (\rightarrow): F=N \sin \theta \\ & \mathrm{M}(A): 20 g \times 4 \cos \theta=5 N \\ & \mathrm{M}(B): 3 N+R \times 8 \cos \theta=F \times 8 \sin \theta+20 g \times 4 \cos \theta \\ & \mathrm{M}(C): R \times 5 \cos \theta=F \times 5 \sin \theta+20 g \times \cos \theta \\ & \mathrm{M}(G): R \times 4 \cos \theta=F \times 4 \sin \theta+N \end{aligned}$ | M1 | 3.3 |
|  |  | A1 | 1.1b |
|  |  | M1 | 3.4 |
|  |  | A1 | 1.1b |
|  |  | M1 | 3.4 |
|  |  | A1 | 1.1b |
|  | (The values of the 3 unknowns are: $N=150.528 ; F=42.14784 ; R=51.49312)$ |  |  |
|  | Alternative 1: using cpts along ramp $(X)$ and perp to $\operatorname{ramp}(Y)$ Possible equations:$\begin{aligned} & (\nearrow): X=20 g \sin \theta \\ & (\mathbb{Z}): Y+N=20 g \cos \theta \\ & (\uparrow): X \sin \theta+Y \cos \theta+N \cos \theta=20 g \\ & (\rightarrow): X \cos \theta=Y \sin \theta+N \sin \theta \\ & \mathrm{M}(A): 20 g \times 4 \cos \theta=5 N \\ & \mathrm{M}(B): 20 g \times 4 \cos \theta=8 Y+3 N \\ & \mathrm{M}(C): 20 g \times \cos \theta=5 Y \\ & \mathrm{M}(G): 4 Y=N \times 1 \end{aligned}$ | M1 | 3.3 |
|  |  | A1 | 1.1b |
|  |  | M1 | 3.4 |
|  |  | A1 | 1.1b |
|  |  | M1 | 3.4 |
|  |  | A1 | 1.1b |
|  | (The values of the 3 unknowns are: $N=150.528 ; X=54.88 ; Y=37.632)$ |  |  |


|  | Alternative 2: using horizontal cpt (H) and cpt perp to ramp |  |  |
| :---: | :---: | :---: | :---: |
|  | $(\nearrow): H \cos \theta=20 g \sin \theta$ | M1 | 3.3 |
|  | $(\nwarrow): S+N=H \sin \theta+20 g \cos \theta$ | A1 | 1.1b |
|  | ( $\uparrow$ ): $S \cos \theta+N \cos \theta=20 g$ | M1 | 3.4 |
|  | $\mathrm{M}(A): 20 g \times 4 \cos \theta=5 N$ | A1 | 1.1b |
|  | $\mathrm{M}(B): 20 g \times 4 \cos \theta+H \times 8 \sin \theta=8 S+3 N$ | M1 | 3.4 |
|  | $\mathrm{M}(G): 4 S=N \times 1+H \times 4 \sin \theta$ | A1 | 1.1b |
|  | (The values of the 3 unknowns are: $N=150.528 ; H=57.1666 \ldots ; S=53.638666 \ldots)$ |  |  |
|  | Solve their 3 equations for $F$ and $R$ OR $\quad X$ and $Y \quad$ OR $\quad H$ and $S$ | M1 | 1.1b |
|  | $\begin{array}{rlrl} \mid \text { Force } \mid & =\sqrt{R^{2}+F^{2}} & \text { Main scheme } \\ \text { OR } & =\sqrt{X^{2}+Y^{2}} & \text { Alternative } 1 \\ \text { OR } & =\sqrt{\left(H^{2}+S^{2}-2 H S \cos \left(90^{\circ}-\theta\right)\right.} & & \text { Alternative } 2 \end{array}$ | M1 | 3.1b |
|  | Magnitude $=67$ or $66.5(\mathrm{~N})$ | A1 | 2.2a |
|  |  | (9) |  |
| (c) | Magnitude of the normal reaction (at $C$ ) will decrease. | B1 | 3.5a |
|  |  | (1) |  |
|  |  | (11) |  |
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| Marks |  | Notes |
| :---: | :---: | :---: |
| 4a | B1 | Ignore any extra incorrect comments. |
|  |  | Generally 3 independent equations required so at least one moments equation.: M1A1M1A1M1A1. <br> More than 3 equations, give marks for the best 3. For each: <br> M1 All terms required. Must be dimensionally correct so if a length is missing from a moments equation it's M0 Condone $\sin / \cos$ confusion. <br> A1 For a correct equation (trig ratios do not need to be substituted and allow e.g. $\cos (24 / 25)$ if they recover <br> Enter marks on ePEN in order in which equations appear. <br> N.B. If reaction at $C$ is not perpendicular to the ramp, can only score marks for $\mathrm{M}(C)$ <br> Allow use of $(\mu R)$ for $F$ |
| 4b | M1 | All terms required. Must be dimensionally correct. Condone sin/cos confusion. |
|  | A1 | Correct unsimplified equation |
|  | M1 | All terms required. Must be dimensionally correct. Condone sin/cos confusion. |
|  | A1 | Correct unsimplified equation |
|  | M1 | All terms required, dim correct, condone sin/cos confusion |
|  | A1 | Correct unsimplified equation |
|  |  | N.B. They can find $F$ and $R$ using only TWO equations, the 1st and 7 th in the list. Mark the better equation as M2A2 (-1 each error). Mark the second equation as M1A1 |
| Alt | M1 | All terms required. Must be dimensionally correct. Condone sin/cos confusion. |
|  | A1 | Correct unsimplified equation |
|  | M1 | All terms required. Must be dimensionally correct. Condone sin/cos confusion. |
|  | A1 | Correct unsimplified equation |
|  | M1 | All terms required. Must be dimensionally correct. Condone sin/cos confusion. |
|  | A1 | Correct unsimplified equation |
|  |  | N.B. They can find $X$ and $Y$ using only TWO equations, the $1^{\text {st }}$ and $7^{\text {th }}$ in the list. Mark the better equation as M2A2 (-1 each error). Mark the second equation as M1A1 |
| $\begin{gathered} \text { Alt } \\ 2 \end{gathered}$ | M1 | All terms required. Must be dimensionally correct. Condone sin/cos confusion. |
|  | A1 | Correct unsimplified equation |
|  | M1 | All terms required. Must be dimensionally correct. Condone sin/cos confusion. |


| A1 | Correct unsimplified equation |
| :---: | :---: |
| M1 | All terms required. Must be dimensionally correct. |
| A1 | Correct unsimplified equation |
|  | N.B. They can find $H$ and $S$ using only TWO equations, the $1^{\text {st }}$ and $7^{\text {th }}$ in the list. Mark the better equation as M2A2 ( -1 each error). Mark the second equation as M1A1 |
| M1 | Substitute for trig and solve for their two cpts. <br> This is an independent mark but must use 3 equations (unless it's the special case when 2 is sufficient) |
| M1 | Use Pythagoras to find magnitude (this is an independent M mark but must have found a value for $F($ or $X)$ and a value for $R$ (or $Y$ )) <br> OR a complete method to find magnitude e.g. cosine rule but must have found a value for $H$ and a value for $S$ |
| A1 | Correct answer only |
| B1 | Ignore reasons |


| Question | Scheme | Marks | AO |
| :---: | :---: | :---: | :---: |
|  | In this question mark parts (a) and (b) together. |  |  |
| 5(a) | Horizontal speed $=20 \cos 30^{\circ}$ | B1 | 3.4 |
|  | Vertical velocity at $t=2$ | M1 | 3.4 |
|  | $=20 \sin 30^{\circ}-2 \mathrm{~g}$ | A1 | 1.1b |
|  | $\theta=\tan ^{-1}\left( \pm \frac{9.6}{10 \sqrt{3}}\right)$ | M1 | 1.1b |
|  | $\text { Speed }=\sqrt{100 \times 3+9.6^{2}} \text { or e.g. speed }=\frac{9.6}{\sin \theta}$ | M1 | 1.1b |
|  | 19.8 or $20\left(\mathrm{~m} \mathrm{~s}^{-1}\right)$ at $29.0^{\circ}$ or $29^{\circ}$ to the horizontal oe | A1 | 2.2a |
|  |  | (6) |  |
| (b) | Using sum of horizontal distances $=50$ at $t=2$ | M1 | 3.3 |
|  | $\begin{gathered} (u \cos \theta) \times 2+\left(20 \cos 30^{\circ}\right) \times 2=50 \\ \left(u \cos \theta=25-20 \cos 30^{\circ}\right) \end{gathered}$ | A1 | 1.1b |
|  | Vertical distances equal | M1 | 3.4 |
|  | $\begin{gathered} \Rightarrow\left(20 \sin 30^{\circ}\right) \times 2-\frac{g}{2} \times 4=(u \sin \theta) \times 2-\frac{g}{2} \times 4 \\ \left(20 \sin 30^{\circ}=u \sin \theta\right) \end{gathered}$ | A1 | 1.1b |
|  | Solving for both $\theta$ and $u$ | M1 | 3.1b |
|  | $\begin{aligned} & \theta=52^{\circ} \text { or better }\left(52.47756849 \ldots .^{\circ}\right) \\ & u=13 \text { or better }(12.6085128 \ldots) \end{aligned}$ | A1 | 2.2a |
|  |  | (6) |  |
| (c) | It does not take account of the fact that they are not particles (moving freely under gravity) <br> It does not take account of the size(s) of the balls <br> It does not take account of the spin of the balls <br> It does not take account of the wind <br> $g$ is not exactly $9.8 \mathrm{~m} \mathrm{~s}^{-2}$ <br> N.B. If they refer to the mass or weight of the balls give B0 | B1 | 3.5b |
|  |  | (1) |  |
|  |  | (13) |  |
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| :---: | :---: | :---: |
| Marks |  | Notes |
| 5a | B1 | Seen or implied, possibly on a diagram |
|  | M1 | Use of $v=u+a t$ or any other complete method using $t=2$ Condone sign errors and $\sin / \cos$ confusion. |
|  | A1 | Correct unsimplified equation in $v$ or $v^{2}$ |
|  | M1 | Correct use of trig to find a relevant angle for the direction. Must have found a horizontal and a vertical velocity component |
|  | M1 | Use Pythagoras or trig to find the magnitude <br> Must have found a horizontal and a vertical velocity component |
|  | A1 | Or equivalent. Need magnitude and direction stated or implied in a diagram. ( 0.506 or 0.51 rads) |
| 5b | M1 | First equation, in terms of $u$ and $\theta$ (could be implied by subsequent working), using the horizontal motion with $t=2$ used <br> Condone sign errors and $\sin / \cos$ confusion |
|  | A1 | Correct unsimplified equation - any equivalent form |
|  | M1 | Second equation, in terms of $u$ and $\theta$ (could be implied by subsequent working), using the vertical motion - equating distances or just vertical components of velocities. <br> Condone sign errors and sin/cos confusion |
|  | A1 | Correct unsimplified equation - any equivalent form |
|  | M1 | Complete strategy: all necessary equations formed and solve for $u$ and $\theta$ <br> N.B. This is an independent method mark but can only be earned if 50 m has been used in their solution. |
|  | A1 | Both values correct. (Here we accept 2SF or better, since the $g$ 's cancel) Allow radians for $\theta: 0.92$ or better ( 0.915906 ..) rads. |
| 5c | B1 | Any factor related to the model as stated in the question. <br> Penalise incorrect extras but ignore consequences <br> e.g. ' $A B$ (or the ground) is not horizontal' should be penalised <br> or 'they do not move in a vertical plane' should be penalised |

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