OCR
Oxford Cambridge and RSA

GCE

Mathematics (MEI)

Unit **4754A**: Applications of Advanced Mathematics: Paper A

Advanced GCE

Mark Scheme for June 2017

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Annotations and abbreviations

Annotation in scoris	Meaning
and	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0 , M1	Method mark awarded 0, 1
A0 , A1	Accuracy mark awarded 0, 1
B0 , B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

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M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance
1	(i)		$\frac{5-x}{(2-x)(1+x)} = \frac{A}{2-x} + \frac{B}{1+x}$ $\Rightarrow 5 - x = A(1 + x) + B(2 - x)$ $x = 2 \Rightarrow 3 = 3A,\ A = 1$ $x = -1 \Rightarrow 6 = 3B \Rightarrow B = 2$	M1 A1 A1 **[3]**	Cover up, substitution or equating coefficients
	(ii)		$\frac{A}{2-x} = \frac{A}{2}(1-\frac{1}{2}x)^{-1}$	B1	Or equivalent
			$= \frac{A}{2}(1+(-1)(-\frac{1}{2}x)+\frac{(-1)(-2)}{2!}(-\frac{1}{2}x)^2+...)$	M1	All three correct **unsimplified** binomial coefficients (not nCr) soi **for either expansion** i.e. 1, -1 and $\frac{(-1)(-2)}{2}$. Or correct simplified coefficients seen
			$= A\left(\frac{1}{2}+\frac{1}{4}x+\frac{1}{8}x^2+...\right)$	A1ft	Ignore any subsequent incorrect terms – ft their A from (i) only
			$\frac{B}{1+x} = B(1+x)^{-1} = B(1-x+x^2+...)$	A1ft	Ignore any subsequent incorrect terms – ft their B from (i) only
			$\frac{5-x}{(2-x)(1+x)} = \frac{5}{2}-\frac{7}{4}x+\frac{17}{8}x^2+...$	A1 **[5]**	www cao – ignore any higher order terms stated – isw after correct expansion seen
2	(i)		$x = -1 + 2\lambda,\ y = -3\lambda,\ z = 1 + \lambda$	M1*	Condone a single sign slip – maybe implied by later working
			$\Rightarrow 3(-1 + 2\lambda) + 4(-3\lambda) - (1 + \lambda) = 17$	M1dep*	Substituting their x, y and z into the correct plane equation (condone a further single slip if intention clear)
			$\Rightarrow -7\lambda = 21,\ \lambda = -3$	A1	www cao
			point of intersection is therefore $(-7, 9, -2)$	A1 **[4]**	www cao – condone $x = -7$, $y = 9$, $z = -2$ – condone answer given as a column vector

Question			Answer	Marks	Guidance
	(ii)		$\mathbf{u} = 3\mathbf{i} + 4\mathbf{j} - \mathbf{k}$, $\mathbf{v} = 2\mathbf{i} - 3\mathbf{j} + \mathbf{k}$ $\cos\theta = \frac{3\times2+4\times(-3)+(-1)\times1}{\sqrt{26}\sqrt{14}}$ $\theta = 111.5°$ so acute angle between line and normal is 68.5°	B1 M1 A1 A1 **[4]**	Identifying and using vectors **u** and **v** and no other vectors Correct formula (including cosine), with correct substitution of their **u** and their **v** – with at least one vector correct. Condone either a single numerical slip **or** a single sign slip (but not one of each) if intention is clear. It must be clear where the slip comes from e.g. if the magnitude of one vector is stated incorrectly with no working then M0 Allow 111 or 112 or 1.94 or 1.95 (must come from correct working) – may be implied by the correct answer of 68.5 www cao (or 1.20 in radians) or better (68.47546…, 1.195122…) A0 if choice of answers If sine used then it must be a complete method for finding the correct angle - so for the M mark need to see $\theta = \sin^{-1}\left(\frac{3\times2+4\times(-3)+(-1)\times1}{\sqrt{26}\sqrt{14}}\right)+90°$ (oe) if no attempt to add 90 M0
3	(i)		$T_4 < T_2$ The approximation is an over-estimate, as the trapezia are above the curve therefore the error becomes less when the number of strips increases	B1 B1 **[2]**	oe (e.g. T_4 is less than T_2) Must see mention of 'over-estimate' and 'above' and 'increasing strips'
	(ii)		0.5×0.5 $\{1.0655... + 2.8963... + 2(1.1695... + 1.4142... + 1.9282...)\}$ $T_4 = 3.25$	B1 M1 A1 A1 **[4]**	For using 0.25 oe The M mark requires the correct $\{...\}$ bracket structure. It needs the first bracket to contain the first y value plus the last y value and the second bracket to be multiplied by 2 and to be the summation of the remaining y values with no additional values. Allow an error in one value **or** the omission of one value from the second bracket M0 if using x values. All values given to at least 3sf or exact The A mark is for the correct $\{...\}$ bracket with no errors (12.98… or 13.0 implies M1A1) cao (3.25 with no working scores 0/4) – must be given to 2dp only (for reference correct answer is 3.2465079…) SC: bracketing error $0.25\times(1.0655+2.8963)+2(1.1695...+...)$ scores B1M1A0A0 unless the final answer implies the correct calculation. An answer of 10.014… usually indicates this error

Question			Answer	Marks	Guidance
		OR	0.5587…+ 0.6459… + 0.8356…+ 1.2061…		Separate trapezia B1 – one area correct (implies 0.25) M1 – three correct (equivalent to one error) A1 – all four correct A1 – cao of 3.25
4			$\|\mathbf{v}\| = \sqrt{a^2+b^2+25} = \sqrt{27}$	M1*	Condone one error on either lhs or rhs (but not both)
			$\Rightarrow a^2 + b^2 = 2$	A1	oe - allow unsimplified but square roots must be removed
			$a - 7b - 10 = 0$	B1	cao (oe) – must include = 0 (if all terms collected on one side)
			$a = 7b + 10 \Rightarrow (7b + 10)^2 + b^2 = 2$	M1dep*	Correct method for solving their simultaneous equations – which would result in a three-term quadratic equation (= 0 maybe implied) in either a or b – if correct the equation in a is $49a^2 + (a-10)^2 = 98$
			$b = -\frac{7}{5}$	A1	www cao
			$a = \frac{1}{5}$	A1	www cao - award B1 for each correct answer if no working seen
				[6]	
5			$4\tan\theta\tan 2\theta = 1 \Rightarrow 4\tan\theta.\frac{2\tan\theta}{1-\tan^2\theta} = 1$	M1*	Use of double angle formula for tan to get an equation in tan – allow one sign slip only
			$\Rightarrow 8\tan^2\theta = 1 - \tan^2\theta$		
			$\Rightarrow \tan^2\theta = 1/9$	M1dep*	Re-arranges to $\tan^2\theta = k$ where $k > 0$ or attempt to solve $a\tan^2\theta - b = 0$ where b/a > 0
			$\tan\theta = 1/3$ or $-1/3$		
			$\theta = 18.43°$ or $161.57°$	A1	One correct answer to at least 1dp
			$\theta = 18.43°$ and $161.57°$	A1	Both answers correct to at least 1dp
					SC A1A0 for answers which round to 0.322 **and** 2.82 (radians) Answers with no working can score B1 B1 (max 2/4) if correct Ignore additional solutions outside the range. If any additional solutions given inside the range of $0 < \theta < 180$ and full marks would have been awarded then remove last mark (so 3/4)
				[4]	

Question			Answer	Marks	Guidance
6	(i)		$\frac{dP}{dt} = k\sqrt{P}$	B1 **[1]**	oe – condone $dP = k\sqrt{P}dt$
	(ii)		$P = (A + Bt)^2 \Rightarrow \frac{dP}{dt} = 2(A+Bt)B$	M1	Attempt at chain rule – allow this mark if B absent or incorrect but must include $2(A+Bt)$ - if multiplied out condone one error in differentiation
			$\frac{dP}{dt} = 2B\sqrt{P}$	A1	oe - correctly showing that the rate of increase of P is proportional to the square root of P
			$k = 2B$	A1 **[3]**	Not for $B = \frac{k}{2}$
		OR	$\int \frac{dP}{\sqrt{P}} = k\int dt \Rightarrow 2\sqrt{P} = kt(+c)$	M1	Separates their variables correctly and attempt to integrate for their differential equation given in (i) – for an attempt powers must increase by 1 (oe) but not for k – condone lack of + c
			$P = \left(\frac{1}{2}kt + A\right)^2$	A1	Correct integration - accept any (correct) constant for A – coefficient of t maybe implicit stated e.g. $P = (Ct + A)^2$ with $k = 2C$ seen
			$k = 2B$	A1	Not for B = …
7	(i)		$\int 4(y+3)\,dy = \int 9x\,dx$	M1	Separating variables – if no subsequent work then integral signs needed, but allow omission of dx and/or dy but must be correctly placed if present – allow errors in re-arranging but x and y on separate sides
			$2y^2 + 12y = ...$	A1	oe - Correct integration of y terms (or $2(y+3)^2$ oe)
			$... = \frac{9}{2}x^2(+c)$	A1	oe - Correct integration of x terms – condone omission of + c for these two A marks
			when x = 0, y = 0, $\Rightarrow$ 0 = 0 + c, so c = 0	B1	cao (previous three marks must have been awarded) – need to show that c = 0. As a minimum x = 0, y = 0, c = 0
			$\Rightarrow 4y^2 + 24y = 9x^2$ $\Rightarrow 9x^2 - 4y^2 - 24y = 0$*	A1	AG www (dependent on all previous marks in this part) – must show at least one step of intermediate working after integration – must have = 0
				[5]	SC: working backwards – B1 for $18x - 8y\frac{dy}{dx} - 24\frac{dy}{dx} = 0$ and then B1 for re-arranging correctly to $\frac{dy}{dx} = \frac{9x}{4(y+3)}$ (so max. 2/5)

Question	Answer	Marks	Guidance
(ii)	$V = \pi\int_0^2 x^2 \, dy$	M1	M1 for $k(\pi)\int_0^2 x^2(dy)$ with correct limits and $k = 1$ or $\frac{1}{2}$, allow correct limits seen or implied later. If formula not stated then must substitute for their x^2 to imply this formula – condone lack of π for the M mark and dy throughout (condone incorrect use of dx too)
	$= \pi\int_0^2 \frac{4}{9}(y^2 + 6y) \, dy$	A1	Correct (or with a factor of ½) – limits may be seen or implied through later working
	$\frac{4}{9}\left(\frac{y^3}{3} + 3y^2\right)$	B1	or $\frac{2}{9}\left(\frac{y^3}{3} + 3y^2\right)$ (but only if k = ½)
	$= \frac{176}{27}\pi$ or $6\frac{14}{27}\pi$	A1	Exact (oe) – mark final answer (so no isw if correct answer is e.g. halved) but if exact value seen and is then followed by 20.4785… then isw
		[4]	

Question		Answer	Marks	Guidance
(iii)	(A)	$...=9(2\tan\theta)^2-4(3(\sec\theta-1))^2-24(3(\sec\theta-1))$	M1	Sub. for x and y – condone one slip e.g. using $3\sec\theta-1$
		$=36\tan^2\theta-36\sec^2\theta+72\sec\theta-36-72\sec\theta+72$	A1	cao (multiplied out) – accept if = 0 and multiplied out correctly
		$=36\tan^2\theta-36\left(1+\tan^2\theta\right)+72\sec\theta-36-72\sec\theta+72$	M1	**Use** of correct formula $\sec^2\theta=1+\tan^2\theta$ (or $\sin^2\theta+\cos^2\theta=1$ if converted to sin/cos) soi - maybe awarded before previous A mark – for those that set the lhs = 0 this mark can be awarded for a clear attempt to derive one of these identities
		$=-36-36+72=0$*	A1	AG www – all working must be explicitly shown including use of relevant Pythagorean identity. If candidates set their expression equal to zero then all four marks can be awarded provided all working is correct (so no dividing through by a constant before the expression has been put equal to zero) **and** a conclusion is required (so for those that derive a correct trig. identity then we must see a statement such as 'true' etc. accept as a minimum a tick. If following working from both sides of their equation 0 = 0 seen then either a statement or re-stating the equation $9x^2-4y^2-24y=0$ is required) – no θ missing in tan etc.
(iii)	(B)	$\frac{dy}{dx}=\frac{18\tan\theta}{12\sec\theta}$	B1	Maybe implied by later working
		$=\frac{18\frac{\sin\theta}{\cos\theta}}{12\frac{1}{\cos\theta}}$	M1	Correct use of both $\tan\theta=\frac{\sin\theta}{\cos\theta}$ and $\sec\theta=\frac{1}{\cos\theta}$ give bod if stated somewhere in their solution
		$=\frac{3}{2}\sin\theta$	A1	$k=\frac{3}{2}$ oe - need not be explicitly stated so accept $\frac{3}{2}\sin\theta$
		when x = 2, $\theta=\pi/4$	B1	Must be exact – condone $\theta=45^\circ$ or $y=3\left(\sqrt{2}-1\right)$
		$\frac{dy}{dx}=\frac{3}{2}\sin\frac{\pi}{4}=\frac{3}{2\sqrt{2}}$	A1	oe $\left(\text{eg }\frac{3\sqrt{2}}{4}\right)$ – must be exact (isw if non-exact seen after exact) – correct final answer implies previous B mark
			[9]	

Question	Answer	Marks	Guidance
8 (i)	x = 1 $\theta = 0$ y = cos 0 + 2sin 0 = 1, (so B is (1, 1))	B1 B1 B1 **[3]**	Condone if derived from $\theta = \pi$ Must be explicitly stated www (not from $2\theta = 2\pi$) - condone x = 1 and y = 1
(ii)	$\cos\theta + 2\sin\theta = R(\cos\theta\cos\alpha + \sin\theta\sin\alpha)$ $\Rightarrow R\cos\alpha = 1, R\sin\alpha = 2$ $R^2 = 1^2 + 2^2 = 5 \Rightarrow R = \sqrt{5}$ $\tan\alpha = 2 \Rightarrow \alpha = 1.107$	M1 A1 B1 M1 A1 **[5]**	Correct pairs. Condone sign errors only for the M mark – allow use of θ for α **only** if recovered later by stating a value for $\tan\alpha$ or α not θ Or 2.24 or better, not $\pm\sqrt{5}$ unless $+\sqrt{5}$ chosen For M1 ft their pairs (condone sign errors but division must be the correct way round). A1 for 1.107 or better (accept 1.11), with no errors seen in method for angle – A0 if in degrees SC: If candidates state that $\cos\alpha = 1, \sin\alpha = 2 \Rightarrow \tan\alpha = 2$ this could score M0 A0 B1 M1 A1 (so max 3/5) Note that candidates who state the correct values of R and α with no (wrong) working seen can score full marks
(iii)	At A, $\cos(\theta - \alpha) = 0$, so $\theta - \alpha = \pi/2$, $\theta = 2.678$ Or $\cos\theta + 2\sin\theta = 0 \Rightarrow 1 + 2\tan\theta = 0 \ \therefore \theta = -0.464$ $x = 0.6, y = 0$ At C, $\cos(\theta - \alpha) = 1$, so $\theta - \alpha = 0$, $\theta = 1.107$ $x = -0.6$ $y = \sqrt{5}$	B1ft B1 B1ft B1 B1ft **[5]**	$\theta = \frac{\pi}{2} + \text{their}\,\alpha$ ($\theta = 2.677945\ldots$) or $-0.4636476...$ from tan - allow in degrees ($-26.56505\ldots$) www their x value must be given exactly or correctly rounded to 0.6 (condone trailing 0s) but do not accept 0.600087…must also state y = 0 $\theta = \text{their}\,\alpha$ from (ii) (allow $\theta = 2.03444...$) - allow in degrees (63.4349…) or from $-\sin\theta + 2\cos\theta = 0$ (condone if stated as dy/dx) www exact or correctly rounded to -0.6 (do not accept e.g. $-0.5997...$ as their final answer) Exact or 2.24 or better (2.2360679…) or ft their R

Question	Answer	Marks	Guidance
(iv)		M1*	M1 for their $(\mathrm{dy}/\mathrm{d}\theta)$ / their $\left(\mathrm{dx}/\mathrm{d}\theta\right)$ in terms of θ with at least one term correct (soi)
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y/\mathrm{d}\theta}{\mathrm{d}x/\mathrm{d}\theta} = \frac{2\cos\theta - \sin\theta}{-2\sin 2\theta}$	A1	cao (oe) – allow unsimplified even if subsequently cancelled incorrectly i.e. can isw
	when $\theta = 2.678, \frac{\mathrm{dy}}{\mathrm{dx}} = -1.398$ or $\theta = -0.464, \frac{\mathrm{dy}}{\mathrm{dx}} = 1.398$	A1	Allow $\pm$ 1.40 or better ($\pm$ 1.3975424…)
	tan ½ β = 1.398	M1dep*	$\tan\left(\frac{1}{2}\beta\right) = \pm\left.\frac{\mathrm{dy}}{\mathrm{dx}}\right\|_{\theta \text{ at A}}$ so this mark can be awarded for putting $\tan\left(\frac{1}{2}\beta\right)$ equal to either +/- their numerical gradient at A but must be using θ values and not x values
	angle between branches = 1.90 (radians)	A1 **[5]**	1.90 or better (1.8994312…) – not in degrees

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A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question		Answer	Marks	Guidance
1.	(i)	(All x_0 for which) $0 < x_0 < 1$	B1	Condone as separate inequalities – allow x, x_n, etc. for x_0 Accept (0, 1) (i.e. correct set-notation) – must be strict inequalities
			[1]	
	(ii)	$x_0 = 0$ or 1	B1	Both required - allow x, x_n, etc. for x_0 - condone 0, 1 stated without x_0 (0 and 1 must not appear as part of a range of values)
			[1]	

2.	**(i)**	$x = 1.6x(1-x)$ $x(1.6x-0.6)=0$ $(x=0$ or) $x=\frac{0.6}{1.6}=0.375$ or $\frac{3}{8}$	**M1** **A1**	For correctly replacing x_{n+1} **and** x_n with x - must be using 1.6 Using an iterative approach or stating the correct answer with no working - M0 A0 Accept as a minimum the correct equation stated in terms of x only followed by the correct answer for both marks
			[2]	
2.	**(ii)**	$x^2-x+2=0$ Discriminant $=(-1)^2-4\times1\times2=-7$ or $x=\frac{1\pm\sqrt{(-1)^2-4(1)(2)}}{2}$ so no (real) roots (oe)	**M1** **A1**	Re-arranging **and** correct attempt at either the discriminant or solving using the correct formula/completing the square for their 3-term quadratic equation. Allow consideration of $\sqrt{b^2-4ac}$ with correct substitution of their values only Correct working with a correct conclusion (so if discriminant stated must be -7 but may be awarded for $1-4(2)\Rightarrow$ no (real) roots/no points of equilibrium) No marks for non-algebraic approach
			[2]	

PLEASE NOTE THAT THE MARKS FOR BOTH 3(i) and 3(ii) MUST BE AWARDED TOGETHER (MAXIMUM OF 6)

3. (i)

	$x_{n+1} = kx_n(1-x_n) - 0.25$			
	k = 2	k = 3	k = 4	k = 5
x_0	0.55	0.55	0.55	0.55
x_1	0.245	0.4925	0.74	0.9875
x_2	0.1199...	0.4998...	0.5196	-0.1882...
x_3	-0.0388...	0.4999...	0.7484...	...
x_4	...	0.5	0.5030...	...
x_5	...	...	0.7499...	...
x_6	...	...	0.5000...	...
x_7	...	...	0.7499...	...
x_8	...	...	0.5000...	...
x_9			0.75	
x_{10}			0.5	
...	...	...	...	...

There may be some variation in the columns for k = 3 and k = 4. Some calculators will round to the final values earlier than others. **In general answers must be given to at least 2dp**

Allow correct truncated or rounded answers

One mark for each column taken far enough

B1 $k = 2$ correct to at least x_3

B1 $k = 3$ correct to at least x_3 (accept 0.5)

B1 $k = 4$ correct to at least x_6 (accept 0.5)

B1 $k = 5$ correct to at least x_2

isw after reaching the values for each column as stated above

3.	**(ii)**		**B2**	All 4 correct Allow B1 for 2 correct
		For k = 2 it is **unbounded** or '**dying out**'		Allow equivalent to 'dying out' provided unambiguous
		For k = 3 it **converges** (to x = 0.5) or **stable** or **equilibrium**		Must be one of these three words
		For k = 4 it **oscillates** (between x = 0.5 and x = 0.75) or **alternates** but not **fluctuates**		Must be one of these two words
		For k = 5, it is **unbounded** or '**dying out**'		Allow equivalent to 'dying out' provided unambiguous
			[6]	

4.	**(i)**	$3.5644+\dfrac{0.0203}{4.6692}$	**B1**	Correct method - allow a slip in one value (e.g. 4.692) only – either correct value for k implies this mark
		$=3.5687$	**B1**	Accept 3.569 (3.5687476…) – no ft from incorrect values – accept if given as the upper bound of an interval
		$3.5687+0.0203\times\left(\dfrac{1}{4.6692}\right)^2=3.5696$ to 5 sf	**B1**	Accept 3.5697 or 3.570 but not 3.57 – no ft from incorrect values – accept if given as an upper bound
			[3]	
4	**(ii)(A)**	$S=\dfrac{1}{1-\dfrac{1}{4.6692}}$	**M1**	Use of the correct formula for the sum to infinity of a GP – allow a slip in the value of r only (e.g. 4.692) – allow r stated to 2dp or better (r = 0.21416945…)
		$S=1.2725...$	**A1**	Accept 1.273 not 1.27 (1.2725389…) M0 A0 if formula not used
			[2]	
4	**(ii)(B)**	It is an estimate of a value of k for which chaos is occurring	**B1**	Accept any mention of 'chaos'
			[1]	

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