OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) **4762**

Mechanics 2

Monday 11 January 2010
Morning

Duration: 1 hour 30 minutes

4762

INSTRUCTIONS TO CANDIDATES

*   Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
*   Use black ink. Pencil may be used for graphs and diagrams only.
*   Read each question carefully and make sure that you know what you have to do before starting your answer.
*   Answer **all** the questions.
*   Do **not** write in the bar codes.
*   You are permitted to use a graphical calculator in this paper.
*   Final answers should be given to a degree of accuracy appropriate to the context.
*   The acceleration due to gravity is denoted by $g\,\text{m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

*   The number of marks is given in brackets **[ ]** at the end of each question or part question.
*   You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
*   The total number of marks for this paper is **72**.
*   This document consists of **8** pages. Any blank pages are indicated.

Turn over

1 **(a)** An object P, with mass $6\,\text{kg}$ and speed $1\,\text{m}\,\text{s}^{-1}$, is sliding on a smooth horizontal table. Object P explodes into two small parts, Q and R. Q has mass $4\,\text{kg}$ and R has mass $2\,\text{kg}$ and speed $4\,\text{m}\,\text{s}^{-1}$ in the direction of motion of P before the explosion. This information is shown in Fig. 1.1.

before · after

| 1 m s⁻¹ | | 4 m s⁻¹ |

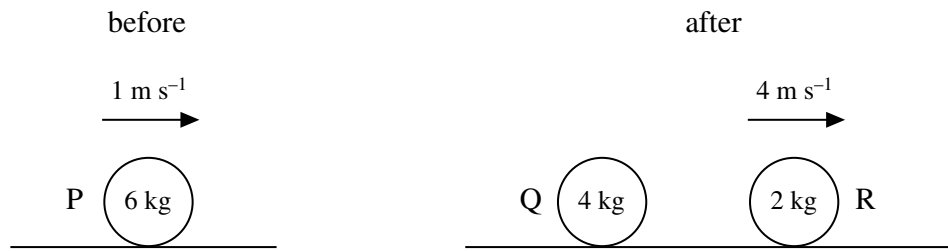P (6 kg)   Q (4 kg)   (2 kg) R

Fig. 1.1

(i) Calculate the velocity of Q. **[4]**

Just as object R reaches the edge of the table, it collides directly with a small object S of mass $3\,\text{kg}$ that is travelling horizontally towards R with a speed of $1\,\text{m}\,\text{s}^{-1}$. This information is shown in Fig. 1.2. The coefficient of restitution in this collision is 0.1.

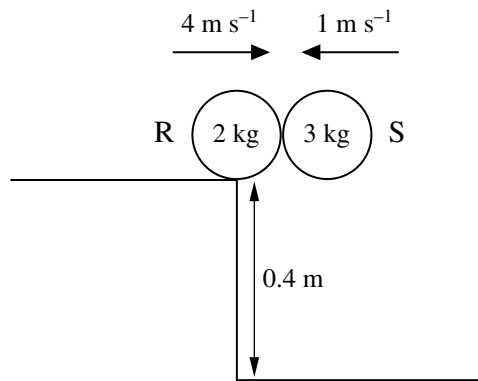4 m s⁻¹   1 m s⁻¹

R (2 kg)(3 kg) S

0.4 m

Fig. 1.2

(ii) Calculate the velocities of R and S immediately after the collision. **[6]**

The table is 0.4 m above a horizontal floor. After the collision, R and S have no contact with the table.

(iii) Calculate the distance apart of R and S when they reach the floor. **[3]**

(b) A particle of mass $m\,\text{kg}$ bounces off a smooth horizontal plane. The components of velocity of the particle just before the impact are $u\,\text{m}\,\text{s}^{-1}$ parallel to the plane and $v\,\text{m}\,\text{s}^{-1}$ perpendicular to the plane. The coefficient of restitution is e.

Show that the mechanical energy lost in the impact is $\frac{1}{2}mv^2(1-e^2)\,\text{J}$. **[4]**

2 A car of mass 1200 kg travels along a road for two minutes during which time it rises a vertical distance of 60 m and does 1.8×10^6 J of work against the resistance to its motion. The speeds of the car at the start and at the end of the two minutes are the same.

  (i) Calculate the average power developed over the two minutes. **[4]**

The car now travels along a straight level road at a steady speed of 18 m s^{-1} while developing constant power of 13.5 kW.

  (ii) Calculate the resistance to the motion of the car.

    How much work is done against the resistance when the car travels 200 m? **[5]**

While travelling at 18 m s^{-1}, the car starts to go **down** a slope inclined at $5°$ to the horizontal with the power removed and its brakes applied. The total resistance to its motion is now 1500 N.

 (iii) Use an energy method to determine how far down the slope the car travels before its speed is halved. **[6]**

Suppose the car is travelling along a straight level road and developing power P W while travelling at $v \text{ m s}^{-1}$ with acceleration $a \text{ m s}^{-2}$ against a resistance of R N.

 (iv) Show that $P = (R + 1200a)v$ and deduce that if P and R are constant then if a is not zero it cannot be constant. **[4]**

3 A side view of a free-standing kitchen cupboard on a horizontal floor is shown in Fig. 3.1. The cupboard consists of: a base CE; vertical ends BC and DE; an overhanging horizontal top AD. The dimensions, in metres, of the cupboard are shown in the figure. The cupboard and contents have a weight of 340 N and centre of mass at G.
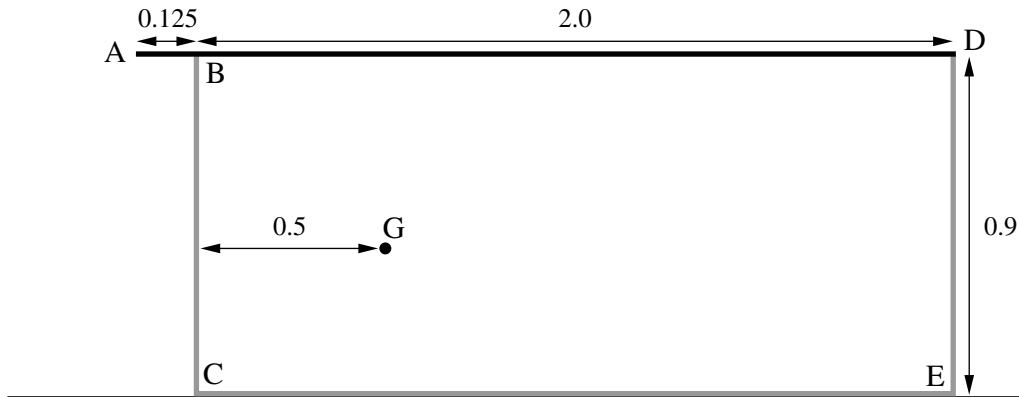

Fig. 3.1

(i) Calculate the magnitude of the vertical force required at A for the cupboard to be on the point of tipping in the cases where the force acts

(A) downwards, **[3]**

(B) upwards. **[3]**

A force of magnitude Q N is now applied at A at an angle of θ to AB, as shown in Fig. 3.2, where $\cos\theta = \frac{5}{13}$ (and $\sin\theta = \frac{12}{13}$).

Fig. 3.2

(ii) By considering the vertical and horizontal components of the force at A, show that the clockwise moment of this force about E is $\dfrac{30Q}{13}$ N m. **[3]**

With the force of magnitude Q N acting as shown in Fig. 3.2, the cupboard is in equilibrium and is on the point of tipping but not on the point of sliding.

(iii) Show that $Q = 221$ and that the coefficient of friction between the cupboard base and the floor must be greater than $\frac{5}{8}$. **[9]**

4 In this question, coordinates refer to the axes shown in the figures and the units are centimetres.

Fig. 4.1 shows a lamina KLMNOP shaded. The lamina is made from uniform material and has the dimensions shown.
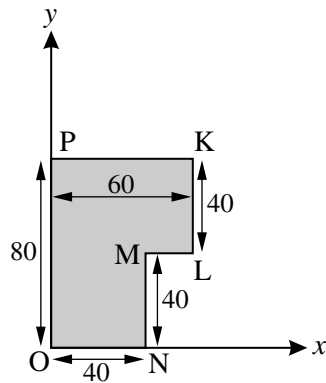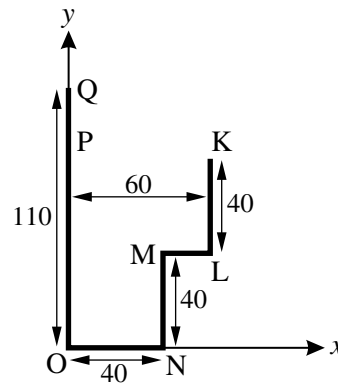

Fig. 4.1   **Fig. 4.2**

(i) Show that the *x*-coordinate of the centre of mass of this lamina is 26 and calculate the *y*-coordinate. **[4]**

A uniform thin heavy wire KLMNOPQ is bent into the shape of part of the perimeter of the lamina KLMNOP with an extension of the side OP to Q, as shown in Fig. 4.2.

(ii) Show that the *x*-coordinate of the centre of mass of this wire is 23.2 and calculate the *y*-coordinate. **[5]**

The wire is freely suspended from Q and hangs in equilibrium.

(iii) Draw a diagram indicating the position of the centre of mass of the hanging wire and calculate the angle of QO with the vertical. **[4]**

A wall-mounted bin with an open top is shown in Fig. 4.3. The centre part has cross-section KLMNOPQ; the two ends are in the shape of the lamina KLMNOP.

The ends are made from the same uniform, thin material and each has a mass of 1.5 kg. The centre part is made from different uniform, thin material and has a total mass of 7 kg.
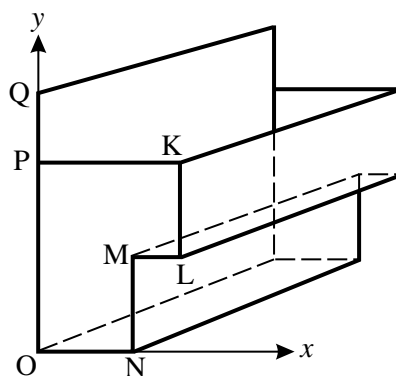

Fig. 4.3

(iv) Calculate the *x*- and *y*-coordinates of the centre of mass of the bin. **[5]**