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A Level Further Mathematics B (MEI)
Y436 Further Pure with Technology
Sample Question Paper — Version 2

Date – Morning/Afternoon
Time allowed: 1 hour 45 minutes

You must have:
- Printed Answer Booklet
- Formulae Further Mathematics B (MEI)

You may use:
- a scientific or graphical calculator
- Computer with appropriate software

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

COMPUTING RESOURCES
- Candidates will require access to a computer with a computer algebra system, a spreadsheet, a programming language and graph-plotting software throughout the examination.

INFORMATION
- The total mark for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **8** pages.

Answer **all** the questions.

1 A family of curves has polar equation $r = \cos^n\left(\frac{\theta}{n}\right)$, $0 \le \theta < n\pi$, where n is a positive even integer.

(i) (A) Sketch the curve for the cases n = 2 and n = 4. **[2]**

(B) State two points which lie on every curve in the family. **[1]**

(C) State one other feature common to all the curves. **[1]**

(ii) (A) Write down an integral for the length of the curve for the case n = 4. **[2]**

(B) Evaluate the integral. **[2]**

(iii) (A) Using $t = \theta$ as the parameter, find a parametric form of the equation of the family of curves. **[1]**

(B) Show that $\frac{dy}{dx} = \frac{\sin t \sin\left(\frac{t}{n}\right) - \cos t \cos\left(\frac{t}{n}\right)}{\sin t \cos\left(\frac{t}{n}\right) + \cos t \sin\left(\frac{t}{n}\right)}$. **[2]**

(iv) Hence show that there are $n+1$ points where the tangent to the curve is parallel to the y-axis. **[6]**

(v) By referring to appropriate sketches, show that the result in part **(iv)** is true in the case n = 4. **[2]**

2 **(i)** (A) Create a program to find all the solutions to $x^2 \equiv -1 \pmod{p}$ where $0 \le x < p$.

Write out your program in full in the Printed Answer Booklet. **[5]**

(B) Use the program to find the solutions to $x^2 \equiv -1 \pmod{p}$ for the primes

- p = 809,
- p = 811 and
- p = 444 001. **[3]**

(ii) State Wilson's Theorem. **[1]**

(iii) The following argument shows that $(4k)! \equiv ((2k)!)^2 \pmod{p}$ for the case $p = 4k + 1$.

$$(4k)! \equiv 1 \times 2 \times 3 \times \ldots \times (2k-1) \times 2k \times (2k+1) \times (2k+2) \times \ldots \times (4k-1) \times 4k \pmod{p} \qquad (1)$$

$$\equiv 1 \times 2 \times 3 \times \ldots \times (2k-1) \times 2k \times (-2k) \times (-(2k-1)) \times \ldots \times (-2) \times (-1) \pmod{p} \qquad (2)$$

$$\equiv ((2k)!)^2 \pmod{p} \qquad (3)$$

(A) Explain why $(2k+2)$ can be written as $(-(2k-1))$ in line (2). **[1]**

(B) Explain how line (3) has been obtained. **[2]**

(C) Explain why, if p is a prime of the form $p = 4k+1$, then $x^2 \equiv -1 \pmod{p}$ will have at least one solution. **[1]**

(D) Hence find a solution of $x^2 \equiv -1 \pmod{29}$. **[2]**

(iv) (A) Create a program that will find all the positive integers n, where n < 1000, such that $(n-1)! \equiv -1 \pmod{n^2}$. Write out your program in full. **[3]**

(B) State the values of n obtained. **[2]**

(C) A Wilson prime is a prime p such that $(p-1)! \equiv -1 \pmod{p^2}$. Write down all the Wilson primes p where p < 1000. **[1]**

3 This question explores the family of differential equations $\frac{dy}{dx} = \sqrt{1 + ax + 2y}$ for various values of the parameter a. Fig. 3 shows the tangent field in the case a = 1.

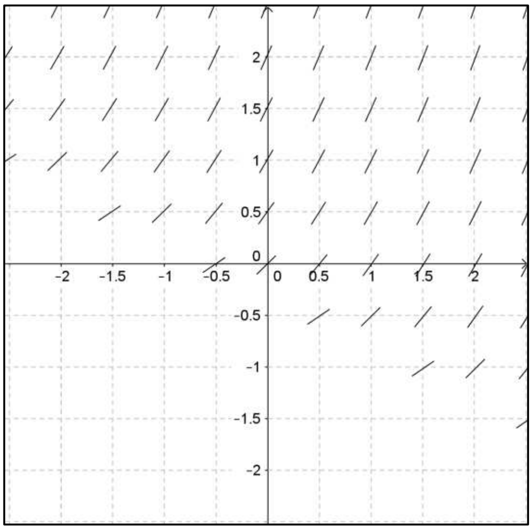


Fig. 3

(i) (A) Sketch the tangent field in the case $a = -2$. **[2]**

(B) Explain why the tangent field is not defined for the whole coordinate plane. **[1]**

(C) Give an inequality which describes the region in which the tangent field is defined. **[1]**

(D) Find a value of a such that the region for which the tangent field is defined includes the entire x-axis. **[1]**

(ii) (A) For the case $a = 1$, with $y = 1$ when $x = 0$, construct a spreadsheet for the Runge-Kutta method of order 2 with formulae as follows, where $f(x, y) = \frac{dy}{dx}$.

$$k_1 = hf(x_n, y_n)$$
$$k_2 = hf(x_n + h, y_n + k_1)$$
$$y_{n+1} = y_n + \frac{1}{2}(k_1 + k_2)$$

State the formulae you have used in your spreadsheet. **[3]**

(B) Use your spreadsheet to obtain the value of y correct to 4 decimal places when $x = 1$ for

- h = 0.1

and

- h = 0.05. **[2]**

(iii) (A) For the case $a = 0$ find the analytical solution that passes through the point $(0, 1)$. **[1]**

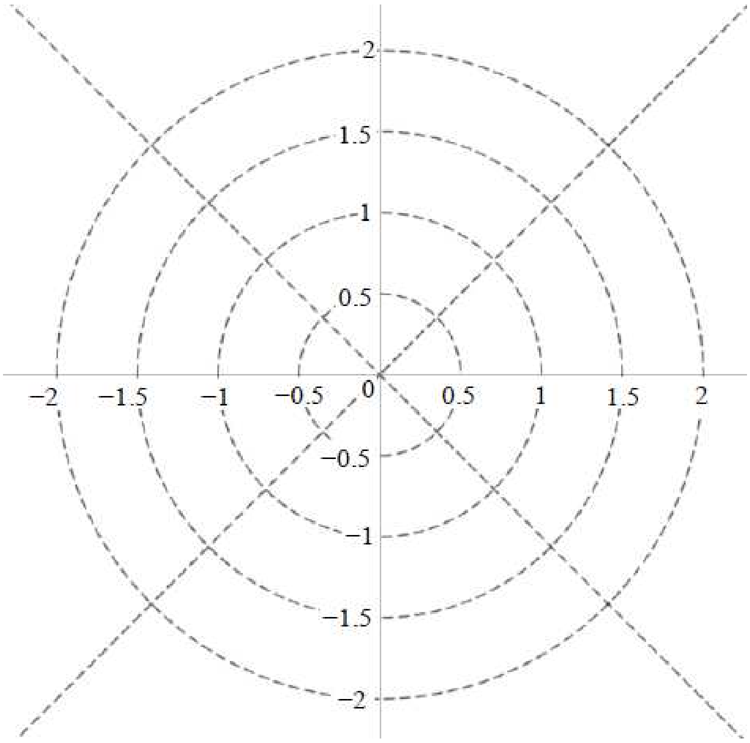
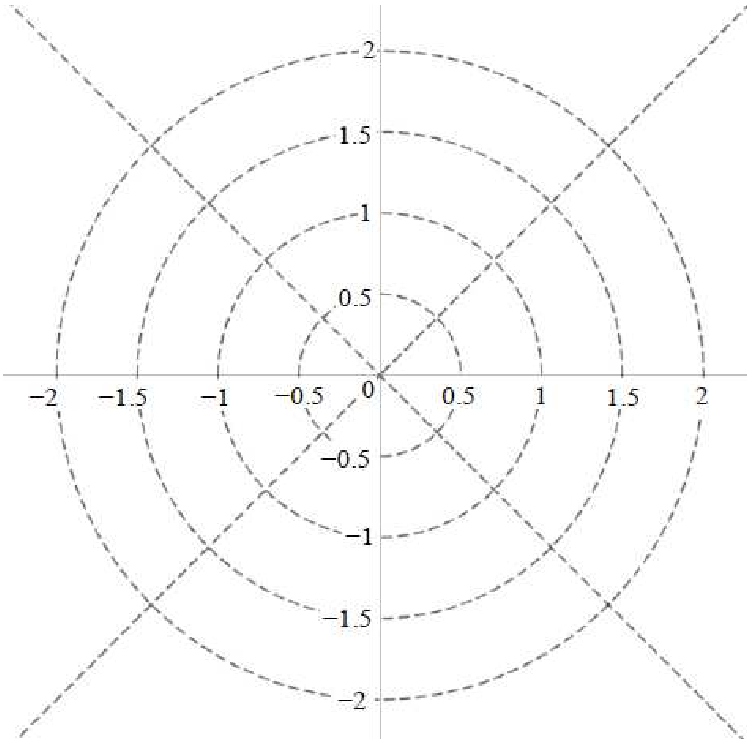
(B) Verify that the solution in part **(iii)** (A) is a solution to the differential equation. **[2]**

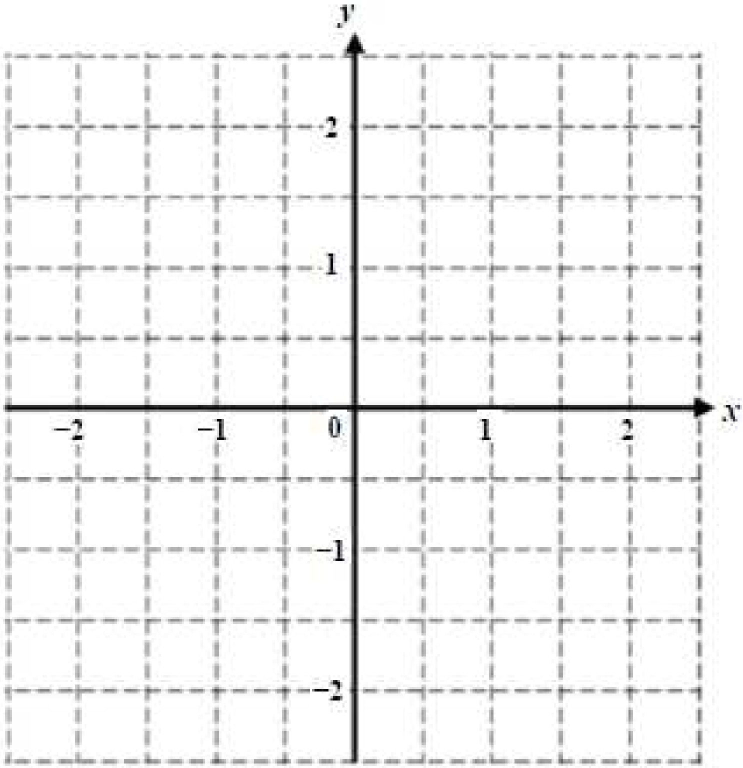
(C) Use the solution in part **(iii)** (A) to find the value of y correct to 4 decimal places when $x = 1$. **[1]**

(iv) (A) Verify that $y = -\frac{a}{2}x + \frac{a^2}{8} - \frac{1}{2}$ is a solution for all cases when $a \leq 0$. **[2]**

(B) Show that this is the only straight line solution in these cases. **[4]**

END OF QUESTION PAPER

1 (i) (A)	n = 2  n = 4 

3 (i) (A)	a = −2 
3 (i) (B)	
3 (i) (C)	
3 (i) (D)	