RECOGNISING ACHIEVEMENT

Oxford Cambridge and RSA Examinations

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS

METHODS FOR ADVANCED MATHEMATICS, C3

4753

MARK SCHEME

Qu	Answer	Mark	Comment
Section A			
1	Take a counter-example, e.g. $x=1 \Rightarrow \mathrm{f}(x)=4, \mathrm{f}(-x)=0$ $\therefore \mathrm{f}(x) \neq \mathrm{f}(-x)$	M1 E1 **[2]**	Use must be shown
2	Integration by parts with: $u=x$ and $\dfrac{\mathrm{d}v}{\mathrm{d}x}=\sin 2x$ $v=-\dfrac{1}{2}\cos 2x$ $\left[x \times (-\dfrac{1}{2}\cos 2x)\right] - \int(-\dfrac{1}{2}\cos 2x)\mathrm{d}x$ $-\dfrac{1}{2}x\cos 2x + \dfrac{1}{4}\sin 2x + c$	 M1 A1 A1 A1 **[4]**	 Use of parts
3	$\dfrac{P}{P_0}=\mathrm{e}^{0.1(t-3)}$ $\ln P - \ln P_0 = 0.1(t-3)$ $t-3=10(\ln P - \ln P_0)$ $t=3+10\ln\dfrac{P}{P_0}$	M1 M1,A1 A1 A1 **[5]**	Separation of e M for use of ln
4	Graph: Segment to right of (-1.5, 0) Segment to left of (-1.5, 0) $2x+3=2-x \Rightarrow x=-\dfrac{1}{3}$ $-(2x+3)=2-x \Rightarrow x=-5$	B1 B1 B1 M1 A1 **[5]**	 Use of $-(2x+3)$
5	Let $u=2x-1$ $\Rightarrow x=\dfrac{1}{2}(u+1),\ \mathrm{d}x=\dfrac{1}{2}\mathrm{d}u$ Limits become –1 and 0 $\int_{-1}^{0}(u+1)u^7\mathrm{d}u=\left\|\dfrac{u^9}{9}+\dfrac{u^8}{8}\right\|_{-1}^{0}$ $-\dfrac{1}{72}$	 M1 M1 M1,A1 A1 **[5]**	 Change of variable Change of limits Integration

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Qu	Answer	Mark	Comment
Section A (continued)			
6	**Either** by inspection $2\times\frac{1}{2}\times(2x+1)^{-\frac{1}{2}}$ $=\frac{1}{\sqrt{2x+1}}$ (**Or** Chain rule: Let $t=2x+1$, $\frac{dt}{dx}=2$ $y=t^{\frac{1}{2}}$, $\frac{dy}{dt}=\frac{1}{2}t^{-\frac{1}{2}}$ $\frac{dy}{dx}=\frac{dy}{dt}\times\frac{dt}{dx}=\frac{1}{\sqrt{2x+1}}$) Product rule: $x^2\times\frac{1}{\sqrt{2x+1}}+2x\times\sqrt{2x+1}=\frac{5x^2+2x}{\sqrt{2x+1}}$	M1 A1 A1 (M1 A1 A1) M1,A1 A1,E1 **[7]**	Dealing with $\sqrt{\ }$ Use of 2 and ½ Chain rule Product rule
7	$\cos x\geq 0$ for $-\frac{\pi}{2}\leq x\leq\frac{\pi}{2}$ $e^x>0$ for all x $\therefore f(x)\geq 0$ for $-\frac{\pi}{2}\leq x\leq\frac{\pi}{2}$. $\therefore f(x)=0$ for $x=-\frac{\pi}{2}$ and $x=\frac{\pi}{2}$. $f'(x)=\frac{e^x(-\sin x)-\cos x\, e^x}{(e^x)^2}$ $=-\frac{(\sin x+\cos x)}{e^x}$ For maximum: $f'(x)=0$ $\Rightarrow \tan x=-1$ $x=-\frac{\pi}{4}$ $f(x)=1.5508...\rightarrow 1.55$	 B1 B1 M1,A1 A1 M1 A1 E1 **[8]**	
			Section A Total: 36

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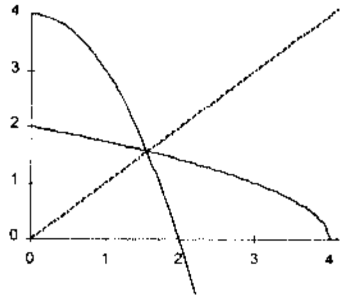
Qu	Answer	Mark	Comment
Section B			
8(i)	Domain $0 \le x \le 4$ Range $0 \le \mathrm{f}(x) \le 2$	B1 B1 **[2]**	
8(ii)(*A*)	$y^2 = 4 - x$ $x = 4 - y^2$ $\mathrm{f}^{-1}(x) = 4 - x^2$	M1 A1 A1 **[3]**	Solving for *y* or reversing flowchart method
8(ii)(*B*)		B1 **[1]**	Correct shape through (0,4) and (2,0)
8(ii)(*C*)	Reflection in $y = x$	B1 **[1]**	
8(iii)	$\mathrm{f}_1(x) = -\sqrt{4-x}$ $\mathrm{f}_2(x) = \sqrt{4-2x}$ $\mathrm{f}_3(x) = \sqrt{(-x)} - 2$	B1 B1 B1,B1 **[4]**	cao cao
8(iv)	$\mathrm{g}(x) = -x^2$ $\mathrm{g}^2(x) = -x^4$	B1,B1 B1 **[3]**	1 for – sign cao
8(v)	$\mathrm{f}^2(x) = \sqrt{4 - \sqrt{4-x}}$ Range is $\sqrt{2} \le \mathrm{f}^2(x) \le 2$ since $y = x$ goes from (0, 0) to (4, 4) and $y = \mathrm{f}^2(x)$ from $(0, \sqrt{2})$ to (4, 2), the two intersect. $\therefore \mathrm{f}^2(x) = x$ has a solution.	M1 A1 E1 E1 **[4]**	

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Qu	Answer	Mark	Comment
Section B (continued)			
9(i)	$\ln x = 0$ $\Rightarrow x = 1$ coordinates are (1,0)	M1 A1 **[2]**	$x = 1$
9(ii)	**Either:** $f'(x) = \dfrac{x.\frac{1}{x} - \ln x.1}{x^2} = \dfrac{1-\ln x}{x^2}$	M1 A1 A1	Quotient rule (consistent with their derivatives) Correct numerator $\dfrac{1-\ln x}{x^2}$ cao
	(**Or:** $x^{-1}.\dfrac{1}{x} - x^{-2}\ln x = x^{-2} - x^{-2}\ln x$ **)**	(M1 A1 A1)	(Product rule Correct expression Simplified correctly (allow –ve indices))
	$f''(x) = \dfrac{x^2.(-\frac{1}{x}) - (1-\ln x).2x}{x^4}$	M1	Any expression for $f''(x)$ consistent with their $f'(x)$ (condone missing brackets)
	$= \dfrac{-x - 2x + 2x\ln x}{x^4}$ $= \dfrac{2\ln x - 3}{x^3}$	A1 **[5]**	$\dfrac{2\ln x - 3}{x^3}$ or $\dfrac{2x\ln x - 3x}{x^4}$ or $2x^{-3}\ln x - 3x^{-3}$
9(iii)(*A*)	**Either:** $f'(x) = 0 \Rightarrow 1 - \ln x = 0$ $\Rightarrow x = \mathrm{e}$	M1 E1 **[2]**	Their $f'(x) = 0$ soi or calculates $f'(\mathrm{e}) \Rightarrow x = \mathrm{e}$ or $1 - \ln \mathrm{e} = 0$ www
	(Or: $f'(\mathrm{e}) = \dfrac{1-\ln \mathrm{e}}{\mathrm{e}^2} = 0$ **)**		
9(iii)(*B*)	when $x = \mathrm{e}$, $y = \dfrac{\ln \mathrm{e}}{\mathrm{e}} = \dfrac{1}{\mathrm{e}}$	B1 **[1]**	$y = \dfrac{1}{\mathrm{e}}$
9(iii)(*C*)	$f''(\mathrm{e}) = \dfrac{2\ln \mathrm{e} - 3}{\mathrm{e}^3} = -\dfrac{1}{\mathrm{e}^3} < 0$ $f''(\mathrm{e}) < 0 \Rightarrow$ Q is a maximum point	M1 A1 **[5]**	Substituting e into their $f''(x)$ cao $f''(x) = -\dfrac{1}{\mathrm{e}^3}$ or –0.04989…<0 $\Rightarrow$ Q is a maximum point [must evaluate $f''(\mathrm{e})$]

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Qu	Answer	Mark	Comment
Section B (continued)			
9(iv)	$A=\int_1^2 \frac{\ln x}{x}\mathrm{d}x$	M1	Correct integral and limits
	let $u=\ln x$ $\Rightarrow \mathrm{d}u=\frac{1}{x}\mathrm{d}x$	M1	Using substitution $u=\ln x$
	$A=\int_{\ln 1}^{\ln 2} u\,\mathrm{d}u$	A1	Integral in terms of u with limits
	$=\left[\frac{1}{2}u^2\right]_0^{\ln 2}$	A1	
	$=\frac{1}{2}(\ln 2)^2 = \ln 2$	M1 A1 **[6]**	Use of limits cao
			Section B Total: 36
			Total: 72

AO	Range	Total	Question Number									CWk
			1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	
1	36-41	38	-	1	2	3	3	3	2	10	10	4
2	36-41	37	1	3	3	2	2	4	5	6	7	4
3	0-9	0	-	-	-	-	-	-	-	-	-	-
4	0-9	4	1	-	-	-	-	-	-	2	-	1
5	9-18	11	-	-	-	-	-	-	1	-	1	9
	Totals	**90**	2	5	4	5	5	7	8	18	18	18

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