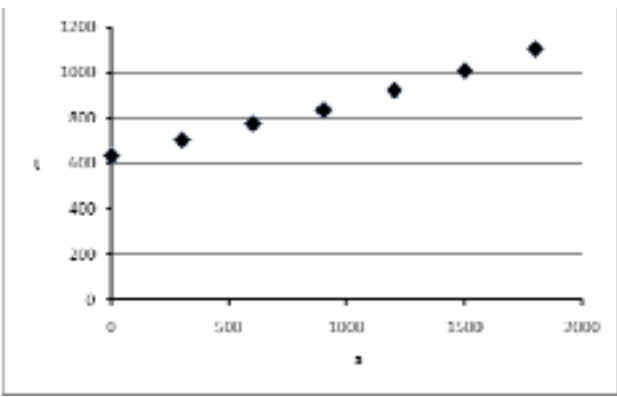


## 4767 Statistics 2

1	(i)		<p>G1 For values of <math>a</math></p> <p>G1 for values of <math>t</math></p> <p>G1 for axes</p>	[3]
(ii)		<p><math>a</math> is independent, <math>t</math> is dependent since the values of <math>a</math> are not subject to random variation, but are determined by the runways which the pilot chooses, whereas the values of <math>t</math> are subject to random variation.</p>	<p>B1</p> <p>E1dep</p> <p>E1dep</p>	[3]
(iii)		<p><math>\bar{a} = 900</math>, <math>\bar{t} = 855.2</math></p> $b = \frac{S_{at}}{S_{aa}} = \frac{6037800 - 5987 \times 6300 / 7}{8190000 - 6300^2 / 7} = \frac{649500}{2520000} = 0.258$ <p>OR <math>b = \frac{6037800 / 7 - 855.29 \times 900}{8190000 / 7 - 900^2} = \frac{92785}{360000} = 0.258</math></p> <p>hence least squares regression line is:</p> $t - \bar{t} = b(a - \bar{a})$ $\Rightarrow t - 855.29 = 0.258(a - 900)$ $\Rightarrow t = 0.258a + 623$	<p>B1 for <math>\bar{a}</math> and <math>\bar{t}</math> used (SOI)</p> <p>M1 for attempt at gradient (<math>b</math>)</p> <p>A1 for 0.258 <b>cao</b></p> <p>M1 for equation of line</p> <p>A1 FT for complete equation</p>	[5]
(iv)		<p>(A) For <math>a = 800</math>, predicted take-off distance = <math>0.258 \times 800 + 623 = 829</math></p> <p>(B) For <math>a = 2500</math>, predicted take-off distance = <math>0.258 \times 2500 + 623 = 1268</math></p> <p>Valid relevant comments relating to the predictions such as: First prediction is interpolation so should be reasonable Second prediction is extrapolation and may not be reliable</p>	<p>M1 for at least one prediction attempted</p> <p>A1 for both answers (FT their equation if <math>b &gt; 0</math>)</p> <p>E1 (first comment)</p> <p>E1 (second comment)</p>	[4]
(v)		<p><math>a = 1200 \Rightarrow</math> predicted <math>t = 0.258 \times 1200 + 623 = 933</math></p> <p>Residual = <math>923 - 933 = -10</math></p> <p>The residual is negative because the observed value is less than the predicted value.</p>	<p>M1 for prediction</p> <p>M1 for subtraction</p> <p>A1 FT</p> <p>E1</p>	[4]
<b>Total</b>				<b>[19]</b>

2	(i)	P(1 of 10 is faulty) $= \binom{10}{1} \times 0.02^1 \times 0.98^9 = 0.1667$	M1 for coefficient M1 for probabilities A1	[3]
(ii)		$n$ is large and $p$ is small	B1, B1 Allow appropriate numerical ranges	[2]
(iii)		$\lambda = 150 \times 0.02 = 3$ (A) $P(X = 0) = e^{-3} \frac{3^0}{0!} = 0.0498$ (3 s.f.) or from tables = 0.0498 (B) Expected number = 3 Using tables: $P(X > 3) = 1 - P(X \leq 3)$ = $1 - 0.6472 = 0.3528$	B1 for mean (soi) M1 for calculation or use of tables A1 B1 expected no = 3 (soi) M1 A1	[3]          [3]
(iv)		(A) Binomial(2000,0.02) (B) Use Normal approx with $\mu = np = 2000 \times 0.02 = 40$ $\sigma^2 = npq = 2000 \times 0.02 \times 0.98 = 39.2$ $P(X \leq 50) = P\left(Z \leq \frac{50.5 - 40}{\sqrt{39.2}}\right)$ = $P(Z \leq 1.677) = \Phi(1.677) = 0.9532$ NB Poisson approximation also acceptable for full marks	B1 for binomial B1 for parameters B1 B1 B1 for continuity corr. M1 for probability using correct tail A1 CAO	[2]          [5]       <b>Total</b> [18]

3	(i)	(A)	$P(X < 50)$ $= P\left(Z < \frac{50 - 45.3}{11.5}\right)$ $= P(Z < 0.4087)$ $= \Phi(0.4087)$ $= 0.6585$	M1 for standardising M1 for correct structure of probability calc'	[3]
		(B)	$P(45.3 < X < 50)$ $= 0.6585 - 0.5$ $= 0.1585$	M1 A1	[2]
	(ii)	<p>From tables <math>\Phi^{-1}(0.9) = 1.282</math></p> $\frac{k - 45.3}{11.5} = 1.282$ $k = 45.3 + 1.282 \times 11.5 = 60.0$	B1 for 1.282 seen M1 for equation in $k$ A1 CAO	[3]	
	(iii)	$P(\text{score} = 111)$ $= P(110.5 < Y < 111.5)$ $= P\left(\frac{110.5 - 100}{15} < Z < \frac{111.5 - 100}{15}\right)$ $= P(0.7 < Z < 0.7667)$ $= \Phi(0.7667) - \Phi(0.7)$ $= 0.7784 - 0.7580$ $= 0.0204$	B1 for both continuity corrections M1 for standardising M1 for correct structure of probability calc' A1 CAO	[4]	
	(iv)	<p>From tables, <math>\Phi^{-1}(0.3) = -0.5244</math>, <math>\Phi^{-1}(0.8) = 0.8416</math></p> $22 = \mu + 0.8416 \sigma$ $15 = \mu - 0.5244 \sigma$ $7 = 1.3660 \sigma$ $\sigma = 5.124, \mu = 17.69$	B1 for 0.5244 or 0.8416 seen M1 for at least one equation in $z$ , $\mu$ & $\sigma$ A1 for both correct M1 for attempt to solve two appropriate equations A1 CAO for both	[5]	
				<b>TOTAL</b>	<b>[17]</b>

4	(i)	<p><math>H_0</math>: no association between size of business and recycling service used.  <math>H_1</math>: some association between size of business and recycling service used.</p>	B1 for both	[1]
	(ii)	<p>Expected frequency = <math>78/285 \times 180 = 49.2632</math>  Contribution = <math>(52 - 49.2632)^2 / 49.2632</math>  = 0.1520</p>	<p>M1 A1  M1 for valid attempt at <math>(O-E)^2/E</math>  A1 <b>NB Answer given</b>  Allow 0.152</p>	[4]
	(iii)	<p>Test statistic <math>X^2 = 0.6041</math></p> <p>Refer to <math>\chi_2^2</math>  Critical value at 5% level = 5.991  Result is not significant</p> <p>There is no evidence to suggest any association between size of business and recycling service used.  NB if <math>H_0</math> <math>H_1</math> reversed, or 'correlation' mentioned in part (i), do not award B1 in part (i) or E1 in part (iii).</p>	<p>B1</p> <p>B1 for 2 deg of f(seen)  B1 CAO for cv  B1 for not significant</p> <p>E1</p>	[5]
	(iv)	<p><math>H_0: \mu = 32.8</math>; <math>H_1: \mu &lt; 32.8</math>  Where <math>\mu</math> denotes the population mean weight of rubbish in the bins.</p> $\text{Test statistic} = \frac{30.9 - 32.8}{3.4/\sqrt{50}} = -\frac{1.9}{0.4808} = -3.951$ <p>5% level 1 tailed critical value of <math>z = -1.645</math></p> <p><math>-3.951 &lt; -1.645</math> so significant.  There is sufficient evidence to reject <math>H_0</math></p> <p>There is evidence to suggest that the weight of rubbish in dustbins has been reduced.</p>	<p>B1 for use of 32.8  B1 for both correct  B1 for definition of <math>\mu</math></p> <p>M1 must include <math>\sqrt{50}</math>  A1</p> <p>B1 for <math>\pm 1.645</math></p> <p>M1 for sensible comparison leading to a conclusion</p> <p>A1 for conclusion in words in context</p> <p style="text-align: right;"><b>TOTAL</b></p>	<p>[8]</p> <p>[18]</p>