

ADVANCED GCE UNIT **MATHEMATICS**

4724/01

Core Mathematics 4

Afternoon

Time: 1 hour 30 minutes

TUESDAY 23 JANUARY 2007

Additional Materials: Answer Booklet (8 pages) List of Formulae (MF1)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer all the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- The total number of marks for this paper is 72.

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- You are reminded of the need for clear presentation in your answers.

1 It is given that

$$f(x) = \frac{x^2 + 2x - 24}{x^2 - 4x} \quad \text{for } x \neq 0, \ x \neq 4.$$

Express f(x) in its simplest form. [3]

- 2 Find the exact value of $\int_{1}^{2} x \ln x \, dx$. [5]
- 3 The points A and B have position vectors **a** and **b** relative to an origin O, where $\mathbf{a} = 4\mathbf{i} + 3\mathbf{j} 2\mathbf{k}$ and $\mathbf{b} = -7\mathbf{i} + 5\mathbf{j} + 4\mathbf{k}$.
 - (i) Find the length of AB. [3]
 - (ii) Use a scalar product to find angle OAB. [3]
- 4 Use the substitution u = 2x 5 to show that $\int_{\frac{5}{2}}^{3} (4x 8)(2x 5)^7 dx = \frac{17}{72}$. [5]
- 5 (i) Expand $(1-3x)^{-\frac{1}{3}}$ in ascending powers of x, up to and including the term in x^3 . [4]
 - (ii) Hence find the coefficient of x^3 in the expansion of $(1 3(x + x^3))^{-\frac{1}{3}}$. [3]
- 6 (i) Express $\frac{2x+1}{(x-3)^2}$ in the form $\frac{A}{x-3} + \frac{B}{(x-3)^2}$, where A and B are constants. [3]
 - (ii) Hence find the exact value of $\int_{4}^{10} \frac{2x+1}{(x-3)^2} dx$, giving your answer in the form $a+b \ln c$, where a, b and c are integers. [4]
- 7 The equation of a curve is $2x^2 + xy + y^2 = 14$. Show that there are two stationary points on the curve and find their coordinates. [8]
- 8 The parametric equations of a curve are $x = 2t^2$, y = 4t. Two points on the curve are $P(2p^2, 4p)$ and $Q(2q^2, 4q)$.
 - (i) Show that the gradient of the normal to the curve at P is -p. [2]
 - (ii) Show that the gradient of the chord joining the points P and Q is $\frac{2}{p+q}$. [2]
 - (iii) The chord PQ is the normal to the curve at P. Show that $p^2 + pq + 2 = 0$. [2]
 - (iv) The normal at the point R(8, 8) meets the curve again at S. The normal at S meets the curve again at T. Find the coordinates of T.

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9 (i) Find the general solution of the differential equation

$$\frac{\sec^2 y}{\cos^2(2x)} \frac{\mathrm{d}y}{\mathrm{d}x} = 2.$$
 [7]

- (ii) For the particular solution in which $y = \frac{1}{4}\pi$ when x = 0, find the value of y when $x = \frac{1}{6}\pi$. [3]
- 10 The position vectors of the points P and Q with respect to an origin O are $5\mathbf{i} + 2\mathbf{j} 9\mathbf{k}$ and $4\mathbf{i} + 4\mathbf{j} 6\mathbf{k}$ respectively.
 - (i) Find a vector equation for the line *PQ*. [2]

The position vector of the point T is $\mathbf{i} + 2\mathbf{j} - \mathbf{k}$.

- (ii) Write down a vector equation for the line OT and show that OT is perpendicular to PQ. [4] It is given that OT intersects PQ.
- (iii) Find the position vector of the point of intersection of OT and PQ. [3]
- (iv) Hence find the perpendicular distance from O to PQ, giving your answer in an exact form. [2]

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