

Friday 20 January 2012 – Afternoon

**A2 GCE MATHEMATICS (MEI)**

**4753/01** Methods for Advanced Mathematics (C3)

**QUESTION PAPER**

Candidates answer on the Printed Answer Book.

**OCR supplied materials:**

- Printed Answer Book 4753/01
- MEI Examination Formulae and Tables (MF2)

**Other materials required:**

- Scientific or graphical calculator

**Duration:** 1 hour 30 minutes



**INSTRUCTIONS TO CANDIDATES**

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

**INFORMATION FOR CANDIDATES**

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

**INSTRUCTION TO EXAMS OFFICER/INVIGILATOR**

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

## Section A (36 marks)

1 Differentiate  $x^2 \tan 2x$ . [3]

2 The functions  $f(x)$  and  $g(x)$  are defined as follows.

$$f(x) = \ln x, \quad x > 0$$

$$g(x) = 1 + x^2, \quad x \in \mathbb{R}$$

Write down the functions  $fg(x)$  and  $gf(x)$ , and state whether these functions are odd, even or neither. [4]

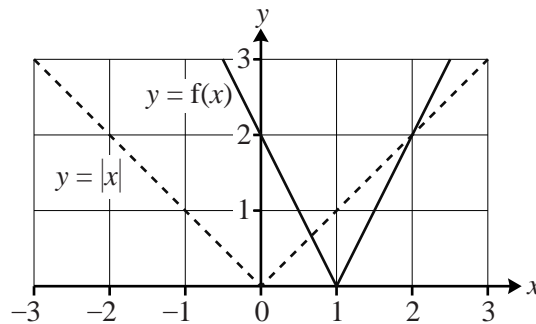
3 Show that  $\int_0^{\frac{\pi}{2}} x \cos^{\frac{1}{2}} x \, dx = \frac{\sqrt{2}}{2} \pi + 2\sqrt{2} - 4$ . [5]

4 Prove or disprove the following statement:

‘No cube of an integer has 2 as its units digit.’ [2]

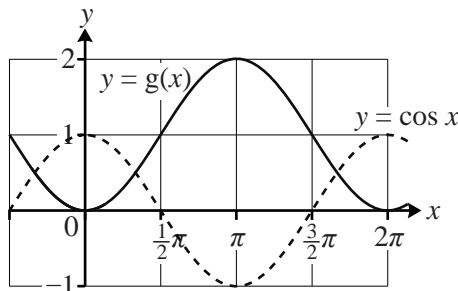
5 Each of the graphs of  $y=f(x)$  and  $y=g(x)$  below is obtained using a sequence of two transformations applied to the corresponding dashed graph. In each case, state suitable transformations, and hence find expressions for  $f(x)$  and  $g(x)$ .

(i)



[3]

(ii)



[3]

- 6 Oil is leaking into the sea from a pipeline, creating a circular oil slick. The radius  $r$  metres of the oil slick  $t$  hours after the start of the leak is modelled by the equation

$$r = 20(1 - e^{-0.2t}).$$

(i) Find the radius of the slick when  $t = 2$ , and the rate at which the radius is increasing at this time. [4]

(ii) Find the rate at which the area of the slick is increasing when  $t = 2$ . [4]

- 7 Fig. 7 shows the curve  $x^3 + y^3 = 3xy$ . The point P is a turning point of the curve.

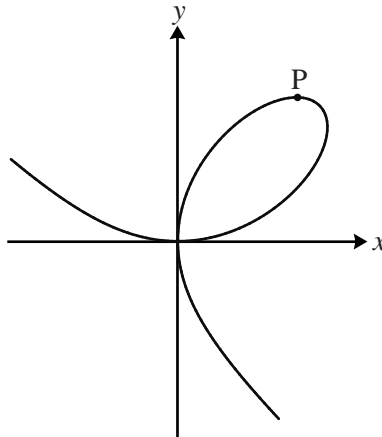


Fig. 7

(i) Show that  $\frac{dy}{dx} = \frac{y - x^2}{y^2 - x}$ . [4]

(ii) Hence find the exact  $x$ -coordinate of P. [4]

## Section B (36 marks)

- 8 Fig. 8 shows the curve  $y = \frac{x}{\sqrt{x-2}}$ , together with the lines  $y = x$  and  $x = 11$ . The curve meets these lines at P and Q respectively. R is the point (11, 11).

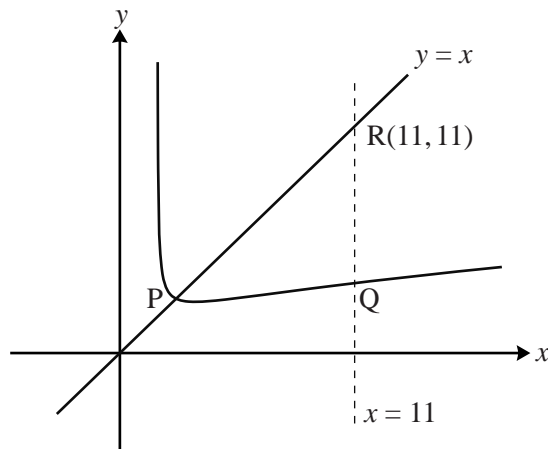


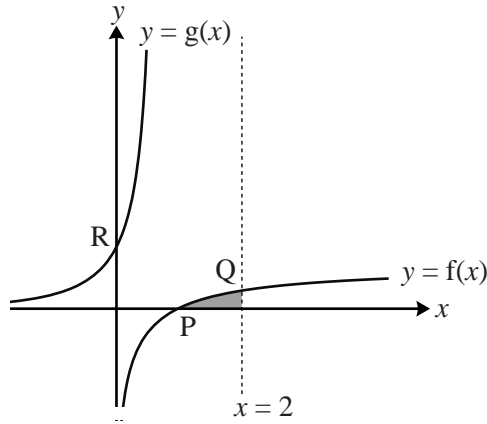
Fig. 8

- (i) Verify that the  $x$ -coordinate of P is 3. [2]
- (ii) Show that, for the curve,  $\frac{dy}{dx} = \frac{x-4}{2(x-2)^{\frac{3}{2}}}$ .  
Hence find the gradient of the curve at P. Use the result to show that the curve is **not** symmetrical about  $y = x$ . [7]
- (iii) Using the substitution  $u = x - 2$ , show that  $\int_3^{11} \frac{x}{\sqrt{x-2}} dx = 25\frac{1}{3}$ .  
Hence find the area of the region PQR bounded by the curve and the lines  $y = x$  and  $x = 11$ . [9]

- 9 Fig. 9 shows the curves  $y = f(x)$  and  $y = g(x)$ . The function  $y = f(x)$  is given by

$$f(x) = \ln \left( \frac{2x}{1+x} \right), \quad x > 0.$$

The curve  $y = f(x)$  crosses the  $x$ -axis at P, and the line  $x = 2$  at Q.



**Fig. 9**

- (i) Verify that the  $x$ -coordinate of P is 1.

Find the exact  $y$ -coordinate of Q.

[2]

- (ii) Find the gradient of the curve at P. [Hint: use  $\ln \frac{a}{b} = \ln a - \ln b$ .]

[4]

The function  $g(x)$  is given by

$$g(x) = \frac{e^x}{2 - e^x}, \quad x < \ln 2.$$

The curve  $y = g(x)$  crosses the  $y$ -axis at the point R.

- (iii) Show that  $g(x)$  is the inverse function of  $f(x)$ .

Write down the gradient of  $y = g(x)$  at R.

[5]

- (iv) Show, using the substitution  $u = 2 - e^x$  or otherwise, that  $\int_0^{\ln \frac{4}{3}} g(x) dx = \ln \frac{3}{2}$ .

Using this result, show that the exact area of the shaded region shown in Fig. 9 is  $\ln \frac{32}{27}$ .  
[Hint: consider its reflection in  $y = x$ .]

[7]

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