

Mathematics (MEI)

Advanced Subsidiary GCE

Unit **4761**: Mechanics 1

Mark Scheme for June 2011

comment

You should expect to follow through from one part to another unless the scheme says otherwise but not follow through within a part unless the scheme specifies this.

Each script must be viewed as a whole at some stage so that

(i) a candidate's writing of letters, digits, symbols on diagrams etc can be better interpreted;

(ii) repeated mistakes can be recognised (e.g. calculator in wrong angle mode throughout – penalty 1 in the script and FT except given answers).

You are advised to 'set height' in *scoris*, particularly for question 7(ii). Questions 5 and 8(v) also spread onto two pages.

Q 1		mark	notes
	$v^2 = 11^2 + 2 \times (-9.8) \times 2.4$ $v = 8.6$ so 8.6 m s^{-1} .	M1 A1 A1	Use of $v^2 = u^2 + 2as$ or complete sequence of correct <i>suvat</i> . Accept sign errors in substitution. All correct cao [Award all marks if 8.6 seen WWW] Do not condone ± 8.6 .
		3	

Q 2	mark	comment
<p>either for u first: $8 = \frac{1}{2}(u + 2.25) \times 32$ $u = -1.75$ so 1.75 m s^{-1} $2.25 = -1.75 + 32a$ $a = 0.125$ so 0.125 m s^{-2} Directions of u and a are defined</p>	<p>M1 A1 M1 F1 F1 5</p>	<p>Using $s = \frac{1}{2}(u + v)t$ Use of any appropriate <i>suvat</i> with their values and correct signs Sign must be consistent with their u, FT from their value of u Establish directions of both u and a in terms of A and B. May be shown by a diagram, eg showing A and B and a line between them together with an arrow to show the positive direction. Without a diagram, the wording must be absolutely clear: eg do not accept left/right, forwards/backwards without a diagram or more explanation. Dependent on both M marks.</p>
<p>Or for a first: $8 = 2.25 \times 32 - \frac{1}{2} \times a \times 32^2$ $a = 0.125$ so 0.125 m s^{-2} $2.25 = u + 32 \times 0.125$ $u = -1.75$ so 1.75 m s^{-1} Directions of u and a are defined</p>	<p>M1 A1 M1 F1 F1 5</p>	<p>Using $s = vt - \frac{1}{2}at^2$ Use of any appropriate <i>suvat</i> with their values and correct signs Sign must be consistent with their a, FT from their value of a Establish directions of both u and a in terms of A and B. May be shown by a diagram, eg showing A and B and a line between them together with an arrow to show the positive direction. Without a diagram, the wording must be absolutely clear: eg do not accept left/right, forwards/backwards without a diagram or more explanation. Dependent on both M marks.</p>
<p>Or using simultaneous equations Set up one relevant equation with a and u. Set up second relevant equation with a and u. Solving to find $u = -1.75$ so 1.75 m s^{-1} Solving to find $a = 0.125$ so 0.125 m s^{-2} Directions of u and a are defined</p>	<p>M1 M1 A1 F1 F1 5</p>	<p>Using one of $v = u + at$, $s = ut + \frac{1}{2}at^2$ and $v^2 = u^2 + 2as$ Using another of $v = u + at$, $s = ut + \frac{1}{2}at^2$ and $v^2 = u^2 + 2as$ FT from their value of u or a, whichever found first Establish directions of both u and a in terms of A and B. May be shown by a diagram, eg showing A and B and a line between them together with an arrow to show the positive direction. Without a diagram, the wording must be absolutely clear: eg do not accept left/right, forwards/backwards without a diagram or more explanation. Dependent on both M marks.</p>
	<p>5</p>	

Q 3		mark	Notes
(i)	$-6 = -2 \times 3$ so $y = 3 \times 3 = 9$ and $z = -4 \times 3 = -12$	M1 A1 2	May be implied Both correct [Award 2 for both correct answers seen WW]
(ii)	$\begin{pmatrix} -2 \\ 3 \\ -4 \end{pmatrix} + \begin{pmatrix} 3 \\ -5 \\ -1 \end{pmatrix} = 5\mathbf{a}$ $\mathbf{a} = \begin{pmatrix} 0.2 \\ -0.4 \\ -1 \end{pmatrix} \text{ so accn is } \begin{pmatrix} 0.2 \\ -0.4 \\ -1 \end{pmatrix} \text{ m s}^{-2}$ Magnitude is $\sqrt{0.2^2 + (-0.4)^2 + (-1)^2}$ $= 1.09544\dots$ so 1.10 m s^{-2} , (3 s. f.)	M1 B1 A1 M1 F1 5	Use of Newton's 2 nd Law in vector form for all 3 cpts of attempted resultant Treat use of wrong vectors as MR. Correct LHS The acceleration may be written as a magnitude in a given direction. FT their values. Condone missing brackets. Condone no – signs. Accept 1.1. Accept surd form. Must come from a vector with 3 non-zero components for a
		7	

Q 4	mark	Comment
(i)	B1 B1 2	Any one force in correct direction correctly labelled with arrow or all forces with correct directions and arrows. A force may be replaced by its components if labelled correctly eg $mg\cos 20^\circ$, $mg\sin 20^\circ$. All correct (Accept words for labels and weight as W , mg , 147 (N)) No extra or duplicate forces. Do not allow force and its components unless components are clearly distinguished, eg by broken lines.
(ii)	M1 A1 A1 3	Either Up the plane $P\cos 20^\circ - 15 \times 9.8 \times \sin 20^\circ = 0$ $P = 53.50362\dots$ so 53.5 (3 s. f.)
	M1 A1 A1 3	Or Vertically and horizontally $R\cos 20^\circ = 15g$, $R\sin 20^\circ = P$ Eliminate R $P = \frac{15g}{\cos 20^\circ} \times \sin 20^\circ$ $P = 53.5$ (3.s.f.)
	M1 A1 A1 3	Or Triangle of forces Triangle drawn and labelled $\frac{P}{15g} = \tan 20^\circ$ $P = 53.5$ (3.s.f.)
	5	

Q 5		mark	notes
	<p>Usual notation either consider height: Attempt to substitute for u and a in $s = ut + \frac{1}{2}at^2$ $y = 30\sin 35 t - 4.9t^2$ Need $y = 0$ for time of flight T giving $T = \frac{30\sin 35}{4.9}$ (= 3.511692...)</p>	<p>M1 A1 B1 A1</p>	<p>Accept: g as $g, \pm 9.8, \pm 9.81, \pm 10; u = 30; s \leftrightarrow c$. Derivation need not be shown cao. Any form. May not be explicit.</p>
	<p>Or Consider time to top Attempt to substitute for u and a in $v = u + at$ $v = 30\sin 35 - 9.8t$ Need $v = 0$ and to double for time of flight T giving $T = \frac{30\sin 35}{4.9}$ (= 3.511692...)</p>	<p>M1 A1 B1 A1</p>	<p>Accept: g as $g, \pm 9.8, \pm 9.81, \pm 10; u = 30; s \leftrightarrow c$. Derivation need not be shown cao. Any form. May not be explicit.</p>
	<p>then $x = 30\cos 35 T$ so $x = 30\cos 35 \times \frac{30\sin 35}{4.9}$ (= 86.29830...) Required time for sound is $x/343$ Total time is $3.511692... + 0.251598... = 3.76329...$ so 3.76 s (3 s. f.)</p>	<p>M1 F1 M1 A1</p>	<p>Accept $s \leftrightarrow c$ if consistent with above FT for their time Condone consistent $s \leftrightarrow c$ error (which could lead to correct answer here). FT from their x cao following fully correct working throughout question.</p>
		8	

Q6		mark	notes
(i)	<p>Either using <i>suvat</i>: Use of $\mathbf{v} = \mathbf{u} + \mathbf{t}\mathbf{a}$ $\mathbf{v} = 4\mathbf{i} - 2t\mathbf{j}$ Use of $\mathbf{r} = (\mathbf{r}_0 +) \mathbf{t}\mathbf{u} + \frac{1}{2} t^2 \mathbf{a}$ $+ 3\mathbf{j}$ $\mathbf{r} = 4t\mathbf{i} + (3 - t^2)\mathbf{j}$</p>	<p>M1 A1 M1 B1 A1 5</p>	<p>Column vectors may be used throughout; lose 1 mark once if j components put at top or if fraction line included. . Notation used must be clear.</p> <p>substitution required. Must be vectors.</p> <p>substitution required. \mathbf{r}_0 not required. Must be vectors. May be seen on either side of a meaningful equation for r Accept $\mathbf{r} = 3\mathbf{j} + 4t\mathbf{i} - \frac{1}{2} \times 2 \times t^2 \mathbf{j}$ oe written in a correct notation. Isw, providing not reduced to scalar: (see 12c in marking instructions)</p>
	<p>Or using integration: $\mathbf{v} = \int \mathbf{a} dt$ $\mathbf{v} = 4\mathbf{i} - 2t\mathbf{j}$ $\mathbf{r} = \int \mathbf{v} dt$ $+ 3\mathbf{j}$ $\mathbf{r} = 4t\mathbf{i} + (3 - t^2)\mathbf{j}$</p>	<p>M1 A1 M1 B1 A1 5</p>	<p>Attempt at integration. Condone no '+c'. Must be vectors.</p> <p>cao</p> <p>Integrate their v but must contain 2 components. Must be vectors.</p> <p>May be seen on either side of a meaningful equation for r Accept $\mathbf{r} = 3\mathbf{j} + 4t\mathbf{i} - \frac{1}{2} \times 2 \times t^2 \mathbf{j}$ oe written in a correct notation. Isw, providing not reduced to scalar: (see 12e in marking instructions)</p>
		5	
(ii)	<p>$\mathbf{v}(2.5) = 4\mathbf{i} - 5\mathbf{j}$ Angle is $(90+) \arctan \frac{5}{4}$ $= 141.34019\dots$ so 141° (3 s. f.)</p>	<p>B1 M1 A1 3</p>	<p>FT their v Award for arctan attempted oe. FT their values. Allow argument to be \pm (their i cpt)/(their j cpt) or \pm (their j cpt)/(their i cpt). Allow this mark if bearing of position vector attempted.</p> <p>cao</p>
		8	

Q7		mark	notes
(i)	$\frac{-20}{2} = -10$ -10 m s^{-2}	M1 A1 2	Use of a suitable triangle to attempt at $\Delta v / \Delta t$ for suitable interval. Accept wrong sign. cao. Allow both marks if correct answer seen.
(ii) (A) (B)	Signed area under graph $\frac{1}{2} \times 2 \times 20 = 20$	M1 A1	Using the relevant area or other complete method
	either using areas Signed area $2 \leq t \leq 5$ is $\frac{1}{2} \times ((5-2) + (4.5-2.4)) \times (-4) = -10.2$ Signed area $5 \leq t \leq 6$ is $\frac{1}{2} \times 1 \times 8 = 4$ Total displacement is 13.8 m	B1 B1 B1	Allow + 10.2. cao but FT from their 20 in part (A)
	or using <i>suvat</i> From $t = 0$ to $t = 2.4$: 19.2 From $t = 4.5$ to $t = 6$: 3.0 From $t = 2.4$ to $t = 4.5$: -8.4 Total : 13.8	B1 B1 B1	Both required and both must be correct.
		5	
(iii)	$a = 4t - 14$ $a(0.5) = -12$ so -12 m s^{-2}	M1 A1 A1 3	Differentiate. Do not award for division by t .
(iv)	Model A gives -4 m s^{-1} For model B we need v when $a = 0$ $v(\frac{7}{2}) = -4.5$ so model B is 0.5 m s^{-1} less	B1 M1 A1 F1 4	May be implied by other working Using (iii) or an argument based on symmetry or sketch graph that $a = 0$ when $t = 3.5$ Accept values without more or less

(v)	<p>Displacement is $\int_0^6 (2t^2 - 14t + 20) dt$</p> $= \left[\frac{2t^3}{3} - 7t^2 + 20t \right]_0^6$ <p>= 12 so 12 m.</p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>4</p>	<p>Do not penalise poor notation</p> <p>Limits not required.</p> <p>Limits not required. Accept 2 terms correct.</p> <p>Substitute limits cao. Accept bottom limit not substituted.</p>
		18	

Q 8		mark	notes
(i)	25 N	B1 1	Condone no units. Do not accept -25 N.
(ii)	50 cos25 = 45.31538... so 45.3 N (3 s. f.)	M1 A1 2	Attempt to resolve 50 N. Accept $s \leftrightarrow c$. No extra forces. cao but accept -45.3.
(iii)	Resolving vertically $R + 50 \sin 25 - 8 \times 9.8 = 0$ $R = 57.26908...$ so 57.3 N (3 s. f.)	M1 A1 A1 3	All relevant forces with resolution of 50 N. No extras. Accept $s \leftrightarrow c$. All correct.
(iv)	Newton's 2 nd Law in direction DC $50 \cos 25 - 20 = 18a$ $a = 1.4064105...$ so 1.41 m s ⁻² (3 s. f.)	M1 A1 A1 3	Newton's 2nd Law with $m = 18$. Accept $F = mga$. Attempt at resolving 50 N. Allow 20 N omitted and $s \leftrightarrow c$. No extra forces. Allow only sign error and $s \leftrightarrow c$. cao
Q8	continued		
(v)	Resolution of weight down the slope	B1	$mg \sin 5^\circ$ where $m = 8$ or 10 or 18, wherever first seen
	either Newton's 2 nd Law down slope overall $18 \times 9.8 \times \sin 5 - 20 = 18a$ $a = -0.2569...$ Newton's 2 nd Law down slope. Force in rod can be taken as tension or thrust. Taking it as tension T gives For D: $10 \times 9.8 \times \sin 5 - 15 - T = 10a$ (For C: $8 \times 9.8 \times \sin 5 - 5 + T = 8a$) $T = -3.888...$ = -3.89 N (3 s. f.) The force is a thrust	M1 A1 M1 F1 A1 A1	$F = ma$. Must have 20 N and $m = 18$. Allow weight not resolved and use of mass. Accept $s \leftrightarrow c$ and sign errors (including inconsistency between the 15 N and the 5 N). cao $F = ma$. Must consider the motion of either C or D and include: component of weight, resistance and T . No extra forces. Condone sign errors and $s \leftrightarrow c$. Do not condone inconsistent value of mass. FT only applies to a , and only if direction is consistent. '+ T ' if T taken as a thrust '- T ' if T taken as a thrust If T taken as thrust, then $T = +3.89$. Dependent on T correct

<p>or Newton's 2nd Law down slope. Force in rod can be taken as tension or thrust. Taking it as tension T gives</p> <p>For C: $8 \times 9.8 \times \sin 5 - 5 + T = 8a$ For D: $10 \times 9.8 \times \sin 5 - 15 - T = 10a$ $a = -0.2569\dots$ $T = -3.888\dots = -3.89$ N (3s.f.)</p> <p>The force is a thrust</p>	<p>M1 M1 A1 A1 F1 A1</p>	<p>$F = ma$. Must consider the motion of C and include: component of weight, resistance and T. No extra forces. Condone sign errors and $s \leftrightarrow c$. Do not condone inconsistent value of mass.</p> <p>$F = ma$. Must consider the motion of D and include: component of weight, resistance and T. No extra forces. Condone sign errors and $s \leftrightarrow c$. Do not condone inconsistent value of mass.</p> <p>Award for either the equation for C or the equation for D correct. '-T' if T taken as a thrust '+T' if T taken as a thrust</p> <p>First of a and T found is correct. If T taken as thrust, then $T = +3.89$.</p> <p>The second of a and T found is FT</p> <p>Dependent on T correct</p>
<p>then After 2 s: $v = 3 + 2 \times a$ $v = 2.4860303\dots$ so 2.49 m s⁻¹ (3 s. f.)</p>	<p>M1 F1 9</p>	<p>Allow sign of a not followed. FT their value of a. Allow change to correct sign of a at this stage. FT from magnitude of their a but must be consistent with its direction.</p>
	<p>18</p>	