

1.

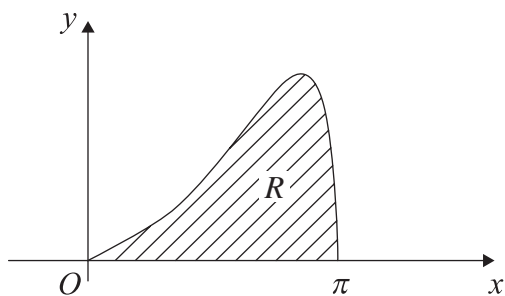


Figure 1

The curve shown in Figure 1 has equation $y = e^x \sqrt{(\sin x)}$, $0 \leq x \leq \pi$. The finite region R bounded by the curve and the x -axis is shown shaded in Figure 1.

(a) Complete the table below with the values of y corresponding to $x = \frac{\pi}{4}$ and $\frac{\pi}{2}$, giving your answers to 5 decimal places.

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
y	0			8.87207	0

(2)

(b) Use the trapezium rule, with all the values in the completed table, to obtain an estimate for the area of the region R . Give your answer to 4 decimal places.

(4)



2. (a) Use the binomial theorem to expand

$$(8 - 3x)^{\frac{1}{3}}, \quad |x| < \frac{8}{3},$$

in ascending powers of x , up to and including the term in x^3 , giving each term as a simplified fraction.

(5)

(b) Use your expansion, with a suitable value of x , to obtain an approximation to $\sqrt[3]{7.7}$. Give your answer to 7 decimal places.

(2)



3.

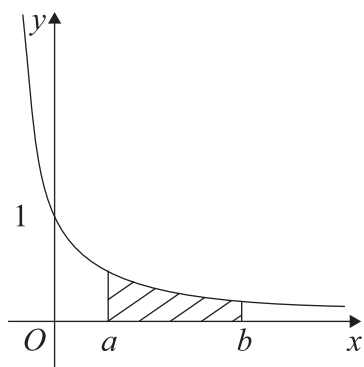


Figure 2

The curve shown in Figure 2 has equation $y = \frac{1}{(2x+1)}$. The finite region bounded by the

curve, the x -axis and the lines $x = a$ and $x = b$ is shown shaded in Figure 2. This region is rotated through 360° about the x -axis to generate a solid of revolution.

Find the volume of the solid generated. Express your answer as a single simplified fraction, in terms of a and b .

(5)



5. A curve is described by the equation

$$x^3 - 4y^2 = 12xy.$$

- (a) Find the coordinates of the two points on the curve where $x = -8$. (3)

- (b) Find the gradient of the curve at each of these points. (6)



7.

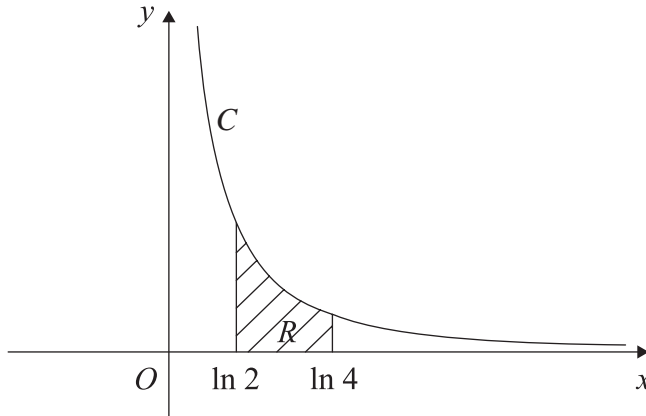


Figure 3

The curve C has parametric equations

$$x = \ln(t+2), \quad y = \frac{1}{(t+1)}, \quad t > -1.$$

The finite region R between the curve C and the x -axis, bounded by the lines with equations $x = \ln 2$ and $x = \ln 4$, is shown shaded in Figure 3.

(a) Show that the area of R is given by the integral

$$\int_0^2 \frac{1}{(t+1)(t+2)} dt. \tag{4}$$

(b) Hence find an exact value for this area. (6)

(c) Find a cartesian equation of the curve C , in the form $y = f(x)$. (4)

(d) State the domain of values for x for this curve. (1)

8. Liquid is pouring into a large vertical circular cylinder at a constant rate of $1600\text{ cm}^3\text{ s}^{-1}$ and is leaking out of a hole in the base, at a rate proportional to the square root of the height of the liquid already in the cylinder. The area of the circular cross section of the cylinder is 4000 cm^2 .

(a) Show that at time t seconds, the height h cm of liquid in the cylinder satisfies the differential equation

$$\frac{dh}{dt} = 0.4 - k\sqrt{h}, \text{ where } k \text{ is a positive constant.} \quad (3)$$

When $h = 25$, water is leaking out of the hole at $400\text{ cm}^3\text{ s}^{-1}$.

(b) Show that $k = 0.02$ (1)

(c) Separate the variables of the differential equation

$$\frac{dh}{dt} = 0.4 - 0.02\sqrt{h},$$

to show that the time taken to fill the cylinder from empty to a height of 100 cm is given by

$$\int_0^{100} \frac{50}{20 - \sqrt{h}} dh. \quad (2)$$

Using the substitution $h = (20 - x)^2$, or otherwise,

(d) find the exact value of $\int_0^{100} \frac{50}{20 - \sqrt{h}} dh$. (6)

(e) Hence find the time taken to fill the cylinder from empty to a height of 100 cm, giving your answer in minutes and seconds to the nearest second. (1)



