Mark Scheme 4754
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Section A

1 $\sin\theta - 3\cos\theta = R\sin(\theta - \alpha)$ $= R(\sin\theta\cos\alpha - \cos\theta\sin\alpha)$ $\Rightarrow R\cos\alpha = 1$, $R\sin\alpha = 3$ $\Rightarrow R^2 = 1^2 + 3^2 = 10 \Rightarrow R = \sqrt{10}$ $\tan\alpha = 3 \Rightarrow \alpha = 71.57°$ $\sqrt{10}\sin(\theta - 71.57°) = 1$ $\Rightarrow \theta - 71.57° = \sin^{-1}(1/\sqrt{10})$ $\theta - 71.57° = 18.43°, 161.57°$ $\Rightarrow \theta = 90°$, $233.1°$	M1 B1 M1 A1 M1 B1 A1 [7]	equating correct pairs oe ft www cao (71.6°or better) oe ft R, α www and no others in range (MR-1 for radians)
2 Normal vectors are $\begin{pmatrix}2\\3\\4\end{pmatrix}$ and $\begin{pmatrix}1\\-2\\1\end{pmatrix}$ $\Rightarrow \begin{pmatrix}2\\3\\4\end{pmatrix}.\begin{pmatrix}1\\-2\\1\end{pmatrix} = 2-6+4=0$ $\Rightarrow$ planes are perpendicular.	B1 B1 M1 E1 [4]	
3 **(i)** $y = \ln x \Rightarrow x = e^y$ $\Rightarrow V = \int_0^2 \pi x^2 dy$ $= \int_0^2 \pi(e^y)^2 dy = \int_0^2 \pi e^{2y} dy$ *	B1 M1 E1 [3]	
(ii) $\int_0^2 \pi e^{2y} dy = \pi\left[\frac{1}{2}e^{2y}\right]_0^2$ $= \frac{1}{2}\pi(e^4 - 1)$	B1 M1 A1 [3]	$\frac{1}{2}e^{2y}$ substituting limits in $k\pi e^{2y}$ or equivalent, but must be exact and evaluate e^0 as 1.
4 $x = \frac{1}{t} - 1 \Rightarrow \frac{1}{t} = x + 1$ $\Rightarrow t = \frac{1}{x+1}$ $\Rightarrow y = \frac{2 + \frac{1}{x+1}}{1 + \frac{1}{x+1}} = \frac{2x+2+1}{x+1+1} = \frac{2x+3}{x+2}$	M1 A1 M1 E1	Solving for t in terms of x or y Subst their t which must include a fraction, clearing subsidiary fractions/ changing the subject oe www
or $\frac{3+2x}{2+x} = \frac{3 + \frac{2-2t}{t}}{2 + \frac{1-t}{t}}$ $= \frac{3t+2-2t}{2t+1-t}$ $= \frac{t+2}{t+1} = y$	M1 A1 M1 E1 [4]	substituting for x or y in terms of t clearing subsidiary fractions/changing the subject

5 $\mathbf{r}=\begin{pmatrix}1\\2\\-1\end{pmatrix}+\lambda\begin{pmatrix}-1\\2\\3\end{pmatrix} \Rightarrow \begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}1-\lambda\\2+2\lambda\\-1+3\lambda\end{pmatrix}$		
When $x = -1$, $1 - \lambda = -1$, $\Rightarrow \lambda = 2$	M1	Finding λ or μ
$\Rightarrow y = 2 + 2\lambda = 6$, $z = -1 + 3\lambda = 5$ $\Rightarrow$ point lies on first line	E1	checking other two coordinates
$\mathbf{r}=\begin{pmatrix}0\\6\\3\end{pmatrix}+\mu\begin{pmatrix}1\\0\\-2\end{pmatrix} \Rightarrow \begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}\mu\\6\\3-2\mu\end{pmatrix}$		
When $x = -1$, $\mu = -1$, $\Rightarrow y = 6$, $z = 3 - 2\mu = 5$ $\Rightarrow$ point lies on second line	E1	checking other two co-ordinates
Angle between $\begin{pmatrix}-1\\2\\3\end{pmatrix}$ and $\begin{pmatrix}1\\0\\-2\end{pmatrix}$ is θ, where	M1	Finding angle between correct vectors
$\cos\theta=\dfrac{-1\times1+2\times0+3\times-2}{\sqrt{14}.\sqrt{5}}$	M1	use of formula
$= -\dfrac{7}{\sqrt{70}}$	A1	$\pm\dfrac{7}{\sqrt{70}}$
$\Rightarrow \theta = 146.8°$ $\Rightarrow$ acute angle is 33.2°	A1cao [7]	Final answer must be acute angle
6(i) $A \approx 0.5[\dfrac{(1.1696+1.0655}{2}+1.1060]$ = 1.11 (3 s.f.)	M1 A1 cao [2]	Correct expression for trapezium rule
(ii) $(1+e^{-x})^{1/2}=1+\dfrac{1}{2}e^{-x}+\dfrac{\frac{1}{2}.-\frac{1}{2}}{2!}(e^{-x})^2+...$ $\approx 1+\dfrac{1}{2}e^{-x}-\dfrac{1}{8}e^{-2x}$ *	M1 A1 E1 [3]	Binomial expansion with p = ½ Correct coeffs
(iii) $I=\int_1^2(1+\dfrac{1}{2}e^{-x}-\dfrac{1}{8}e^{-2x})dx$		
$=\left[x-\dfrac{1}{2}e^{-x}+\dfrac{1}{16}e^{-2x}\right]_1^2$	M1	integration
$=(2-\dfrac{1}{2}e^{-2}+\dfrac{1}{16}e^{-4})-(1-\dfrac{1}{2}e^{-1}+\dfrac{1}{16}e^{-2})$	A1	substituting limits into correct expression
= 1.9335 − 0.8245 = 1.11 (3 s.f.)	A1 [3]	

Section B

7 (a) (i) $P_{max} = \frac{2}{2-1} = 2$ $P_{min} = \frac{2}{2+1} = 2/3.$	B1 B1 [2]	
(ii) $P = \frac{2}{2-\sin t} = 2(2-\sin t)^{-1}$ $\Rightarrow \frac{dP}{dt} = -2(2-\sin t)^{-2}.-\cos t$ $= \frac{2\cos t}{(2-\sin t)^2}$ $\frac{1}{2}P^2\cos t = \frac{1}{2}\frac{4}{(2-\sin t)^2}\cos t$ $= \frac{2\cos t}{(2-\sin t)^2} = \frac{dP}{dt}$	M1 B1 A1 DM1 E1 [5]	chain rule $-1(\ldots)^{-2}$ soi (or quotient rule M1,numerator A1,denominator A1) attempt to verify or by integration as in (b)(ii)
(b)(i) $\frac{1}{P(2P-1)} = \frac{A}{P} + \frac{B}{2P-1}$ $= \frac{A(2P-1)+BP}{P(2P-1)}$ $\Rightarrow 1 = A(2P-1) + BP$ $P = 0 \Rightarrow 1 = -A \Rightarrow A = -1$ $P = ½ \Rightarrow 1 = A.0 + ½ B \Rightarrow B = 2$ So $\frac{1}{P(2P-1)} = -\frac{1}{P} + \frac{2}{2P-1}$	M1 M1 A1 A1 [4]	correct partial fractions substituting values, equating coeffs or cover up rule $A = -1$ $B = 2$
(ii) $\frac{dP}{dt} = \frac{1}{2}(2P-P^2)\cos t$ $\Rightarrow \int \frac{1}{2P^2-P}dP = \int \frac{1}{2}\cos t dt$ $\Rightarrow \int (\frac{2}{2P-1} - \frac{1}{P})dP = \int \frac{1}{2}\cos t dt$ $\Rightarrow \ln(2P-1) - \ln P = ½ \sin t + c$ When $t = 0$, $P = 1$ $\Rightarrow \ln 1 - \ln 1 = ½ \sin 0 + c \Rightarrow c = 0$ $\Rightarrow \ln(\frac{2P-1}{P}) = \frac{1}{2}\sin t$ *	M1 A1 A1 B1 E1 [5]	separating variables $\ln(2P-1) - \ln P$ ft their A,B from (i) ½ sin t finding constant = 0
(iii) $P_{max} = \frac{1}{2-e^{1/2}} = 2.847$ $P_{min} = \frac{1}{2-e^{-1/2}} = 0.718$	M1A1 M1A1 [4]	www www

8 (i) $\frac{dy}{dx} = \frac{10\cos\theta + 10\cos 2\theta}{-10\sin\theta - 10\sin 2\theta}$	M1	*dy/dθ ÷dx/dθ*
$= -\frac{\cos\theta + \cos 2\theta}{\sin\theta + \sin 2\theta}$ *	E1	
When $\theta = \pi/3$, $\frac{dy}{dx} = -\frac{\cos\pi/3 + \cos 2\pi/3}{\sin\pi/3 + \sin 2\pi/3}$ = 0 as cos π/3 = ½ , cos 2π/3 = −½	B1	or solving cos*θ*+cos2*θ*=0
At *A* *x* = 10 cos π/3 + 5 cos 2π/3	M1	substituting π/3 into *x* or *y*
= 2½	A1	2½
y = 10 sin π/3 + 5 sin 2π/3 = 15√3/2	A1 [6]	15√3/2 (condone 13 or better)
(ii) $x^2 + y^2 = (10\cos\theta + 5\cos 2\theta)^2 + (10\sin\theta + 5\sin 2\theta)^2$	B1	
$= 100\cos^2\theta + 100\cos\theta\cos 2\theta + 25\cos^2 2\theta$ $+100\sin^2\theta + 100\sin\theta\sin 2\theta + 25\sin^2 2\theta$	M1	expanding
= 100 + 100cos(2*θ* − *θ*) + 25	DM1	cos 2*θ* cos *θ* + sin 2*θ* sin *θ* = cos(2*θ* − *θ*) or substituting for sin 2*θ* and cos 2*θ*
= 125 + 100cos *θ* *	E1 [4]	
(iii) Max $\sqrt{125+100}$ = 15	B1	
min $\sqrt{125-100}$ = 5	B1 [2]	
(iv) 2cos² *θ* +2cos *θ* − 1 = 0 $\cos\theta = \frac{-2\pm\sqrt{12}}{4} = \frac{-2\pm 2\sqrt{3}}{4}$	M1	quadratic formula
At B, $\cos\theta = \frac{-1+\sqrt{3}}{2}$	A1	or *θ*=68.53° or 1.20radians, correct root selected
OB² = 125 + 50(−1 + √3) = 75 + 50√3 = 161.6…	M1	or OB=10sin*θ*+5sin2*θ* ft their *θ*/cos*θ*
⇒ OB = √161.6… = 12.7 (m)	A1 [4]	oe cao

Paper B Comprehension

1)	M $(a\pi, 2a)$, $\theta = \pi$ N $(4a\pi, 0)$, $\theta = 4\pi$	B1 B1	
2)	Compare the equations with equations given in text, $x = a\theta - b\sin\theta$, $y = b\cos\theta$	M1	Seeing a=7, b=0.25
	Wavelength = $2\pi a$ = 14π (≈ 44) Height = $2b$ = 0.5	A1 B1	
3i)	Wavelength = 20 ⇒ $a = \frac{10}{\pi}$ (=3.18…) Height = 2 ⇒ b = 1	B1 B1	
ii)	In this case, the ratio is observed to be 12:8 Trough length : Peak length = $\pi a + 2b : \pi a - 2b$ and this is $(10 + 2 \times 1) : (10 - 2 \times 1)$ So the curve is consistent with the parametric equations	B1 M1 A1	substituting
4i)	$x = a\theta$, $y = b\cos\theta$ is the sine curve V and $x = a\theta - b\sin\theta$, $y = b\cos\theta$ is the curtate cycloid U. The sine curve is above mid-height for half its wavelength (or equivalent)	B1	
ii)	$d = a\theta - (a\theta - b\sin\theta)$ $\theta = \pi/2$, $d = \left(\frac{\pi a}{2}\right) - \left(\frac{\pi a}{2} - b\right) = b$	M1 E1	Subtraction Using $\theta = \pi/2$
iii)	Because b is small compared to a, the two curves are close together.	M1 E1	Comparison attempted Conclusion
5)	Measurements on the diagram give Wavelength ≈ 3.5cm, Height ≈ 0.8cm $\frac{\text{Wavelength}}{\text{Height}} \approx \frac{3.5}{0.8} = 4.375$ Since 4.375 < 7, the wave will have become unstable and broken.	B1 M1 E1	measurements/reading ratio [18]