

General Certificate of Education
January 2007
Advanced Subsidiary Examination



MATHEMATICS
Unit Pure Core 2

MPC2

Wednesday 10 January 2007 1.30 pm to 3.00 pm

For this paper you must have:

- an 8-page answer book
 - the **blue** AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC2.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information

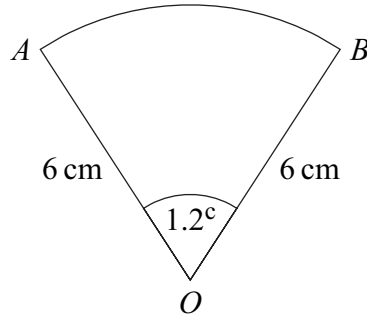
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

- 1 The diagram shows a sector OAB of a circle with centre O .



The radius of the circle is 6 cm and the angle AOB is 1.2 radians.

- (a) Find the area of the sector OAB . (2 marks)
- (b) Find the perimeter of the sector OAB . (3 marks)

- 2 Use the trapezium rule with four ordinates (three strips) to find an approximate value for

$$\int_0^3 \sqrt{2^x} \, dx$$

giving your answer to three decimal places. (4 marks)

- 3 (a) Write down the values of p , q and r given that:

(i) $64 = 8^p$;

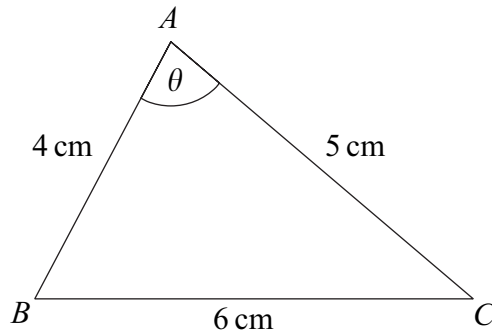
(ii) $\frac{1}{64} = 8^q$;

(iii) $\sqrt{8} = 8^r$. (3 marks)

- (b) Find the value of x for which

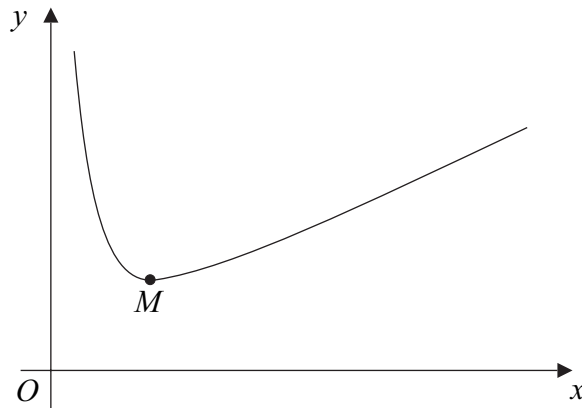
$$\frac{8^x}{\sqrt{8}} = \frac{1}{64} \quad (2 \text{ marks})$$

- 4 The triangle ABC , shown in the diagram, is such that $BC = 6$ cm, $AC = 5$ cm and $AB = 4$ cm. The angle BAC is θ .



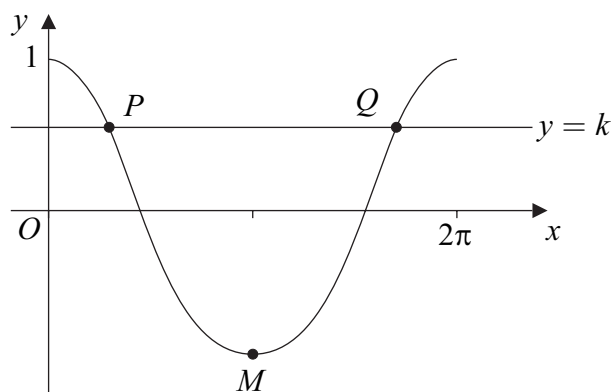
- (a) Use the cosine rule to show that $\cos \theta = \frac{1}{8}$. (3 marks)
- (b) Hence use a trigonometrical identity to show that $\sin \theta = \frac{3\sqrt{7}}{8}$. (3 marks)
- (c) Hence find the area of the triangle ABC . (2 marks)
- 5 The second term of a geometric series is 48 and the fourth term is 3.
- (a) Show that one possible value for the common ratio, r , of the series is $-\frac{1}{4}$ and state the other value. (4 marks)
- (b) In the case when $r = -\frac{1}{4}$, find:
- (i) the first term; (1 mark)
- (ii) the sum to infinity of the series. (2 marks)

- 6 A curve C is defined for $x > 0$ by the equation $y = x + 1 + \frac{4}{x^2}$ and is sketched below.



- (a) (i) Given that $y = x + 1 + \frac{4}{x^2}$, find $\frac{dy}{dx}$. (3 marks)
- (ii) The curve C has a minimum point M . Find the coordinates of M . (4 marks)
- (iii) Find an equation of the normal to C at the point $(1, 6)$. (4 marks)
- (b) (i) Find $\int \left(x + 1 + \frac{4}{x^2} \right) dx$. (3 marks)
- (ii) Hence find the area of the region bounded by the curve C , the lines $x = 1$ and $x = 4$ and the x -axis. (2 marks)
- 7 (a) The first four terms of the binomial expansion of $(1 + 2x)^8$ in ascending powers of x are $1 + ax + bx^2 + cx^3$. Find the values of the integers a , b and c . (4 marks)
- (b) Hence find the coefficient of x^3 in the expansion of $\left(1 + \frac{1}{2}x\right)(1 + 2x)^8$. (3 marks)

- 8 (a) Solve the equation $\cos x = 0.3$ in the interval $0 \leq x \leq 2\pi$, giving your answers in radians to three significant figures. (3 marks)
- (b) The diagram shows the graph of $y = \cos x$ for $0 \leq x \leq 2\pi$ and the line $y = k$.



The line $y = k$ intersects the curve $y = \cos x$, $0 \leq x \leq 2\pi$, at the points P and Q . The point M is the minimum point of the curve.

- (i) Write down the coordinates of the point M . (2 marks)
- (ii) The x -coordinate of P is α .
Write down the x -coordinate of Q in terms of π and α . (1 mark)
- (c) Describe the geometrical transformation that maps the graph of $y = \cos x$ onto the graph of $y = \cos 2x$. (2 marks)
- (d) Solve the equation $\cos 2x = \cos \frac{4\pi}{5}$ in the interval $0 \leq x \leq 2\pi$, giving the values of x in terms of π . (4 marks)

Turn over for the next question

Turn over ►

9 (a) Solve the equation $3 \log_a x = \log_a 8$. *(2 marks)*

(b) Show that

$$3 \log_a 6 - \log_a 8 = \log_a 27 \quad (3 \text{ marks})$$

(c) (i) The point $P(3, p)$ lies on the curve $y = 3 \log_{10} x - \log_{10} 8$.

$$\text{Show that } p = \log_{10} \left(\frac{27}{8} \right). \quad (2 \text{ marks})$$

(ii) The point $Q(6, q)$ also lies on the curve $y = 3 \log_{10} x - \log_{10} 8$.

$$\text{Show that the gradient of the line } PQ \text{ is } \log_{10} 2. \quad (4 \text{ marks})$$

END OF QUESTIONS

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