Mark Scheme (Results)

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Pearson Edexcel International GCSE
In Mathematics A (4MA1)
Paper 1H

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme.
  Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

- **Types of mark**
  - M marks: method marks
  - A marks: accuracy marks
  - B marks: unconditional accuracy marks (independent of M marks)

- **Abbreviations**
  - cao – correct answer only
  - ft – follow through
  - isw – ignore subsequent working
  - SC - special case
  - oe – or equivalent (and appropriate)
  - dep – dependent

- o indep – independent
- o awrt – answer which rounds to
- o eeoo – each error or omission

- **No working**

  If no working is shown then correct answers normally score full marks
  If no working is shown then incorrect (even though nearly correct) answers score no marks.

- **With working**

  If there is a wrong answer indicated on the answer line always check the working in the body of the script (and on any diagrams), and award any marks appropriate from the mark scheme.
  If it is clear from the working that the "correct" answer has been obtained from incorrect working, award 0 marks.
  If a candidate misreads a number from the question. Eg. Uses 252 instead of 255; method marks may be awarded provided the question has not been simplified. Examiners should send any instance of a suspected misread to review. If there is a choice of methods shown, mark the method that leads to the answer on the answer line; where no answer is given on the answer line, award the lowest mark from the methods shown.
  If there is no answer on the answer line then check the working for an obvious answer.

- **Ignoring subsequent work**

  It is appropriate to ignore subsequent work when the additional work does not change the answer in a way that is inappropriate for the question: eg. Incorrect cancelling of a fraction that would otherwise be correct.
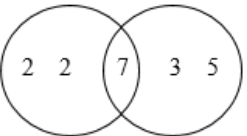 It is not appropriate to ignore subsequent work when the additional work essentially makes the answer incorrect eg algebra.
  Transcription errors occur when candidates present a correct answer in working, and write it incorrectly on the answer line; mark the correct answer.

- **Parts of questions**

  Unless allowed by the mark scheme, the marks allocated to one part of the question CANNOT be awarded to another.

Question		Working	Answer	Mark	Notes
Apart from questions (where the mark scheme states otherwise) the correct answer, unless clearly obtained by an incorrect method, should be taken to imply a correct method					
1	(a)	$\dfrac{5+13}{2}$ or $\dfrac{-4+1}{2}$		2	M1 for a correct method to find one coordinate **or** for one coordinate correct **or** for (−1.5, 9)
			(9, −1.5)		A1 oe
	(b)		−3	1	B1
	(c)		No with reason	1	B1 No (oe) and e.g. line goes through (100, −298) or (101.3(3..), −302) or $\left(\dfrac{304}{3}, -302\right)$ or (3 × 100) − 302 = −2 not (+)2
					Total 4 marks

2		28, 56, 84, 112… **and** 105, 210, 315, 420… **or** 2, 2, 7 **and** 3, 5, 7 **or** **or** $\dfrac{28 \times 105}{7}$ **or** 2, 2, 3, 5, 7 oe		2	M1 for any correct valid method e.g. for starting to list at least **four** multiples of each number or 2, 2, 7 **and** 3, 5, 7 seen (may be in a factor tree and ignore 1) or a fully correct Venn diagram
			420		A1 cao
					Total 2 marks

3		E.g. 12×9 (=108) **or** $(9 - 6) \times x$ (= 3x)		4	M1 for one correct relevant area
		E.g. $129 - '108'$ (= 21) **or** '108' + '3x' = 129			M1 (dep on M1) for 129 used correctly with another area **or** for a correct equation (ft) with bracket(s) expanded
		E.g. '21' ÷ (9 – 6) **or** $x = \dfrac{129 - '108'}{9 - 6}$			M1 for a complete method
			7		A1 Accept 7 cm
					Total 4 marks

4	(a)		$3 < w \leq 4$	1	B1
	(b)	$(12 \times 2.5) + (16 \times 3.5) + (9 \times 4.5) +$ $(2 \times 5.5) + (1 \times 6.5)$ **or** $30 + 56 + 40.5 + 11 + 6.5 \ (= 144)$		4	M2 for at least **4** correct products added (need not be evaluated) **or** If not M2 then award M1 for consistent use of value within interval (including end points) for at least **4** products which must be added **or** correct midpoints used for at least **4** products and not added
		$[(12 \times 2.5) + (16 \times 3.5) + (9 \times 4.5) +$ $(2 \times 5.5) + (1 \times 6.5)] \div 40$ **or** '144' $\div$ 40			M1 (dep on at least M1) Allow division by their Σf provided addition or total under column seen
			3.6		A1 oe
	(c)	$\dfrac{2}{40} + \dfrac{1}{40}$		2	M1 for $\dfrac{a}{40}$ where $0 < a < 40$ or $\dfrac{3}{b}$ where $b > 3$ where a and b are integers
			$\dfrac{3}{40}$		A1 0.075 oe
					Total 7 marks

5		$120 \div (3 + 5)$ (= 15)		6	M1	M2 for
		'15' × 3 (= 45) **or** '15' × 5 (= 75)			M1	$\dfrac{3}{8} \times 120$ (= 45) **or** $\dfrac{5}{8} \times 120$ (= 75) oe
		'45' ÷ 3 (= 15) **or** '45' ÷ 3 × 2 (= 30)			M1	
		'75' × $\dfrac{16}{25}$ (= 48) **or** '75' × $\dfrac{9}{25}$ (= 27)			M1	
		E.g. ('45' ÷ 3 × 2) + ('75' × $\dfrac{9}{25}$) oe **or** '27' + '30' **or** ('75' − '48') + ('45' − '15')			M1 for a complete method	
			57		A1	
						Total 6 marks

6	(a)		0.000 78	1	B1	
	(b)	22 500 000 oe e.g. 22.5×10^6 **or** 2.25×10^n $n \neq 7$		2	M1	
			2.25×10^7		A1	
						Total 3 marks

7	(a)	$m^2 - 8m + 5m - 40$		2	M1 for any 3 correct terms **or** for 4 out of 4 correct terms ignoring signs for $m^2 - 3m$ … **or** for …$- 3m - 40$
			$m^2 - 3m - 40$		A1
	(b)		$5y(1 + 4y)$	2	B2 If not B2 then award B1 for $5(y + 4y^2)$ **or** $y(5 + 20y)$ **or** $5y(a + 4y)$ where a is an integer and $a \neq 0$ **or** $5y(1 + by)$ where b is an integer and $b \neq 0$
	(c)		1	1	B1
	(d)	E.g. $6x - 15$ or $12x - 30$ oe		4	M1 for expansion of a correct bracket
		$2 \times 3(2x - 5) = 9 - x$ oe **or** $2('6x - 15') = 9 - x$ oe **or** $3(2x - 5) = \dfrac{9}{2} - \dfrac{x}{2}$ oe			M1 for removal of fraction or separating fraction (RHS) in an equation
		$12x + x = 9 + 30$ oe **or** $6x + \dfrac{x}{2} = \dfrac{9}{2} + 15$ oe			M1 ft (dep on 4 terms) for terms in x on one side of equation; number terms on the other
			3		A1 dep on at least M2 awarded
					Total 9 marks

8			Trapezium with vertices at (6, 3) (8, 3) (8, 6) (4, 6)	2	B2 If not B2 then award B1 for shape of correct size and orientation **or** 3 or 4 points plotted correctly	
						Total 2 marks

9		$\cos 63 = \dfrac{24.3}{(PQ)}$ **or** $\sin 27 = \dfrac{24.3}{(PQ)}$ **or** $\dfrac{(PQ)}{\sin 90} = \dfrac{24.3}{\sin 27}$ **or** $\dfrac{\sin 90}{(PQ)} = \dfrac{\sin 27}{24.3}$ oe		3	M1 for a correct trigonometric ratio	M2 for $(RQ =) 24.3 \times \tan 63 (= 47.6914..)$ **and** $(PQ =) \sqrt{'47.6914'^2 + 24.3^2}$ oe
		$(PQ =) \dfrac{24.3}{\cos 63}$ **or** $(PQ =) \dfrac{24.3}{\sin 27}$ **or** $(PQ) = \dfrac{24.3}{\sin 27} \times \sin 90$			M1 for a correct rearrangement for PQ	
			53.5		A1 Accept 53.5 - 53.53	
						Total 3 marks

10			$x \geq -1$ oe $x + y \leq 4$ oe $y \geq \dfrac{1}{3}x - 2$ oe	3	B3 for all 3 correct inequalities (B2 for two correct inequalities B1 for one correct inequality) (SC B3 for $x \leq -1$, $x + y \geq 4$ and $y \leq \dfrac{1}{3}x - 2$ oe) (If no marks gained B1 for understanding of equation $x + y = 4$ e.g. $y > 4 - x$) Accept $<$ for $\leq$ and $>$ for $\geq$ throughout
					Total 3 marks

11		$6000 \times 1.015^2 \ (= 6181.35)$ **or** $6000 + (0.015 \times 6000) + (0.015 \times (6000 +$ '90')) $(= 6181.35)$ **or** $(1.015)^2 \ (= 1.030225)$ **or** $\dfrac{6311.16}{6000}(=1.05186)$		3	M1 for working out the total amount after **two** years **or** working out the compound interest multiplier after two years **or** working out the compound interest multiplier after three years
		$6311.16 \div$ '6181.35' $(= 1.021)$ $(\times 100)$ **or** $\dfrac{6311.16 - '6181.35'}{'6181.35'}(= 1.021) \ (\times 100)$ **or** '1.05186' $\div$ '1.030225' $(= 1.021)$ $(\times 100)$			M1 (dep on M1) for a complete method to find the compound interest multiplier ($\times 100$)
			2.1		A1 awrt 2.1
					Total 3 marks

12	(a)	E.g. 56 – 38		2	M1 for subtracting readings from 60 and 20 oe
			18		A1 for answer in the range 17 – 19
	(b)	[40.5, 43]		3	B1
		'42' ÷ 0.6 oe			M1 for complete method to find the number of men
			70		A1
					Total 5 marks

13		$0.14 = \dfrac{56}{w^2}$ oe **or** $56 \div 0.14 \, (= 400)$		4	M1 for using the given formula correctly
		$\sqrt{\dfrac{56}{0.14}}$ **or** $\sqrt{'400'}$ (=20)			M1 for a method to find w
		'20' × '20' × '20' oe			M1 (dep on M2) for a method to find the volume of the cube
			8000		A1
					Total 4 marks

14	(a)	(0.5 ×) 9.3 × 14.7 × sin106 **or** (9.3 × cos 16) × 14.7 **or** (9.3 × sin 74) × 14.7		2	M1 for applying the area of a triangle formula using correct values (to find half of the area of the parallelogram) **or** for a correct method to find the area of the parallelogram
			131		A1 awrt 131
	(b)	$(GE^2 =) 9.3^2 + 14.7^2 - 2 \times 9.3 \times 14.7 \times \cos106$		3	M1 for the correct use of the cosine rule
		377(.9….) **or** 378 **or** 86.49 + 216.09 + 75.3… **or** 302.58 + 75.3….			M1 (dep on M1)for the correct order of operations
			19.4		A1 for 19.4 – 19.5
					Total 5 marks

15	(a)	$(2x + 5)(x + 1) = 2x^2 + 2x + 5x + 5$ $(= 2x^2 + 7x + 5)$ **or** $(x + 1)(3 - x) = -x^2 + 3x - x + 3$ $(= -x^2 + 2x + 3)$ **or** $(3 - x)(2x + 5) = -2x^2 + 6x - 5x + 15$ $(= -2x^2 + x + 15)$		3	M1 for multiplying out two brackets correctly at least 3 terms correct	M2 for at least 4 terms correct out of a maximum of 8 terms $6x^2 - 2x^3 + 6x - 2x^2 + 15x - 5x^2 + 15 - 5x$
		E.g. $[(2x^2 + 7x + 5)(3 - x) =]$ $-2x^3 - 7x^2 - 5x + 6x^2 + 21x + 15$ **or** $[(-x^2 + 2x + 3)(2x + 5) =]$ $-2x^3 - 5x^2 + 10x + 4x^2 + 6x + 15$ **or** $[(-2x^2 + x + 15)(x + 1) =]$ $-2x^3 - 2x^2 + 15x + x^2 + x + 15$			M1 for at least 3 terms correct out of a maximum of 6 terms **or** for at least 4 terms correct out of a maximum of 8 terms	
			Shown		A1	

15	(b)	$\left(\dfrac{dV}{dx}=\right)16-2x+\left(3\times-2x^2\right)$ oe		5	M1 for the correct differentiation of at least 2 correct terms from
					16 **or** $-2x$ **or** $(3 \times -2x^2)$
		$\left(\dfrac{dV}{dx}=\right)16-2x-6x^2$ oe			A1 for a correct differentiated expression
		$'16-2x-6x^2\,'=0$ oe			M1 (dep on M1) for equating their differentiated expression to zero
		E.g. $(x=)\,\dfrac{-2\pm\sqrt{2^2-4\times6\times-16}}{2\times6}$ oe (accept + in place of $\pm$) or E.g. $6\!\left(\left(x+\dfrac{1}{6}\right)^2-\left(\dfrac{1}{6}\right)^2\right)-16(=0)$ oe			M1 (dep on M1) for a complete method to solve their 3-term quadratic equation (allow one sign error and some simplification – allow as far as $\dfrac{-2\pm\sqrt{4+384}}{12}$)
			1.47		A1 dep on M1 for answer in range 1.47 – 1.5 from correct working (Must reject −1.80 to −1.81 if calculated)
					Total 8 marks

16		58.35 or 58.45 or 19.5 or 20.5 or 3.55 or 3.65		3	B1 for any correct bound Accept $58.44\dot{9}$ for 58.45 or $20.4\dot{9}$ for 20.5 or $3.64\dot{9}$ for 3.65
		$\dfrac{2\times58.45-19.5}{3.55}$ $(= 27.4366...)$			M1 for correct substitution into $\dfrac{2\times a_{UB}-c_{LB}}{d_{LB}}$ where $58.4 < a_{UB} \leq 58.45$ **and** $19.5 \leq c_{LB} < 20$ **and** $3.55 \leq d_{LB} < 3.6$
			27.44		A1 from correct working allow 27.4 – 27.5
					Total 3 marks

17	(a)	$6 \times 6 + 6 \times 2\sqrt{12} + 6 \times 2\sqrt{12} + \left(2 \times \sqrt{12}\right)^2$ or $36 + 12\sqrt{12} + 12\sqrt{12} + 4\sqrt{12}\sqrt{12}$ or $36 + 12\sqrt{12} + 12\sqrt{12} + \left(4 \times 12\right)$ or $36 + 24\sqrt{3} + 24\sqrt{3} + 48$ or $36 + 2 \times 24\sqrt{3} + 48$ or $36 + 6 \times 2 \times 2\sqrt{12} + 48$		3	M1 for correct expansion of brackets showing **four** terms (need not be simplified) **or** for the use of $(a + b)^2 = a^2 + 2ab + b^2$ **or** for showing or stating $\sqrt{12} = 2\sqrt{3}$ oe
		$84 + 48\sqrt{3}$			M1 (dep on M1)
			Shown		A1 for fully correct working leading to given expression

17	(b)	E.g. $$\left(\dfrac{3a^4}{t^5}\right)^{-2} \text{ or } \left(\dfrac{t^{15}}{27a^{12}}\right)^{\frac{2}{3}} \text{ or } \left(\dfrac{729a^{24}}{t^{30}}\right)^{-\frac{1}{3}}$$		3	M1 for **one** of cube rooting **or** inverting **or** squaring **or** $\dfrac{ka^{-8}}{t^{-10}}$ where k is an integer $\neq 0$
		E.g. $$\left(\dfrac{9a^8}{t^{10}}\right)^{-1} \text{ or } \dfrac{3^{-2}a^{-8}}{t^{-10}} \text{ or } \dfrac{\frac{1}{9}a^{-8}}{t^{-10}} \text{ or } \left(\dfrac{t^5}{3a^4}\right)^{2} \text{ or }$$ $$\left(\dfrac{t^{30}}{729a^{24}}\right)^{\frac{1}{3}} \text{ or } \dfrac{a^{-8}}{9t^{-10}}$$			M1 for **two** of cube rooting **or** inverting **or** squaring **or** $\dfrac{t^{10}}{ka^8}$ where k is an integer $\neq 0$
			$\dfrac{t^{10}}{9a^8}$		A1 Allow $\dfrac{t^{10}a^{-8}}{9}$ or $\dfrac{1}{9}t^{10}a^{-8}$
					Total 6 marks

18	$\dfrac{4}{16}\times\dfrac{3}{15}\times\dfrac{2}{14}\left(=\dfrac{24}{3360}=\dfrac{1}{140}\right)$ oe **or** $\dfrac{7}{16}\times\dfrac{6}{15}\times\dfrac{5}{14}\left(=\dfrac{210}{3360}=\dfrac{1}{16}\right)$ oe **or** $\dfrac{5}{16}\times\dfrac{4}{15}\times\dfrac{3}{14}\left(=\dfrac{60}{3360}=\dfrac{1}{56}\right)$ oe		4	M1 for finding *BBB* **or** *OOO* or *LLL*	M3 for $\dfrac{11}{16}\times\dfrac{10}{15}\times\dfrac{9}{14}$ oe
	$\dfrac{4}{16}\times\dfrac{7}{15}\times\dfrac{6}{14}\left(=\dfrac{168}{3360}=\dfrac{1}{20}\right)$ oe **or** $\dfrac{4}{16}\times\dfrac{3}{15}\times\dfrac{7}{14}\left(=\dfrac{84}{3360}=\dfrac{1}{40}\right)$ oe **or** $\dfrac{5}{16}\times\dfrac{4}{15}\times\dfrac{4}{14}\left(=\dfrac{80}{3360}=\dfrac{1}{42}\right)$ oe **or** $\dfrac{5}{16}\times\dfrac{4}{15}\times\dfrac{7}{14}\left(=\dfrac{140}{3360}=\dfrac{1}{24}\right)$ oe **or** $\dfrac{5}{16}\times\dfrac{4}{15}\times\dfrac{3}{14}\left(=\dfrac{60}{3360}=\dfrac{1}{56}\right)$ oe **or** $\dfrac{5}{16}\times\dfrac{7}{15}\times\dfrac{6}{14}\left(=\dfrac{210}{3360}=\dfrac{1}{16}\right)$ oe **or** $\dfrac{5}{16}\times\dfrac{7}{15}\times\dfrac{4}{14}\left(=\dfrac{140}{3360}=\dfrac{1}{24}\right)$ oe **or** $\dfrac{5}{16}\times\dfrac{4}{15}\times\dfrac{11}{14}\left(=\dfrac{220}{3360}=\dfrac{11}{168}\right)$ oe or $\dfrac{5}{16}\times\dfrac{11}{15}\times\dfrac{10}{14}\left(=\dfrac{550}{3360}=\dfrac{55}{336}\right)$ oe			M1 for finding the following in any order *BOO* **or** *BBO* **or** *LLB* **or** *LLO* **or** *LBB* **or** *LOO* **or** *LOB* **or** *LLX* **or** *LXX* (*X* = not *L*)	
	$'\dfrac{24}{3360}'+3\times'\dfrac{84}{3360}'+'\dfrac{210}{3360}'+3\times'\dfrac{168}{3360}'$ oe **or** $1-\left('\dfrac{60}{3360}'+3\times'\dfrac{80}{3360}'+3\times'\dfrac{140}{3360}'+3\times'\dfrac{60}{3360}'3\times'\dfrac{210}{3360}'6\times'\dfrac{140}{3360}'\right)$ oe **or** $1-\left('\dfrac{60}{3360}'+3\times'\dfrac{220}{3360}'+3\times'\dfrac{550}{3360}'\right)$ oe			M1 for a complete method	
		$\dfrac{990}{3360}$		A1 for $\dfrac{990}{3360}$ oe e.g. $\dfrac{33}{112}$ or 0.29(464...)	
					Total 4 marks

19		$(AH =)\sqrt{6^2 + 5^2 + 9^2} \;\; (=\sqrt{142}\;)$ **or** $(FH = GE =)\;\sqrt{5^2 + 9^2}\;\;(=\sqrt{106}\;)$		4	M1 for working out *AH* or *FH* or *GE*
		E.g. $\sin AHF = \dfrac{6}{'\sqrt{142}'}$ **or** $\tan AHF = \dfrac{6}{'\sqrt{106}'}$ **or** $\cos AHF = \dfrac{'\sqrt{106}'}{'\sqrt{142}'}$ **or** $\sin FAH = \dfrac{'\sqrt{106}'}{'\sqrt{142}'}$ **or** $\cos FAH = \dfrac{6}{'\sqrt{142}'}$ **or** $\tan FAH = \dfrac{'\sqrt{106}'}{6}$			M1 for a correct method for finding angle *AHF* **or** finding angle *FAH* Allow $\cos AHF = \left(\dfrac{'\sqrt{142}\,'^2 + '\sqrt{106}\,'^2 - 6^2}{2 \times '\sqrt{142}\,' \times '\sqrt{106}\,'}\right)$ oe **or** $\sin AHF = \dfrac{\sin 90}{'\sqrt{142}\,'} \times 6$ oe
		E.g. $\sin^{-1}\left(\dfrac{6}{'\sqrt{142}'}\right)$ **or** $\tan^{-1}\left(\dfrac{6}{'\sqrt{106}'}\,'\right)$ **or** $\cos^{-1}\left(\dfrac{'\sqrt{106}'}{'\sqrt{142}'}\right)$ **or** $90 - \sin^{-1}\left(\dfrac{'\sqrt{106}'}{'\sqrt{142}'}\right)$ **or** $90 - \cos^{-1}\left(\dfrac{6}{'\sqrt{142}'}\right)$ **or** $90 - \tan^{-1}\left(\dfrac{'\sqrt{106}'}{6}\right)$			M1 for a complete method Allow $\cos^{-1}\left(\dfrac{'\sqrt{142}\,'^2 + '\sqrt{106}\,'^2 - 6^2}{2 \times '\sqrt{142}\,' \times '\sqrt{106}\,'}\right)$ oe **or** $\sin^{-1}\left(\dfrac{\sin 90}{'\sqrt{142}\,'} \times 6\right)$ oe
			30.2		A1 for $30.2 - 30.3$
					Total 4 marks

20		graph drawn in shape of a quadratic with a minimum in any quadrant		4	M1 for a quadratic with a minimum
		$x = 1$, $y = 4\,(1 - 1)^2 - a$			M1 for finding the turning point (may be seen marked on the graph as $(1, -a)$)
		$x = 1 \pm \sqrt{\dfrac{a}{4}}$ oe **or** $y = 4 - a$			M1 for finding one of the intercepts (or award for any one correct coordinate shown on graph) $(0, 4 - a)$ or $(1 + \dfrac{\sqrt{a}}{2}, 0)$ or $(1 - \dfrac{\sqrt{a}}{2}, 0)$ Note: The 0's can be ignored (as shown in the diagram)
			Correct graph		A1 for a fully correct graph - quadratic shape with minimum in the fourth quadrant and marked as $(1, -a)$ oe x-axis intercepts marked as $(1 + \dfrac{\sqrt{a}}{2}, 0)$ oe on the positive x-axis **and** $(1 - \dfrac{\sqrt{a}}{2}, 0)$ oe on the negative x-axis y-axis intercept marked as $(0, 4 - a)$ oe Note: The 0's can be ignored (as shown in the diagram)
					Total 4 marks

21		$(\mathrm{fg}(x) =)\ (x + 3)^2 - 2(x + 3)$ oe		5	M1 for substituting g(x) into f(x)
		$(\mathrm{fg}(x) =)\ x^2 + 4x + 3$			A1 Allow $y^2 + 4y + 3$
		$(x + 2)^2 - 4 + 3$ **or** $(x + 2)^2 - 1$ **or** $x^2 + 4x + (3 - y) = 0$ **or** $y^2 + 4y + (3 - x) = 0$			M1 ft (dep on M1) for correctly completing the square on their 3 term quadratic **or** Correctly setting up an equation
		$(x + 2)^2 = y + 1$ **or** $(y + 2)^2 = x + 1$ **or** $x = \dfrac{-4 \pm \sqrt{16 - 4(3 - y)}}{2}$ or $x = -2 \pm \sqrt{1 + y}$			M1 ft (dep on M2) for a correct rearrangement for their completed the square quadratic **or** correctly substituting into the quadratic formula Allow same equations with x and y swapped
			$-2 + \sqrt{x + 1}$		A1 oe
					Total 5 marks

22	gradient of $JK = -0.5$ **or** $m \times 2 = -1$		6	M1 for finding the gradient of JK using $m_1 \times m_2 = -1$
	$\dfrac{k-15}{6-j} = -\dfrac{1}{2}$ **or** $2k - j = 24$ **or** $j = 2k - 24$ **or** $k = \dfrac{j+24}{2}$ oe			M1 for expressing the gradient of JK in terms of j and k or a correct equivalent equation
	$(j-6)^2 + (k-15)^2 = 80$ oe **or** $\left(\dfrac{j+6}{2}, \dfrac{k+15}{2}\right)$ oe **or** $(j+4)^2 + 196 = 100 + (k-1)^2$ oe			M1 for finding equation of JK in terms of j and k **or** for finding the midpoint of M **or** for equating length HJ with length HK
	eg $3k^2 - 78k + 495 = 0$ oe **or** $5j^2 - 60j - 140 = 0$ oe **or** $5k^2 - 150k + 1045 = 0$ oe **or** $3j^2 - 12j - 36 = 0$ **oe** **or** gradient HM: eg $\dfrac{\dfrac{k+15}{2} - 1}{\dfrac{j+6}{2} + 4} = 2$ **or** $k = 2j + 15$ **or** $j = \dfrac{k-15}{2}$ oe			M1 (dep on M3) writing a correct quadratic expression in the form $ax^2 + bx + c\ (=0)$ (allow $ax^2 + bx = c$) **or** A correct equation for the gradient of HM in terms of j and k or a correct equivalent equation
	eg $(k-15)(k-11)(=0)$ **or** $\dfrac{78 \pm \sqrt{(-78)^2 - 4 \times 3 \times 495}}{2 \times 3}$ **or** $(k-13)^2 - 169 + 165(=0)$	eg $(j-6)(j+2)(=0)$ **or** $\dfrac{12 \pm \sqrt{(-12)^2 - 4 \times 3 \times -36}}{2 \times 3}$ **or** $(j-2)^2 - 4 - 12(=0)$		M1 (dep on M3) for a complete method to solve their 3-term quadratic equation (allow one sign error in the use of the quadratic formula) **or** a correct method to eliminate either j or k eg $2k - 24 = \dfrac{k-15}{2}$ oe **or** $\dfrac{j+24}{2} = 2j + 15$ oe
		$j = -2, k = 11$		A1
				Total 6 marks

22		$\left(\dfrac{j+6}{2},\dfrac{k+15}{2}\right)$ oe			6	M1 for finding the midpoint of M
ALT		$\dfrac{\dfrac{k+15}{2}-1}{\dfrac{j+6}{2}+4}=2$ **or** $k-2j=15$ **or** $k=2j+15$ **or** $j=\dfrac{k-15}{2}$ oe				M1 for expressing the gradient of JK in terms of j and k or a correct equivalent equation
		$(j-6)^2+(k-15)^2=80$ oe **or** $(j+4)^2+196=100+(k-1)^2$ oe				M1 for finding the length of JK in terms of j and k **or** for equating length HJ with length HK
		E.g. $5j^2-12j-44=0$ **or** $3j^2+48j+84=0$ oe	E.g. $5k^2-174k+1309=0$ **or** $3k^2+6k-429=0$ oe			M1 (dep on M3) writing the correct quadratic expression in form $ax^2+bx+c\,(=0)$ allow $ax^2+bx=c$
		E.g. $(5j-22)(j+2)(=0)$ **or** $\dfrac{12\pm\sqrt{(-12)^2-4\times5\times-44}}{2\times5}$ **or** $(j+8)^2-64+28(=0)$	E.g. $(5k-119)(k-11)(=0)$ **or** $\dfrac{174\pm\sqrt{(-174)^2-4\times5\times1309}}{2\times5}$ **or** $(k+1)^2-1-143(=0)$			M1 (dep on M3) for a complete method to solve their 3-term quadratic equation (allow one sign error in the use of the quadratic formula)
				$j=-2,k=11$		A1
						Total 6 marks