

A-level  
**FURTHER MATHEMATICS**  
**7367/1**

Paper 1

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**Mark scheme**

June 2022

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Version: 1.0 Final Mark Scheme



Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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## Mark scheme instructions to examiners

### General

The mark scheme for each question shows:

- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

### Key to mark types

|   |   |
|---|---|
| M | mark is for method  |
| R | mark is for reasoning   |
| A | mark is dependent on M marks and is for accuracy              |
| B | mark is independent of M marks and is for method and accuracy |
| E | mark is for explanation                                       |
| F | follow through from previous incorrect result                 |

### Key to mark scheme abbreviations

|         |   |
|---------|---|
| CAO     | correct answer only   |
| CSO     | correct solution only   |
| ft      | follow through from previous incorrect result                     |
| 'their' | indicates that credit can be given from previous incorrect result |
| AWFW    | anything which falls within                                       |
| AWRT    | anything which rounds to  |
| ACF     | any correct form  |
| AG      | answer given  |
| SC      | special case  |
| OE      | or equivalent   |
| NMS     | no method shown   |
| PI      | possibly implied  |
| sf      | significant figure(s)   |
| dp      | decimal place(s)  |

Examiners should consistently apply the following general marking principles:

### **No Method Shown**

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

**Otherwise we require evidence of a correct method for any marks to be awarded.**

### **Diagrams**

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

### **Work erased or crossed out**

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

### **Choice**

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

**AS/A-level Maths/Further Maths assessment objectives**

| AO         |        | Description   |
|------------|--------|---|
| <b>A01</b> | AO1.1a | Select routine procedures   |
|            | AO1.1b | Correctly carry out routine procedures  |
|            | AO1.2  | Accurately recall facts, terminology and definitions                              |
| <b>A02</b> | AO2.1  | Construct rigorous mathematical arguments (including proofs)                      |
|            | AO2.2a | Make deductions   |
|            | AO2.2b | Make inferences   |
|            | AO2.3  | Assess the validity of mathematical arguments                                     |
|            | AO2.4  | Explain their reasoning   |
|            | AO2.5  | Use mathematical language and notation correctly                                  |
| <b>A03</b> | AO3.1a | Translate problems in mathematical contexts into mathematical processes           |
|            | AO3.1b | Translate problems in non-mathematical contexts into mathematical processes       |
|            | AO3.2a | Interpret solutions to problems in their original context                         |
|            | AO3.2b | Where appropriate, evaluate the accuracy and limitations of solutions to problems |
|            | AO3.3  | Translate situations in context into mathematical models                          |
|            | AO3.4  | Use mathematical models   |
|            | AO3.5a | Evaluate the outcomes of modelling in context                                     |
|            | AO3.5b | Recognise the limitations of models   |
|            | AO3.5c | Where appropriate, explain how to refine models                                   |

| Q            | Marking instructions   | AO   | Marks    | Typical solution |
|--------------|------------------------|------|----------|------------------|
| 1            | Circles correct answer | 2.2a | B1       | $\frac{2\pi}{3}$ |
| <b>Total</b> |                        |      | <b>1</b> |                  |

| Q            | Marking instructions | AO   | Marks    | Typical solution  |
|--------------|----------------------|------|----------|---|
| 2            | Ticks correct answer | 1.1b | B1       | $\cos\left(\frac{8\pi}{13}\right) + i \sin\left(\frac{8\pi}{13}\right)$ |
| <b>Total</b> |                      |      | <b>1</b> |   |

| Q            | Marking instructions | AO   | Marks    | Typical solution                 |
|--------------|----------------------|------|----------|----------------------------------|
| 3            | Ticks correct answer | 2.2a | B1       | $-\operatorname{sech} x \tanh x$ |
| <b>Total</b> |                      |      | <b>1</b> |                                  |

| Q            | Marking instructions   | AO   | Marks    | Typical solution |
|--------------|------------------------|------|----------|------------------|
| 4            | Circles correct answer | 2.2a | B1       | 36               |
| <b>Total</b> |                        |      | <b>1</b> |                  |

| Q           | Marking instructions   | AO   | Marks    | Typical solution   |
|-------------|--|------|----------|--|
| <b>5(a)</b> | States complex conjugate   | 1.1b | B1       | $z = -\frac{3}{2} - i\frac{\sqrt{11}}{2}$ is another root<br>Sum of complex roots = $-3$<br>Product of complex roots = $5$<br>So $z^2 + 3z + 5$ is a factor of<br>$z^4 - 3z^3 - 5z^2 + kz + 40$<br><br>The other quadratic factor is<br>$z^2 - 6z + 8$<br>$= (z - 2)(z - 4)$<br><br>The other roots are<br>$2, 4, -\frac{3}{2} - i\frac{\sqrt{11}}{2}$ |
|             | Obtains a quadratic factor<br>or<br>Obtains sum and product of their two complex roots<br>or<br>Substitutes given root or their conjugate into the quartic | 1.1a | M1       |  |
|             | Obtains $z^2 + 3z + 5$<br>or<br>Obtains $k = -6$   | 1.1b | A1       |  |
|             | Forms a second quadratic<br>or<br>Solves the quartic equation with their value of $k$  | 1.1a | M1       |  |
|             | Obtains 2 and 4  | 1.1b | A1       |  |
|             | <b>Total</b>   |      | <b>5</b> |  |

| Q           | Marking instructions  | AO   | Marks    | Typical solution |
|-------------|---|------|----------|------------------|
| <b>5(b)</b> | Deduces correct solution<br>(ft from their exactly two distinct real roots) | 2.2a | B1F      | $2 < x < 4$      |
|             | <b>Total</b>  |      | <b>1</b> |                  |

|  |                       |  |          |  |
|--|-----------------------|--|----------|--|
|  | <b>Question total</b> |  | <b>6</b> |  |
|--|-----------------------|--|----------|--|

| Q            | Marking instructions   | AO   | Marks    | Typical solution  |
|--------------|--|------|----------|---|
| <b>6(a)</b>  | Recalls exponential definition of tanh   | 1.2  | B1       | Let $y = \tanh^{-1} x$  |
|              | Multiplies by their denominator and multiplies by $e^y$ (or $e^x$ )                  | 3.1a | M1       | Then $x = \tanh y$  |
|              | Obtains $e^{2y} = \frac{1+x}{1-x}$<br>or<br>Obtains $e^{2x} = \frac{1+y}{1-y}$<br>OE | 1.1b | A1       | $x = \frac{e^y - e^{-y}}{e^y + e^{-y}}$ $x(e^y + e^{-y}) = e^y - e^{-y}$ $e^y(x - 1) + e^{-y}(x + 1) = 0$ $e^{2y}(x - 1) + (x + 1) = 0$ $e^{2y} = \frac{1+x}{1-x}$ $2y = \ln\left(\frac{1+x}{1-x}\right)$ |
|              | Completes a rigorous argument to show the required result                            | 2.1  | R1       | $\tanh^{-1} x = \frac{1}{2} \ln\left(\frac{1+x}{1-x}\right)$ as required.   |
| <b>Total</b> |  |      | <b>4</b> |   |

| Q            | Marking instructions  | AO   | Marks    | Typical solution  |
|--------------|---|------|----------|---|
| <b>6(b)</b>  | Uses appropriate hyperbolic identity (condone sign errors) to obtain an equation in one hyperbolic function<br>or<br>substitutes exponential form, condone error in sum/difference of exponential terms | 1.1a | M1       | $20(1 - \tanh^2 x) - 11 \tanh x = 16$ $0 = 20 \tanh^2 x + 11 \tanh x - 4$ $(5 \tanh x + 4)(4 \tanh x - 1) = 0$ $\tanh x = -\frac{4}{5} \text{ or } \frac{1}{4}$                   |
|              | Solves their quadratic equation to obtain two solutions in $\tanh x$<br>or<br>quadratic in $e^{2x}$ to obtain two solutions<br>or<br>quartic in $e^x$ to obtain at least two solutions                  | 1.1a | M1       | $x = \tanh^{-1}\left(-\frac{4}{5}\right) = \frac{1}{2} \ln\left(\frac{1}{9}\right)$ $\text{or } x = \tanh^{-1}\left(\frac{1}{4}\right) = \frac{1}{2} \ln\left(\frac{5}{3}\right)$ |
|              | Obtains $\tanh x = -\frac{4}{5}$ and $\frac{1}{4}$ OE<br>or<br>Obtains $e^{2x} = \frac{5}{3}$ and $\frac{1}{9}$ OE<br>or<br>Obtains $e^x = \frac{\sqrt{15}}{3}$ and $\frac{1}{3}$ OE                    | 1.1b | A1       |   |
|              | Obtains correct solutions (any correct exact log form)<br>ISW   | 1.1b | A1       |   |
| <b>Total</b> |   |      | <b>4</b> |   |

|                       |  |  |          |  |
|-----------------------|--|--|----------|--|
| <b>Question total</b> |  |  | <b>8</b> |  |
|-----------------------|--|--|----------|--|



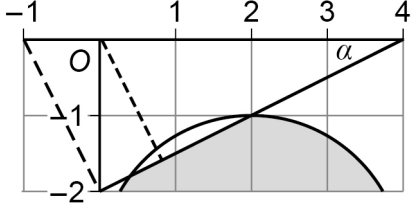
| Q       | Marking instructions  | AO   | Marks    | Typical solution  |
|---------|---|------|----------|---|
| 7(a)(i) | Expands $ \mathbf{M} $ to get a linear expression in $k$ , using any row or column.   | 1.1a | M1       | $ \mathbf{M}  = (12 - 3k - 3) - 7(6 - k - 1) - 3(9 - 6)$<br>$= 4k - 35$<br>Cofactors are<br>$\begin{bmatrix} 9 - 3k & k - 5 & 3 \\ -23 & 5 & 4 \\ 7k + 25 & -k - 10 & -15 \end{bmatrix}$<br>$\mathbf{M}^{-1} =$<br>$\frac{1}{4k - 35} \begin{bmatrix} 9 - 3k & -23 & 7k + 25 \\ k - 5 & 5 & -k - 10 \\ 3 & 4 & -15 \end{bmatrix}$ |
|         | Obtains matrix of minors/cofactors with at least four correct elements<br>PI transposed form.<br>Condone overall sign error on each element.  | 1.1b | B1       |   |
|         | Obtains matrix of minors/cofactors with at least seven correct elements<br>PI transposed form.<br>Condone overall sign error on each element. | 1.1b | B1       |   |
|         | Obtains correct matrix of minors/cofactors<br>PI transposed form.<br>Condone overall sign error on each element.                              | 1.1b | B1       |   |
|         | Obtains fully correct, simplified answer  | 1.1b | A1       |   |
|         | <b>Total</b>  |      | <b>5</b> |   |

| Q        | Marking instructions  | AO   | Marks    | Typical solution      |
|----------|---|------|----------|-----------------------|
| 7(a)(ii) | Obtains correct answer for their linear expression for $ \mathbf{M} $ | 1.1b | B1F      | $k \neq \frac{35}{4}$ |
|          | <b>Total</b>  |      | <b>1</b> |                       |

| Q    | Marking instructions  | AO   | Marks    | Typical solution  |
|------|---|------|----------|---|
| 7(b) | Obtains their correct $\mathbf{M}^{-1}$ , need not be simplified.               | 3.1a | B1F      | $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{-1}{15} \begin{bmatrix} -6 & -23 & 60 \\ 0 & 5 & 15 \\ 3 & 4 & -15 \end{bmatrix} \begin{bmatrix} 6 \\ 3 \\ 1 \end{bmatrix}$ $= \frac{-1}{15} \begin{bmatrix} -45 \\ 0 \\ 15 \end{bmatrix}$ $= \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}$ $x = 3, y = 0, z = -1$ |
|      | Forms their product $\mathbf{M}^{-1} \begin{bmatrix} 6 \\ 3 \\ 1 \end{bmatrix}$ | 1.1a | M1       |   |
|      | Obtains<br>$x = 3, y = 0, z = -1$ ACF<br>CSO                                    | 1.1b | A1       |   |
|      | <b>Total</b>  |      | <b>3</b> |   |
|      | <b>Question total</b>   |      | <b>9</b> |   |

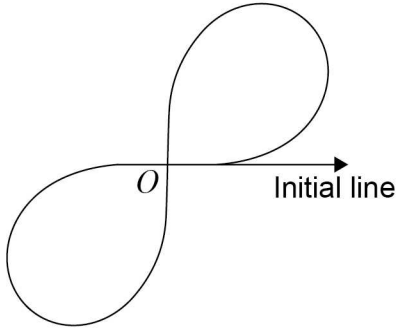
| Q            | Marking instructions                             | AO   | Marks    | Typical solution  |
|--------------|--|------|----------|---|
| 8(a)         | Deduces correct gradient or intercept PI         | 2.2a | M1       | Gradient = $\frac{1}{2}$                                |
|              | Obtains correct equation with $y$ as the subject | 1.1b | A1       | Line passes through $(0, -2)$<br>$y = \frac{1}{2}x - 2$ |
| <b>Total</b> |  |      | <b>2</b> |   |

| Q            | Marking instructions  | AO   | Marks    | Typical solution |
|--------------|---|------|----------|------------------|
| 8(b)         | Draws a half line from $-2i$ passing through $4$ .<br>Condone full line<br>ft their linear equation in part (a) | 1.1b | B1F      |                  |
|              | Draws circle or arc of a circle, with centre at $2-3i$ or radius 2  | 2.2a | M1       |                  |
|              | Draws circle or arc of a circle, centre at $2-3i$ and radius 2  | 1.1b | A1       |                  |
|              | Correct region indicated  | 2.2a | A1       |                  |
| <b>Total</b> |   |      | <b>4</b> |                  |

| Q            | Marking instructions   | AO   | Marks    | Typical solution   |
|--------------|--|------|----------|--|
| 8(c)(i)      | Identifies the point in their region nearest to the origin.<br><br>For example draws the perpendicular from the half-line to the origin<br>or<br>Finds $y = -2x$   | 3.1a | B1F      |  $\sin \alpha = \frac{\sqrt{5}}{5}$ $\frac{ z_1 }{4} = \sin \alpha$ $ z_1  = \frac{4\sqrt{5}}{5}$ |
|              | Finds the distance between their valid point and the origin<br><br>For example uses $\sin(\tan^{-1} \frac{1}{2})$<br>or<br>Finds the distance between the origin and the point of intersection of the lines $y = \frac{1}{2}x - 2$ and $y = -2x$ | 3.1a | M1       |  |
|              | Obtains correct exact value of $ z_1 $   | 1.1b | A1       |  |
| <b>Total</b> |  |      | <b>3</b> |  |

| Q            | Marking instructions   | AO   | Marks    | Typical solution   |
|--------------|--|------|----------|--|
| 8(c)(ii)     | Uses their values from part (c)(i) to obtain value of $a$ or $b$ | 1.1a | M1       | $a =  z_1  \sin \alpha \quad b = - z_1  \cos \alpha$ $= \frac{4\sqrt{5}}{5} \times \frac{\sqrt{5}}{5} \quad = \frac{-4\sqrt{5}}{5} \times \frac{2\sqrt{5}}{5}$ $= \frac{4}{5} \quad = \frac{-8}{5}$ $z_1 = \frac{4}{5} - \frac{8}{5}i$ |
|              | Obtains correct value of $z_1$<br>CSO                            | 1.1b | A1       |  |
| <b>Total</b> |  |      | <b>2</b> |  |

|                       |  |  |           |  |
|-----------------------|--|--|-----------|--|
| <b>Question total</b> |  |  | <b>11</b> |  |
|-----------------------|--|--|-----------|--|

| Q            | Marking instructions   | AO   | Marks    | Typical solution  |
|--------------|--|------|----------|---|
| 9(a)         | Draws at least one loop in the correct place                 | 1.1b | B1       |  |
|              | Draws both loops correctly (approx equal size) and no others | 1.1b | B1       |   |
| <b>Total</b> |  |      | <b>2</b> |   |

| Q            | Marking instructions   | AO  | Marks    | Typical solution   |
|--------------|--|-----|----------|--|
| 9(b)         | Criticises limits of integration used by Roberto<br>PI                                     | 2.3 | M1       | Roberto has used incorrect limits of integration.  |
|              | Explains what is wrong with Roberto's range of values, including reference to $r^2 \geq 0$ | 2.4 | R1       | He should only have included values of $\theta$ which make $\sin 2\theta$ positive, because $r^2$ must be positive |
| <b>Total</b> |  |     | <b>2</b> |  |

| Q            | Marking instructions   | AO   | Marks    | Typical solution   |
|--------------|--|------|----------|--|
| 9(c)         | Forms an expression for an area using valid limits<br>PI by $9/4$ , $9/2$ or $9$ | 2.2a | M1       | $A = \frac{1}{2} \int_{-\pi}^{-\frac{\pi}{2}} 9 \sin 2\theta \, d\theta$ $+ \frac{1}{2} \int_0^{\frac{\pi}{2}} 9 \sin 2\theta \, d\theta$ $= \int_0^{\frac{\pi}{2}} 9 \sin 2\theta \, d\theta$ by symmetry $A = \left[ -\frac{9}{2} \cos 2\theta \right]_0^{\frac{\pi}{2}}$ $= -\frac{9}{2} (\cos \pi - \cos 0)$ $= 9$ |
|              | Obtains correct answer   | 1.1b | A1       |  |
| <b>Total</b> |  |      | <b>2</b> |  |

| Q            | Marking instructions   | AO   | Marks    | Typical solution   |
|--------------|--|------|----------|--|
| 9(d)         | Deduces at least one correct value of $\theta$   | 2.2a | M1       | Max. value of $r = 3$<br>For $r$ maximum, $\sin 2\theta = 1$<br>$2\theta = \frac{\pi}{2}, -\frac{3\pi}{2}$ $\theta = \frac{\pi}{4}, -\frac{3\pi}{4}$ $P\left(3, \frac{\pi}{4}\right)$ and $Q\left(3, -\frac{3\pi}{4}\right)$ |
|              | Obtains both correct solutions (not $r = -3$ )<br>Condone $r$ and $\theta$ transposed.<br>accept $\frac{5\pi}{4}$ etc<br>OE decimals to 3sig fig or better | 1.1b | A1       |  |
| <b>Total</b> |  |      | <b>2</b> |  |

| Q            | Marking instructions   | AO   | Marks    | Typical solution  |
|--------------|--|------|----------|---|
| 9(e)(i)      | Finds cartesian coordinates or position vector of their P, ACF   | 1.1b | B1F      | Cartesian coordinates of $P$ are<br>$\left(\frac{3\sqrt{2}}{2}, \frac{3\sqrt{2}}{2}\right)$<br>$\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3\sqrt{2} \\ 2 \end{bmatrix} = \begin{bmatrix} 9\sqrt{2} \\ 2 \end{bmatrix}$ For $P'$ , $r^2 = \frac{81}{2} + \frac{9}{2} = 45 \Rightarrow r = 3\sqrt{5}$<br>$\tan \theta = \frac{1}{3} \Rightarrow \theta = \tan^{-1}\left(\frac{1}{3}\right)$<br>$P'\left(3\sqrt{5}, \tan^{-1}\left(\frac{1}{3}\right)\right)$ |
|              | Multiplies their cartesian position vector by matrix M, must be $Mv$ , to obtain an image              | 3.1a | M1       |   |
|              | Obtains their correct value for $r$ or $\theta$ , ACF, for their image                                 | 1.1a | M1       |   |
|              | Obtains their correct answer, either exact or to at least 2 sig fig AWRT (6.7, 0.32)<br>Must have M1M1 | 1.1b | A1F      |   |
| <b>Total</b> |  |      | <b>4</b> |   |

| Q            | Marking instructions   | AO   | Marks    | Typical solution  |
|--------------|--|------|----------|---|
| 9(e)(ii)     | Explains correctly why the area is unchanged                 | 2.4  | E1       | Det $M = 1$<br>Area enclosed by C2 = (Area enclosed by C1) $\times$ det $M$<br>$= 9 \times 1 = 9$ |
|              | Obtains their correct area from their part (c) and their det | 2.2a | B1F      |   |
| <b>Total</b> |  |      | <b>2</b> |   |

|                       |  |  |           |  |
|-----------------------|--|--|-----------|--|
| <b>Question total</b> |  |  | <b>14</b> |  |
|-----------------------|--|--|-----------|--|

| Q            | Marking instructions  | AO   | Marks    | Typical solution   |
|--------------|---|------|----------|--|
| 10(a)        | Obtains correct normal vector to the plane  | 2.2a | B1       | Normal to plane $\mathbf{n} = \begin{bmatrix} 4 \\ p \\ 5 \end{bmatrix}$   |
|              | Obtains correct expression for $\overline{AB}$ or $\overline{BA}$   | 1.2  | B1       | Let $\overline{AB} = \mathbf{c}$ then<br>$\mathbf{c} = \begin{bmatrix} 2 \\ -5 \\ 1 \end{bmatrix}$   |
|              | Obtains their correct scalar (or vector) product  | 1.1b | B1F      | $\mathbf{n} \cdot \mathbf{c} = 13 - 5p$  |
|              | Uses scalar (or vector) product to obtain an equation in $p$  | 3.1a | M1       | $ \mathbf{c}  = \sqrt{30}$ and $ \mathbf{n}  = \sqrt{41 + p^2}$  |
|              | Forms an equation in $p$ , by squaring and removing any rational functions<br>Eg $82 + 2p^2 = 5^2(13 - 5p)^2$         | 1.1a | M1       | As $\alpha$ is acute, $\sin \alpha = \cos \theta = \frac{\sqrt{15}}{75}$   |
|              | Solves quadratic and selects correct answer, discarding the other root<br>Condone lack of modulus sign in the working | 2.1  | R1       | Hence $\left  \frac{13 - 5p}{\sqrt{30}\sqrt{41 + p^2}} \right  = \frac{\sqrt{15}}{75}$<br><br>$82 + 2p^2 = 5^2(13 - 5p)^2$<br>$623p^2 - 3250p + 4143 = 0$<br>$p = 3$ or $p = \frac{1381}{623}$<br>$p \in \mathbb{Z}$<br>$\therefore p = 3$ |
| <b>Total</b> |   |      | <b>6</b> |  |

| Q            | Marking instructions  | AO   | Marks    | Typical solution  |
|--------------|---|------|----------|---|
| 10(b)        | Recognises need to divide constant term of the plane equation by $ \mathbf{n} $                           | 1.1a | M1       | $ \mathbf{n}  = 5\sqrt{2}$<br>Distance is   |
|              | Finds correct distance for their $p$ , exact or decimal at least 2sf (condone 2)<br>Condone missing units | 1.1b | A1F      | $\frac{9}{5\sqrt{2}} + \frac{5}{5\sqrt{2}} = \frac{7\sqrt{2}}{5} = 1.98 \text{ cm}$ |
| <b>Total</b> |   |      | <b>2</b> |   |

| Q     | Marking instructions  | AO   | Marks     | Typical solution  |
|-------|---|------|-----------|---|
| 10(c) | Obtains correct equation of the line through A & A' for their $p$<br>Condone lack of $r =$                  | 3.1a | B1F       | $\mathbf{r} = \begin{bmatrix} 7 \\ 2 \\ -3 \end{bmatrix} + \mu \begin{bmatrix} 4 \\ 3 \\ 5 \end{bmatrix}$ At $\Pi_1$<br>$4(7 + 4\mu) + 3(2 + 3\mu) + 5(-3 + 5\mu) = 9$ $\mu = \frac{-1}{5}$ At image point<br>$\mu = \frac{-2}{5}$ $\left(\frac{27}{5}, \frac{4}{5}, -5\right)$ |
|       | Forms an equation to find the value of $\mu$ for their line<br>Condone use of $\Pi_2$                       | 3.1a | M1        |   |
|       | Doubles their value of $\mu$ and uses it to find image point for their line<br>Condone use of $\Pi_2$       | 3.2a | M1        |   |
|       | Obtains correct coordinates for their $p$<br>Do not accept position vector<br>Do not condone use of $\Pi_2$ | 1.1b | A1F       |   |
|       | <b>Total</b>  |      | <b>4</b>  |   |
|       | <b>Question total</b>   |      | <b>12</b> |   |



| Q     | Marking instructions  | AO   | Marks    | Typical solution   |
|-------|---|------|----------|--|
| 11(a) | Forms equilibrium force equation with three correct terms.<br>Condone sign errors   | 3.1b | B1       | In equilibrium position<br>$7e_A = 9e_B + 0.32g\sin 30$<br>$1.2 = e_A + e_B$<br>$e_A = 0.775, e_B = 0.425$   |
|       | Forms at least one correct expression in $x$ for the tension<br>ie $9(e_B - x)$ or $7(e_A + x)$<br>Condone their incorrect $e_A$ or $e_B$   | 1.1a | B1F      | After release<br>$9(e_B - x) + 0.32g\sin 30 - 7(e_A + x) = 0.32\ddot{x}$<br>$9(0.425 - x) + 0.32g\sin 30 - 7(0.775 + x) = 0.32\ddot{x}$<br>$3.825 - 9x + 1.6 - 5.425 - 7x = 0.32\ddot{x}$<br>$-16x = 0.32\ddot{x}$<br>$\ddot{x} + 50x = 0$ |
|       | Forms general force equation with four terms (with at least two terms correct).<br>Condone "a" for $\ddot{x}$<br>Condone sign errors on the terms<br>Condone their incorrect $e_A$ or $e_B$ | 3.1b | M1       | as required  |
|       | Forms correct force equation.<br>Can be in terms of $e_A$ & $e_B$<br>Condone "a" for $\ddot{x}$<br>Condone their incorrect $e_A$ or $e_B$   | 1.1b | A1F      |  |
|       | Constructs a rigorous argument to show the required result  | 2.1  | R1       |  |
|       | <b>Total</b>  |      | <b>5</b> |  |

| Q        | Marking instructions   | AO   | Marks    | Typical solution  |
|----------|--|------|----------|---|
| 11(b)(i) | Obtains fully correct 2 <sup>nd</sup> order DE or auxiliary equation, any correct form.<br>Condone $-k$<br>Allow $m$ instead of 0.32 | 2.2a | B1       | $9(0.425 - x) + 0.32g \sin 30 - 7(0.775 + x) - k\dot{x} = 0.32\ddot{x}$ $0.32\ddot{x} + k\dot{x} + 16x = 0$ $0.32\lambda^2 + k\lambda + 16 = 0$ |
|          | Sets up $b^2 - 4ac = 0$ from their 2 <sup>nd</sup> order DE or Auxiliary Equation  | 1.2  | M1       | Critical Damping so:<br><br>$b^2 - 4ac = 0$ $k^2 - 4 \times 0.32 \times 16 = 0$   |
|          | Obtains correct value of $k$ from correct working  | 2.1  | R1       | $k^2 = \frac{512}{25}$ $k = \frac{16\sqrt{2}}{5}$   |
|          | <b>Total</b>   |      | <b>3</b> |   |

| Q         | Marking instructions  | AO   | Marks    | Typical solution   |
|-----------|---|------|----------|--|
| 11(b)(ii) | Obtains correct solution from their three term Auxiliary Equation                       | 3.1a | M1       | $0.32\lambda^2 + \frac{16\sqrt{2}}{5}\lambda + 16 = 0$ $\lambda = -5\sqrt{2} \text{ twice}$ $x = Ae^{-5\sqrt{2}t} + Bte^{-5\sqrt{2}t}$ |
|           | Obtains correct solution of their three term differential equation                      | 1.1b | A1F      | $x = 0.2, t = 0 \Rightarrow A = 0.2$   |
|           | Uses $x = 0.2$ when $t = 0$ to obtain correct $A$                                       | 3.3  | B1       | $\dot{x} = 5\sqrt{2}Ae^{-5\sqrt{2}t} - 5\sqrt{2}Bte^{-5\sqrt{2}t}$ $\dot{x} = 0, t = 0 \Rightarrow B = \sqrt{2}$                       |
|           | Sets their correct $\dot{x} = 0$ when $t = 0$   | 3.3  | M1       | $x = 0.2e^{-5\sqrt{2}t} + \sqrt{2}te^{-5\sqrt{2}t}$  |
|           | Obtains their correct $B$<br>Ft from $\lambda^2 + \frac{16\sqrt{2}}{5}\lambda + 50 = 0$ | 1.1b | A1F      |  |
|           | Completes a rigorous argument to obtain the correct result                              | 2.1  | R1       |  |
|           | <b>Total</b>  |      | <b>6</b> |  |

| Q            | Marking instructions   | AO   | Marks    | Typical solution   |
|--------------|--|------|----------|--|
| 11(b)(iii)   | Forms an equation to find the value of $t$ at the maximum speed<br>Must start from a damped harmonic model | 3.1a | M1       | $\dot{x} = \sqrt{2}e^{-5\sqrt{2}t} - \sqrt{2}e^{-5\sqrt{2}t} - 10te^{-5\sqrt{2}t}$ $= -10te^{-5\sqrt{2}t}$ $\ddot{x} = -10e^{-5\sqrt{2}t} - 50\sqrt{2}te^{-5\sqrt{2}t}$ <p>At Max <math>\dot{x}</math></p> $\ddot{x} = 0 \quad 10e^{-5\sqrt{2}t} - 50\sqrt{2}te^{-5\sqrt{2}t}$ |
|              | Obtains a correct equation to find the value of $t$ at the maximum speed                                   | 1.1b | A1F      | $5\sqrt{2}t = 1$ $t = \frac{1}{5\sqrt{2}} = \frac{\sqrt{2}}{10}$   |
|              | Obtains their correct $t$ , any form   | 1.1b | A1F      | $\dot{x} = \sqrt{2}e^{-1} = 0.52026\dots$ <p>Max Speed = 0.5 m s<sup>-1</sup></p>  |
|              | Uses their value of $t$ to obtain the velocity<br>Must start from a damped harmonic model                  | 3.4  | M1       |  |
|              | Obtains correct max speed, to at least 1sf or exact<br>Condone missing units (-0.5 m s <sup>-1</sup> = A0) | 3.2a | A1       |  |
| <b>Total</b> |  |      | <b>5</b> |  |

|                       |  |           |  |
|-----------------------|--|-----------|--|
| <b>Question total</b> |  | <b>19</b> |  |
|-----------------------|--|-----------|--|

| Q            | Marking instructions   | AO   | Marks    | Typical solution  |
|--------------|--|------|----------|---|
| 12(a)        | Obtains at least one correct non-zero argument/solution                              | 1.1a | M1       | $\begin{aligned} \cos 5\theta &= 1 \\ 5\theta &= 2n\pi \\ z &= \cos 0 + i \sin 0, \\ &\cos \frac{2\pi}{5} + i \sin \frac{2\pi}{5}, \\ &\cos \frac{4\pi}{5} + i \sin \frac{4\pi}{5}, \\ &\cos \frac{6\pi}{5} + i \sin \frac{6\pi}{5}, \\ &\cos \frac{8\pi}{5} + i \sin \frac{8\pi}{5} \end{aligned}$ |
|              | Obtains completely correct solutions must be $0 \leq \theta < 2\pi$ (condone $z=1$ ) | 1.1b | A1       |   |
| <b>Total</b> |  |      | <b>2</b> |   |

| Q            | Marking instructions  | AO  | Marks    | Typical solution   |
|--------------|---|-----|----------|--|
| 12(b)        | States that all the points are the same distance from the origin                      | 2.4 | E1       | Each of the solutions has modulus 1, and their arguments increase in steps of $2\pi/5$ |
|              | States that the angles between lines from the origin to adjacent points are all equal | 2.1 | E1       |  |
| <b>Total</b> |   |     | <b>2</b> |  |

| Q            | Marking instructions   | AO   | Marks    | Typical solution  |
|--------------|--|------|----------|---|
| 12(c)        | Expands $(\cos \theta + i \sin \theta)^5$<br>Condones errors in or omissions of Imaginary part | 1.1a | M1       | $(\cos \theta + i \sin \theta)^5 = 1$<br><br>Let $c = \cos \theta, s = \sin \theta$<br><br>$c^5 + 5c^4is - 10c^3s^2 - 10c^2is^3 + 5cs^4 + is^5 = 1$                             |
|              | Obtains correct unsimplified Real part of expansion  | 1.1b | A1       | Real parts: $c^5 - 10c^3s^2 + 5cs^4 = 1$<br>$c^5 - 10c^3(1 - c^2) + 5c(1 - c^2)^2 - 1 = 0$<br>$c^5 - 10c^3 + 10c^5 + 5c - 10c^3 + 5c^5 - 1 = 0$<br>$16c^5 - 20c^3 + 5c - 1 = 0$ |
|              | Equates real parts   | 3.1a | M1       | as required   |
|              | Uses appropriate trig identity to express real part in terms of $c$                            | 3.1a | M1       |   |
|              | Completes a rigorous argument to obtain the required result                                    | 2.1  | R1       |   |
| <b>Total</b> |  |      | <b>5</b> |   |

| Q            | Marking instructions  | AO   | Marks    | Typical solution  |
|--------------|---|------|----------|---|
| 12(d)        | Explains that $z_3$ is the complex conjugate of $z_4$ and that $z_2$ is the complex conjugate of $z_5$      | 2.4  | E1       | By symmetry<br>$z_4^* = z_3$ so<br>$\cos(\arg z_3) = \cos(\arg z_4) = a$<br>and<br>$z_5^* = z_2$ so<br>$\cos(\arg z_2) = \cos(\arg z_5) = b$  |
|              | Explains that the Real parts of the points on the diagram are the solutions of $16c^5 - 20c^3 + 5c - 1 = 0$ | 2.2a | M1       | So $c = a$ and $c = b$ are both double roots of the equation $16c^5 - 20c^3 + 5c - 1 = 0$<br>and, by the factor theorem, $(c - a)(c - b)$ is a repeated quadratic factor of<br>$16c^5 - 20c^3 + 5c - 1$ |
|              | Completes a rigorous argument to obtain the required result   | 2.1  | R1       |   |
| <b>Total</b> |   |      | <b>3</b> |   |

| Q     | Marking instructions   | AO   | Marks    | Typical solution   |
|-------|--|------|----------|--|
| 12(e) | Deduces that $h$ is a solution of the equation<br>This may appear anywhere in the solution | 2.2a | B1       | $h$ is a solution to the equation<br>$16c^5 - 20c^3 + 5c - 1 = 0$ $(c - 1)(16c^4 + 16c^3 - 4c^2 - 4c + 1) = 0$ |
|       | Factorises to obtain a linear factor and a quartic factor or better                        | 3.1a | M1       | Discard $c = 1$<br>$(4c^2 + 2c - 1)^2 = 0$ $4c^2 + 2c - 1 = 0$ $c = \frac{-1 \pm \sqrt{5}}{4}$                 |
|       | Solves the quartic or quadratic equation correctly to get only two solutions               | 1.1b | A1       | Select the solution with the greater absolute value; so  |
|       | Selects the correct solution   | 3.2a | E1       | as required<br>$h = \left  \frac{-1 - \sqrt{5}}{4} \right  = \frac{\sqrt{5} + 1}{4}$                           |
|       | Completes a rigorous argument to explain the required result                               | 2.1  | R1       |  |
|       | <b>Total</b>   |      | <b>5</b> |  |

|  |                       |  |           |  |
|--|-----------------------|--|-----------|--|
|  | <b>Question total</b> |  | <b>17</b> |  |
|--|-----------------------|--|-----------|--|

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|--|--------------------|--|------------|--|
|  | <b>Paper total</b> |  | <b>100</b> |  |
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