

GCE

Mathematics (MEI)

Unit **4767**: Statistics 2

Advanced GCE

Mark Scheme for June 2014

1. Annotations and abbreviations

Annotation in scoris	Meaning
BP	Blank Page – this annotation must be used on all blank pages within an answer booklet (structured or unstructured) and on each page of an additional object where there is no candidate response.
✓ and ✕	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	Meaning
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

2. Subject-specific Marking Instructions for GCE Mathematics (MEI) Statistics strand

- a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

- b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

- c The following types of marks are available.

M

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B

Mark for a correct result or statement independent of Method marks.

E

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation *isw*. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

- d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e The abbreviation *ft* implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

- f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

Candidates are expected to give numerical answers to an appropriate degree of accuracy. 3 significant figures may often be the norm for this, but this always needs to be considered in the context of the problem in hand. For example, in quoting probabilities from Normal tables, we generally expect *some* evidence of interpolation and so quotation to 4 decimal places will often be appropriate. But even this does not always apply – quotations of the standard critical points for significance tests such as 1.96, 1.645, 2.576 (maybe even 2.58 – but not 2.57) will commonly suffice, especially if the calculated value of a test statistic is nowhere near any of these values. Sensible discretion *must* be exercised in such cases.

Discretion must also be exercised in the case of small variations in the degree of accuracy to which an answer is given. For example, if 3 significant figures are expected (either because of an explicit instruction or because the general context of a problem demands it) but only 2 are given, loss of an accuracy ("A") mark is likely to be appropriate; but if 4 significant figures

are given, this should not normally be penalised. Likewise, answers which are slightly deviant from what is expected in a very minor manner (for example a Normal probability given, after an attempt at interpolation, as 0.6418 whereas 0.6417 was expected) should not be penalised. However, answers which are *grossly* over- or under-specified should normally result in the loss of a mark. This includes cases such as, for example, insistence that the value of a test statistic is (say) 2.128888446667 merely because that is the value that happened to come off the candidate's calculator. Note that this applies to answers that are given as final stages of calculations; intermediate working should usually be carried out, and quoted, to a greater degree of accuracy to avoid the danger of premature approximation.

The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h Genuine misreading (of numbers or symbols, occasionally even of text) occurs. If this results in the object and/or difficulty of the question being considerably changed, it is likely that all the marks for that question, or section of the question, will be lost. However, misreads are often such that the object and/or difficulty remain substantially unaltered; these cases are considered below.

The simple rule is that *all* method ("M") marks [and of course all independent ("B") marks] remain accessible but at least some accuracy ("A") marks do not. It is difficult to legislate in an overall sense beyond this global statement because misreads, even when the object and/or difficulty remains unchanged, can vary greatly in their effects. For example, a misread of 1.02 as 10.2 (perhaps as a quoted value of a sample mean) may well be catastrophic; whereas a misread of 1.6748 as 1.6746 may have so slight an effect as to be almost unnoticeable in the candidate's work.

A misread should normally attract *some* penalty, though this would often be only 1 mark and should rarely if ever be more than 2. Commonly in sections of questions where there is a numerical answer either at the end of the section or to be obtained and commented on (eg the value of a test statistic), this answer will have an "A" mark that may actually be designated as "cao" [correct answer only]. This should be interpreted *strictly* – if the misread has led to failure to obtain this value, then this "A" mark must be withheld even if all method marks have been earned. It will also often be the case that such a mark is implicitly "cao"

even if not explicitly designated as such.

On the other hand, we commonly allow "fresh starts" within a question or part of question. For example, a follow-through of the candidate's value of a test statistic is generally allowed (and often explicitly stated as such within the marking scheme), so that the candidate may exhibit knowledge of how to compare it with a critical value and draw conclusions. Such "fresh starts" are not affected by any earlier misreads.

A misread may be of a symbol rather than a number – for example, an algebraic symbol in a mathematical expression. Such misreads are more likely to bring about a considerable change in the object and/or difficulty of the question; but, if they do not, they should be treated as far as possible in the same way as numerical misreads, *mutatis mutandis*. This also applied to misreads of text, which are fairly rare but can cause major problems in fair marking.

The situation regarding any particular cases that arise while you are marking for which you feel you need detailed guidance should be discussed with your Team Leader.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question	Answer	Marks	Guidance
1 (ii)	<p>H_0: no association between diastolic blood pressure and systolic blood pressure in the population of young adults</p> <p>H_1: positive association between diastolic blood pressure and systolic blood pressure in the population of young adults</p> <p>One tail test critical value at 5% level is 0.5636</p> <p>Since $0.758 > 0.5636$, there is sufficient evidence to reject H_0,</p> <p>i.e. conclude that there is sufficient evidence to suggest that there is <u>positive association between diastolic blood pressure and systolic blood pressure</u> (in the population of young adults).</p>	<p>B1</p> <p>B1</p> <p>B1</p> <p>B1*</p> <p>M1dep*</p> <p>A1</p> <p>[6]</p>	<p>NB Hypotheses must be in context.</p> <p>H_0: no association, H_1: positive association, earns SC1</p> <p>Hypotheses must not be given in terms of ρ or mention correlation. Ignore references to ρ if hypotheses also given in words.</p> <p>For population of young adults seen at least once. Do not allow underlying population. B0 for population correlation coefficient.</p> <p>for 0.5636 cv from pmcc test = 0.5494 gets B0</p> <p>For a sensible comparison, leading to a conclusion, of their r_s with 0.5636, provided $0 < r_s < 1$ The comparison may be in the form of a diagram.</p> <p>For non-assertive, <u>correct conclusion in context</u>. f.t. their r_s. Allow “support” in place of “suggest”. Do not allow “show”, “imply”, “conclude” or “prove” in place of “suggest”. If a two-tailed test is carried out then award maxB1B0B1B1(for 0.6485)M0A0</p>
1 (iii)	<p>The scatter diagram does not appear to be roughly elliptical</p> <p>so the (population) may not have a bivariate Normal distribution.</p>	<p>E1</p> <p>E1</p> <p>[2]</p>	<p>For not elliptical. Allow “not oval”.</p> <p>For not bivariate Normal. Do not allow “not Normal bivariate”. Do not allow “the data does not have a bivariate Normal distribution”.</p>

Question		Answer	Marks	Guidance
1	(iv)	$H_0: \rho = 0$ $H_1: \rho > 0$ where ρ is the (population) correlation coefficient For $n = 10$, 1% critical value = 0.7155 Since $0.707 < 0.7155$ the result is not significant There is insufficient evidence at the 1% level to suggest that there is positive correlation between diastolic blood pressure and systolic blood pressure (in this population).	B1 B1 B1* M1dep* A1 [5]	Do not allow other symbols unless clearly defined as population correlation coefficient. Do not allow hypotheses solely in words. For defining ρ For 0.7155 For sensible comparison leading to a conclusion. Conclusion soi. For non-assertive correct conclusion in context . If a two-tailed test is carried out then award maxB0B1B1(for 0.7646)M0A0
2	(i)	‘Independently’ means that the occurrence of one defect does not affect the probability of another defect occurring. Poisson distribution	E1 E1 [2]	Allow e.g. “event” for “defect” Allow Po(...)
2	(ii)	$P(2 \text{ defects}) = \frac{e^{-1.7} 1.7^2}{2!}$ OR from tables $P(2 \text{ defects}) = 0.7572 - 0.4932$ $= 0.2640$	M1 A1 [2]	For calculation of $P(X = 2)$ CAO Allow 0.264 Do not allow 0.2639

Question		Answer	Marks	Guidance
2	(iii)	<p>New $\lambda = 7 \times 1.7 = 11.9$</p> $P(12 \text{ defects}) = \frac{e^{-11.9} 11.9^{12}}{12!}$ <p>= 0.1143 cao</p>	<p>B1</p> <p>M1</p> <p>A1</p> <p>[3]</p>	<p>For mean (SOI)</p> <p>For calculation with their $\lambda \neq 1.7$</p> <p>Allow 0.114 www</p> <p>Note B0M1A0 for 0.114 from use of $\lambda = 11.7$</p>
2	(iv)	<p>New $\lambda = (2 \times 1.7) + (2 \times 0.8) = 5.0$</p> <p>P(At least 8 defects) = $1 - P(7 \text{ or fewer defects})$</p> <p>= $1 - 0.8666$</p> <p>(= 0.1334) NB Answer given</p>	<p>B1*</p> <p>B1*</p> <p>B1dep*</p> <p>[3]</p>	<p>For new mean = 5 seen</p> <p>For $1 - P(X \leq 7)$ or $1 - P(X < 8)$ seen</p> <p>For $1 - 0.8666$ seen</p>
2	(v)	<p>Binomial(100, 0.1334) or B(100, 0.1334)</p>	<p>B1*</p> <p>B1dep*</p> <p>[2]</p>	<p>For binomial.</p> <p>For parameters</p>
2	(vi)	<p>Mean = 13.34</p> <p>Variance = $100 \times 0.1334 \times 0.8666 = 11.56$</p> <p>Using Normal approx. to the binomial, $X \sim N(13.34, 11.56)$</p> $P(X \geq 20) = P\left(Z \geq \frac{19.5 - 13.34}{\sqrt{11.56}}\right)$ <p>= $P(Z > 1.812) = 1 - \Phi(1.812) = 1 - 0.9650$</p> <p>= 0.035</p>	<p>B1</p> <p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>[5]</p>	<p>For mean (soi)</p> <p>For variance (soi)</p> <p>For B(13.34, 11.56) seen, award B0B0 unless used correctly as part of a Normal calculation.</p> <p>For 19.5 seen</p> <p>For probability using correct tail and a sensible calculation with their mean and variance. e.g. using standard deviation = 11.56 or finding $P(Z < 1.812)$ gets M0A0</p> <p>CAO, (Do not FT wrong or omitted CC)</p>

Question		Answer	Marks	Guidance
3	(i)	$P(X > 135) = P\left(Z > \frac{135 - 130.5}{\sqrt{11.84}}\right)$ $= P(Z > 1.308) = 1 - \Phi(1.308) = 1 - 0.9045$ $= 0.0955 \text{ (3s.f.)}$	<p>M1</p> <p>M1</p> <p>A1</p> <p>[3]</p>	<p>For standardising. Penalise use of “continuity corrections”</p> <p>For correct structure i.e. finding the area to the right of their z</p> <p>CAO inc use of diff tables Allow 0.0954 and 0.0956 If numerator reversed, give BOD only if $P(Z < -1.308)$ is used</p>
3	(ii)	<p>From tables $\Phi^{-1}(0.99) = 2.326$</p> $\frac{k - 130.5}{\sqrt{11.84}} = 2.326 \text{ oe}$ $k = 130.5 + 2.326 \times \sqrt{11.84} = 138.50$	<p>B1</p> <p>M1</p> <p>A1</p> <p>[3]</p>	<p>± 2.326 or (better) seen, not ± 2.33</p> <p>For sensible equation in k with their z value. Note that use of $z = 0.8389$ from $\Phi(0.99)$ gets B0M0A0, as 0.8389 is clearly a probability. Allow use of -2.326 (or their negative z) with numerator reversed. Condone use of $\sigma = 11.84$ if also used in part (i). Condone use of “$k \pm 0.5$” for k in equation. 0/3 for trial and improvement</p> <p>CAO Allow 138.504. Accept 138.5 Do not accept final answers of 139 or 138.</p>

Question	Answer	Marks	Guidance
3 (iii)	$P(\text{Wing length} = 131) = P\left(\frac{130.5 - 130.5}{\sqrt{11.84}} \leq Z \leq \frac{131.5 - 130.5}{\sqrt{11.84}}\right)$ $= P(0 < Z < 0.2906)$ $= \Phi(0.2906) - \Phi(0)$ $= 0.6143 - 0.5$ $= 0.1143$	B1 M1 A1 [3]	For both limits correct, soi. e.g. use of 0.5 in probability calculation implies correct lower limit. For correct structure using their standardised values. i.e. Finding the area between their z values found using $\mu = 130.5$ Condone use of $\sigma = 11.84$ if also used in part (i) or part (ii). CAO inc use of diff tables Allow 0.1145 Allow 0.114 www

Question	Answer	Marks	Guidance
3 (iv)	<p>$H_0: \mu = 130.5$ $H_1: \mu > 130.5$</p> <p>Where μ denotes the mean wing length (in the population) (of Scandinavian male blackbirds).</p> $\text{Test statistic} = \frac{132.4 - 130.5}{\sqrt{11.84} / \sqrt{20}} = \frac{1.90}{0.7694}$ <p>= 2.469</p> <p>Upper 5% level 1 tailed critical value of $z = 1.645$</p> <p>$2.469 > 1.645$</p> <p>The result is significant. There is sufficient evidence to reject H_0</p> <p>There is sufficient evidence to suggest that the <u>mean wing length</u> (of this population of birds) <u>is greater</u> (than 130.5mm).</p>	<p>B1</p> <p>B1</p> <p>M1*</p> <p>A1</p> <p>B1</p> <p>M1dep*</p> <p>A1</p> <p>A1</p> <p>[8]</p>	<p>For both correct</p> <p>Hypotheses in words must refer to population. Do not allow other symbols unless clearly defined as population mean</p> <p>For definition of μ in context. Do not allow “sample mean wing length” or “mean wing length of English blackbirds”</p> <p>must include $\sqrt{20}$</p> <p>Condone use of $\sigma = 11.84$ if also used in part (i), part (ii) or part (iii). Condone numerator reversed for max M1*A0B1M0depA0A0 (max 4/8)</p> <p>Allow 2.47</p> <p>For 1.645. Must be positive. B0 if -1.645 seen. No further A marks from here if wrong.</p> <p>For sensible comparison leading to a conclusion.</p> <p>For correct conclusion. e.g. for “significant” oe FT only candidate’s test statistic if cv = 1.645</p> <p>For non-assertive conclusion in <u>context</u>, consistent with their result Condone use of “average” for “mean” FT only candidate’s test statistic if cv = 1.645</p>

Question		Answer	Marks	Guidance
3	(v)	<p>With a 10% significance level rather than a 5% significance level,</p> <p>Advantage: One is less likely to accept the null hypothesis when it is false.</p> <p>Disadvantage: One is more likely to reject the null hypothesis when it is true.</p>	<p>E1</p> <p>E1</p> <p>[2]</p>	<p>Accept equivalent wording.</p> <p>Note – Unless stated otherwise, assume the first comment relates to an advantage and the second comment relates to a disadvantage.</p>

Question	Answer	Marks	Guidance																																
4 (i)	<p>H_0: no association between age and working days lost H_1: some association between age and working days lost</p> <table border="1" data-bbox="349 336 927 587"> <thead> <tr> <th>EXPEC TED</th> <th>0 to 4</th> <th>5 to 9</th> <th>10 or more</th> </tr> </thead> <tbody> <tr> <td>Under 35</td> <td>24.47</td> <td>28.39</td> <td>9.14</td> </tr> <tr> <td>35 to 50</td> <td>26.84</td> <td>31.14</td> <td>10.02</td> </tr> <tr> <td>Over 50</td> <td>23.68</td> <td>27.47</td> <td>8.84</td> </tr> </tbody> </table> <table border="1" data-bbox="349 647 983 871"> <thead> <tr> <th>CONTRIB 'N</th> <th>0 to 4</th> <th>5 to 9</th> <th>10 or more</th> </tr> </thead> <tbody> <tr> <td>Under 35</td> <td>1.7404</td> <td>0.0680</td> <td>2.8880</td> </tr> <tr> <td>35 to 50</td> <td>0.0499</td> <td>0.0239</td> <td>0.4076</td> </tr> <tr> <td>Over 50</td> <td>2.4931</td> <td>0.0101</td> <td>5.7945</td> </tr> </tbody> </table> <p>$X^2 = 13.48$</p> <p>Refer to χ_4^2 Critical value at 1% level = 13.28 $13.48 > 13.28$ so result is significant</p> <p>There is evidence to suggest that there is <u>some association between age and working days lost</u></p>	EXPEC TED	0 to 4	5 to 9	10 or more	Under 35	24.47	28.39	9.14	35 to 50	26.84	31.14	10.02	Over 50	23.68	27.47	8.84	CONTRIB 'N	0 to 4	5 to 9	10 or more	Under 35	1.7404	0.0680	2.8880	35 to 50	0.0499	0.0239	0.4076	Over 50	2.4931	0.0101	5.7945	<p>B1</p> <p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>B1</p> <p>B1</p> <p>B1</p> <p>B1*</p> <p>E1dep*</p> <p>[10]</p>	<p>For hypotheses, at least one of which is in context. Allow hypotheses in terms of independence, in context. Do not allow “relationship” or “correlation” for “association”</p> <p>For at least one row/column of expected values correct (to 1 d.p.) Seen or implied by contributions</p> <p>All correct (to 2 d.p.). Accept fraction equivalents. Seen or implied by contributions</p> <p>For valid attempt at $(O-E)^2/E$ Seen or implied by at least one correct contribution to 2 d.p.. Allow 5.80 for last cell. Condone values in a list.</p> <p>For all correct to 2 d.p. or better. Allow 5.80 for last cell. Condone values in a list. NB: These two marks cannot be implied by a correct final value of X^2</p> <p>For answers rounding to 13.48. Do not penalize over-specification here.</p> <p>For 4 degrees of freedom.</p> <p>No further marks from here if incorrect.</p> <p>For significant oe FT their test statistic</p> <p>For non-assertive conclusion in <u>context</u> Allow conclusion in terms of independence FT their test statistic NB if H_0 H_1 reversed, or ‘correlation’ mentioned, do not award final E1</p>
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4	(ii)	<p>The large contribution/contribution of 2.4931 implies that there are fewer employees than expected who lose 0 to 4 working days.</p> <p>The small contribution/contribution of 0.0101 implies that there are about as many as expected who lose 5 to 9 working days.</p> <p>The large contribution/contribution of 5.7945 implies that there are more employees than expected who lose more than 10 working days.</p>	<p>E1</p> <p>E1</p> <p>E1</p> <p>[3]</p>	<p>For correct interpretation with reference to contribution for 0 to 4 working days</p> <p>For correct interpretation with reference to contribution for 5 to 9 working days</p> <p>For correct interpretation with reference to contribution for more than 10 working days</p> <p>SC1 if all interpretations are correct but with no reference to contributions. i.e. max 1 out of 3</p>
4	(iii)	<p>Critical value at 1% level = 9.21</p> <p>$7.08 < 9.21$ so the result is not significant oe</p> <p>There is insufficient evidence to suggest that there is <u>an association between age and working days lost</u>.</p>	<p>B1*</p> <p>B1dep*</p> <p>E1dep*</p> <p>[3]</p>	<p>CAO for cv</p> <p>For correct conclusion</p> <p>For non-assertive conclusion in <u>context</u></p>
4	(iv)	<p>Conclusion in (iii) is valid if only categorizing working days lost into '0 to 4' and '5 or more'.</p> <p>However, if '5 or more' is subdivided into '5 to 9' and '10 or more', this additional subdivision gives the data more precision and allows the relationship in part (i) to be revealed.</p>	<p>E1</p> <p>E1</p> <p>[2]</p>	<p>As written,</p> <p>or for explaining that combining/subdividing groups leads to a different result.</p> <p>For subdivision allows relationship to be revealed or gives more precision/sensitivity/detail.</p> <p>Allow subdivision gives "a more reliable test" but not "a more accurate test"</p> <p>For both to be awarded, wording must be clear.</p>