

# GCE

## **Mathematics B MEI**

### H640/02: Pure Mathematics and Statistics

A Level

## Mark Scheme for June 2023

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

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#### MARKING INSTRUCTIONS

#### PREPARATION FOR MARKING RM ASSESSOR

- 1. Make sure that you have accessed and completed the relevant training packages for on-screen marking: RM Assessor Online Training; OCR Essential Guide to Marking.
- 2. Make sure that you have read and understood the mark scheme and the question paper for this unit. These are posted on the RM Cambridge Assessment Support Portal <u>http://www.rm.com/support/ca</u>
- 3. Log-in to RM Assessor and mark the required number of practice responses ("scripts") and the number of required standardisation responses.

#### MARKING

- 1. Mark strictly to the mark scheme.
- 2. Marks awarded must relate directly to the marking criteria.
- 3. The schedule of dates is very important. It is essential that you meet the RM Assessor 50% and 100% (traditional 40% Batch 1 and 100% Batch 2) deadlines. If you experience problems, you must contact your Team Leader (Supervisor) without delay.

#### 4. Annotations

| Annotation   | Meaning                       |
|--------------|-------------------------------|
| √and ×       |                               |
| BOD          | Benefit of doubt              |
| FT           | Follow through                |
| ISW          | Ignore subsequent working     |
| M0, M1       | Method mark awarded 0, 1      |
| A0, A1       | Accuracy mark awarded 0, 1    |
| B0, B1       | Independent mark awarded 0, 1 |
| Е            | Explanation mark 1            |
| SC           | Special case                  |
| ^            | Omission sign                 |
| MR           | Misread                       |
| BP           | Blank Page                    |
| Seen         |                               |
| Highlighting |                               |

| Other abbreviations in mark scheme | Meaning   |
|------------------------------------|---|
| E1                                 | Mark for explaining a result or establishing a given result   |
| dep*                               | Mark dependent on a previous mark, indicated by *. The * may be omitted if only one previous M mark |
| сао                                | Correct answer only   |
| oe                                 | Or equivalent   |
| rot                                | Rounded or truncated  |
| soi                                | Seen or implied   |
| www                                | Without wrong working   |
| AG                                 | Answer given  |
| awrt                               | Anything which rounds to  |
| BC                                 | By Calculator   |
| DR                                 | This question included the instruction: In this question you must show detailed reasoning.          |
| BP                                 | Blank Page  |
| Seen                               |   |
| Highlighting                       |   |

#### **Mark Scheme**

#### 5. Subject Specific Marking Instructions

a. Annotations must be used during your marking. For a response awarded zero (or full) marks a single appropriate annotation (cross, tick, M0 or ^) is sufficient, but not required.

For responses that are not awarded either 0 or full marks, you must make it clear how you have arrived at the mark you have awarded and all responses must have enough annotation for a reviewer to decide if the mark awarded is correct without having to mark it independently.

It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

#### Award NR (No Response)

- if there is nothing written at all in the answer space and no attempt elsewhere in the script
- OR if there is a comment which does not in any way relate to the question (e.g. 'can't do', 'don't know')
- OR if there is a mark (e.g. a dash, a question mark, a picture) which isn't an attempt at the question.

Note: Award 0 marks only for an attempt that earns no credit (including copying out the question).

If a candidate uses the answer space for one question to answer another, for example using the space for 8(b) to answer 8(a), then give benefit of doubt unless it is ambiguous for which part it is intended.

b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not always be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

If you are in any doubt whatsoever you should contact your Team Leader.

c. The following types of marks are available.

#### Μ

A suitable method has been selected and applied in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using

#### **Mark Scheme**

some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified. A method mark may usually be implied by a correct answer unless the question includes the DR statement, the command words "Determine" or "Show that", or some other indication that the method must be given explicitly.

#### Α

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

#### В

Mark for a correct result or statement independent of Method marks.

#### Е

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

- d. When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep\*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e. The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

#### **Mark Scheme**

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f. Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.)

We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.

- When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value.
- When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. unless a different level of accuracy has been asked for in the question, or the mark scheme specifies an acceptable range. NB for Specification A the rubric specifies 3 s.f. as standard, so this statement reads "3 s.f".

Follow through should be used so that only one mark in any question is lost for each distinct accuracy error.

Candidates using a value of 9.80, 9.81 or 10 for g should usually be penalised for any final accuracy marks which do not agree to the value found with 9.8 which is given in the rubric.

- g. Rules for replaced work and multiple attempts:
  - If one attempt is clearly indicated as the one to mark, or only one is left uncrossed out, then mark that attempt and ignore the others.
  - If more than one attempt is left not crossed out, then mark the last attempt unless it only repeats part of the first attempt or is substantially less complete.
  - if a candidate crosses out all of their attempts, the assessor should attempt to mark the crossed out answer(s) as above and award marks appropriately.
- h. For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A or B mark in the question. Marks designated as cao may be awarded as long as there are no other errors.

If a candidate corrects the misread in a later part, do not continue to follow through. E marks are lost unless, by chance, the given results are established by equivalent working. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

### **Mark Scheme**

- i. If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers, provided that there is nothing in the wording of the question specifying that analytical methods are required such as the bold "In this question you must show detailed reasoning", or the command words "Show" or "Determine". Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.
- j. If in any case the scheme operates with considerable unfairness consult your Team Leader.

|   | Question   | Answer  | Marks      | AO           | Guidance  |
|---|------------|---|------------|--------------|---|
| 1 |            | $\frac{a}{1-r}$ used                                    | M1         | <b>1.1</b> a | a and r are numerical values; a = 9 and/or $r = \pm \frac{1}{3}$                                |
|   |            | $\frac{9}{1\pm\frac{1}{3}}$ soi                         | M1         | 1.1          |   |
|   |            | $\frac{27}{4}$ or $6\frac{3}{4}$ or 6.75 <b>cao isw</b> | A1         | 1.1          | if unsupported allow SC2 for correct answer   |
|   |            |   | [3]        |              |   |
| 2 | (a)        | $(x \pm 6)^2$ and $(y \pm 4)^2$                         | M1         | 1.1          | completing the square twice <b>soi</b>  |
|   |            | r = 7 not from wrong working                            | A1         | 1.1          | <b>NB</b> $(x - 6)^2 - 36 + (y + 4)^2 - 16 + 3 = 0$ oe<br>allow <b>B2</b> for r = 7 unsupported |
|   |            | Alternatively $\pm 2a = -12$ oe and $\pm 2b = 8$ oe     | M1         |              |   |
|   |            | r = 7 not from wrong working                            | A1         |              | <b>NB</b> $r^2 = 6^2 + 4^2 - 3$   |
|   |            |   | [2]        |              |   |
| 2 | <b>(b)</b> | (6, -4)   | <b>B</b> 1 | 1.1          | <b>FT</b> $(x \pm 6)^2 + (y \pm 4)^2$   |
|   |            |   |            |              | or <b>FT</b> $\pm 2a = -12$ oe and $\pm 2b = 8$ oe  |
|   |            |   | [1]        |              |   |

|   | Question |    | Answer  | Marks          | AO  | Guidance   |
|---|----------|----|---|----------------|-----|--|
| 3 |          |    | take reciprocal<br>calculate cube<br>calculate square root to obtain<br>$m = 343, n = 512$ isw or $\frac{343}{512}$ isw | B1<br>B1<br>B1 |     | operations may be in any order, but 3 <b>distinct numerical</b> steps<br>required for 3 marks<br>if taking <b>reciprocal</b> and <b>one</b> other step are combined into one<br>step, allow <b>B1B1B0</b><br>if <b>B0B0</b> for cubing and square rooting, allow <b>SC1</b> for<br>$\left(\sqrt{\frac{49}{64}}\right)^3 = \frac{343}{512}$ or $\left(\sqrt{\frac{64}{49}}\right)^3 = \frac{512}{343}$ seen |
|   |          |    |   | [3]            |     |  |
|   |          | eg | $\left(\frac{49}{64}\right)^{\frac{3}{2}}$  | B1             | 1.1 | taking reciprocal; may be awarded after simplification   |
|   |          |    | $\left(\frac{7}{8}\right)^3$  | B1             | 1.1 | square roots found; may be seen before taking reciprocal   |
|   |          |    | $\frac{343}{512}$ isw   | <b>B</b> 1     | 1.1 | dependent on award of both preceding marks   |
|   |          |    |   | [3]            |     |  |
|   |          | eg | Alternatively $\left(\frac{64}{49}\right)^{-3} = \left(\frac{m}{n}\right)^2$  |                |     |  |
|   |          |    | $\left(\frac{49}{64}\right)^3 = \left(\frac{m}{n}\right)^2$   | B1             |     |  |
|   |          |    | $117649 = m^2$ and $262144 = n^2$   | B1             |     |  |
|   |          |    | m= 343 and n = 512  | B1             |     | dependent on award of both preceding marks   |
|   |          |    |   |                |     |  |

|   | Question | Answer   | Marks | AO  | Guidance  |
|---|----------|--|-------|-----|---|
| 4 | (a)      | 7p + 3p = 1 oe   | M1    | 1.2 | must see some reasoning; allow explanation in words   |
|   |          | $p = 0.1 \text{ or } \frac{1}{10}$ isw cao   | A1    | 1.1 | if unsupported, allow SC1 for correct answer  |
|   |          |  | [2]   |     |   |
| 4 | (b)      | 0.7 or $\frac{7}{10}$  | B1FT  | 1.1 | their 7p  |
|   |          |  | [1]   |     |   |
| 4 | (c)      | B(30, their 0.3) seen or used  | M1    | 1.1 | <b>FT</b> their 3p; allow <b>M1</b> for $(their 0.3)^2 \times (their 0.7)^{28}$   |
|   |          | <b>awrt</b> 0.0018 <b>isw</b>  | A1    | 1.1 | not from wrong working; if unsupported, allow <b>SC1</b> for correct answer   |
|   |          |  | [2]   |     |   |
| 5 |          | $\left[ \begin{pmatrix} 5\\-3 \end{pmatrix} - \begin{pmatrix} 3\\-1 \end{pmatrix} = \right] \begin{pmatrix} 2\\-2 \end{pmatrix}$ | B1    | 2.1 | may be in coordinate form or may see distances identified or on diagram;  |
|   |          | or $\begin{bmatrix} 3 \\ -1 \end{bmatrix} - \begin{bmatrix} 5 \\ -3 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$       |       |     | may be implied by $\sqrt{(\pm 2)^2 + (\pm 2)^2}$ oe   |
|   |          | $\sqrt{(\pm 2)^2 + (\pm 2)^2}$ oe  | M1    | 1.1 | or <b>FT</b> their evaluation of $\begin{pmatrix} 5 \\ -3 \end{pmatrix} - \begin{pmatrix} 3 \\ -1 \end{pmatrix}$ may be implied by correct answer |
|   |          | $\sqrt{8}$ or $2\sqrt{2}$ isw  | A1    | 1.1 | if <b>B0M0</b> ; allow <b>SC1</b> for $\sqrt{80}$ or $4\sqrt{5}$ (from addition of vectors) if supported by Pythagoras;                           |
|   |          |  |       |     | if <b>B0M0</b> allow <b>SC1</b> for $\sqrt{8}$ or $2\sqrt{2}$ unsupported   |
|   |          |  | [3]   |     |   |

| Question | Answer  | Marks | AO           | Guidance   |
|----------|---|-------|--------------|--|
| 6        | $2\cos\theta = x + 3$ or $\cos\theta = \frac{x+3}{2}$   | B1    | 2.1          |  |
|          | $2\sin\theta = y - 1$ or $\sin\theta = \frac{y-1}{2}$   | B1    | 1.1          |  |
|          | $\left(\frac{x\pm 3}{2}\right)^2 + \left(\frac{y\pm 1}{2}\right)^2 = \cos^2\theta + \sin^2\theta \text{ or}$ $(x\pm 3)^2 + (y\pm 1)^2 = 4\cos^2\theta + 4\sin^2\theta$ oe | M1    | 1.1          | allow sign errors in their expressions for $\sin\theta$ and $\cos\theta$ ; allow if just see brackets expanded, but must be 3 terms in each case |
|          | $(x+3)^2 + (y-1)^2 = 4$   | A1    | 1.1          | allow SC2 for $(x + 3)^2 + (y - 1)^2 = 4$ unsupported  |
|          |   | [4]   |              |  |
|          | Alternatively<br>centre of circle is (-3, 1)  | B1    |              |  |
|          | radius is 2   | B1    |              |  |
|          | $(x+3)^2 + (y-1)^2 = 2^2$   | M1    |              | allow one sign error in bracket;   |
|          | $(x+3)^2 + (y-1)^2 = 4$   | A1    |              |  |
| 7        | $(2x)^8$ soi  | M1    | <b>3.1</b> a | allow recovery from bracket error; may be implied by award of second <b>M1</b>   |
|          | <sup>12</sup> C <sub>8</sub> or <sup>12</sup> C <sub>4</sub> or 495 seen  | B1    | 1.1          |  |
|          | 495×256×k <sup>4</sup> [ $x^8$ ]= 79200000 [ $x^8$ ] oe   | M1    | 1.1          | allow <b>M1</b> for $495 \times 2 \times k^4 = 79\ 200\ 000\ $ <b>oe</b> ;<br>" = 79\ 200\ 000" may be implied by k = 5                          |
|          | k = 5 cao isw   | A1    | 3.2a         | not from wrong working; but allow recovery from $x^8$ on one side of equation only   |
|          |   |       |              | allow <b>SC2</b> for $k = 5$ unsupported   |
|          |   | [4]   |              |  |

|   | Question | Answer   | Marks | AO   | Guidance   |
|---|----------|--|-------|------|--|
| 8 | (a)      | <b>not</b> a simple random sample since <b>each</b><br><b>possible sample</b> does <b>not</b> have an <b>equal</b><br><b>probability</b> of being selected   | B1    | 2.4  | allow " <b>not</b> a simple random sample since <b>each plant</b> does <b>not</b> have an <b>equal probability</b> of being selected" ignore further comments unless contradictory the essential elements of the comment are in bold   |
|   |          |  | [1]   |      |  |
| 8 | (b)      | 18 (shorter than 40 cm) seen   | B1    | 2.1  | may be embedded in calculation   |
|   |          | 110 (shorter than 80 cm) <b>or</b> 10 (taller than 80 cm) seen   | B1    | 1.1  | may be embedded in calculation   |
|   |          | $\frac{92}{120}$ oe or $\frac{28}{120}$ oe   | M1    | 1.1  | <b>FT</b> their 18 and their 110; allow <b>M1</b> for $\frac{91}{120}$ or $\frac{29}{120}$ oe  |
|   |          |  |       |      | or $\frac{75}{100} \times 120 = 90$ oe   |
|   |          | <b>awrt</b> 77% > 75% <b>oe</b> so this supports the (owner's) statement   | A1    | 2.2a | or $92 > 90$ so this supports the (owner's) statement; need comparison <b>and</b> comment  |
|   |          |  | [4]   |      |  |
| 8 | (c)      | <ul> <li>must refer to sample in answer</li> <li>eg since different samples give different results, not all samples would necessarily support owner's statement</li> <li>eg no since another sample may contain all the tallest plants (and/or shortest)</li> <li>eg no since the heights in other samples may be different</li> </ul> | B1    | 2.4  | <ul> <li>ignore superfluous comments referring to eg growing conditions, soil type etc</li> <li>do not allow</li> <li>eg no, conditions different for other plants</li> <li>eg no, different plants on different rows</li> <li>eg no, might be biggest and smallest plants on other rows</li> <li>eg the first sample is not representative</li> </ul> |
|   |          |  | [1]   |      |  |

|   | Question | Answer   | Marks | AO   | Guidance  |
|---|----------|--|-------|------|---|
| 9 | (a)      | eg sample, since only some data used <b>oe</b><br>eg sample, since data for 2009 not used <b>oe</b>  | B1    | 2.2a | sample, since not all data used; ignore further reasoning unless contradictory                            |
|   |          |  | [1]   |      |   |
| 9 | (b)      | £27085 or £27 100 or £27 000   | B1    | 3.4  |   |
|   |          |  | [1]   |      |   |
| 9 | (c)      | it's interpolation <b>oe</b><br>eg because we use 2009 <b>oe</b> which is<br>between 2008 and 2010   | B1    | 2.2a | do not allow<br>eg 27085 lies between 2008 and 2010<br>eg 27085 lies between the values for 2008 and 2010 |
|   |          | <ul> <li>eg a straight line model appears to be justified</li> <li>eg relationship seems to be linear</li> <li>eg median income seems to be directly proportional to time</li> <li>eg positive correlation [between median income and year]</li> </ul> | B1    | 2.2b | do not allow<br>eg positive association<br>eg positive relationship                                       |
|   |          |  | [2]   |      |   |

|    | Question     | Answer  | Marks     | AO           | Guidance  |
|----|--------------|---|-----------|--------------|---|
| 9  | ( <b>d</b> ) | not reasonable oe sinceeg (median taxable) income variesconsiderably across different regions ofLondoneg (median) income different in Croydonand Camdeneg (median) income lower in Croydon(than Camden)eg pattern of change in income over timedifferent in Camden to Croydon | B1        | 2.4          | LDS advantage   |
|    |              |   | [1]       |              |   |
| 10 |              | $4x \times \frac{1}{2}\sin 2x - \int \frac{1}{2}\sin 2x \times 4  \mathrm{d}x$  | M1*       | <b>3.1</b> a | allow sign errors only, condone omission of dx  |
|    |              | $2x\sin 2x - \int 2\sin 2x  \mathrm{d}x  \mathrm{oe}$   | A1        | 1.1          | allow omission of dx; may be unsimplified   |
|    |              | $F[x] = 2x\sin 2x + \cos 2x$  | A1        | 1.1          | ignore + c  |
|    |              | $F\left[\frac{\pi}{4}\right] - F[0]$  | M1dep*    | 1.1          | must see substitution if F[x] incorrect, otherwise may be implied by correct answer;  |
|    |              | $\frac{\pi}{2} - 1$ or $\frac{\pi - 2}{2}$ isw  | A1        | 3.2a         | allow recovery from bracket error   |
|    |              |   | [5]       |              |   |
| 11 | (a)          | $\frac{\mathrm{d}y}{\mathrm{d}x} = k\sqrt{x}$   | <b>B1</b> | 2.1          | may be implied by final answer  |
|    |              | $3 = k \times \sqrt{4}$   | M1        | 1.1          |   |
|    |              | $k = \frac{3}{2}$ or $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3}{2}\sqrt{x}$ isw  | A1        | 2.2a         | if <b>B0M0</b> allow <b>SC1</b> for $\frac{dy}{dx} = \frac{k}{\sqrt{x}}$ and $k = 6$ as final answer<br>or $\frac{dy}{dx} = \frac{6}{\sqrt{x}}$ as final answer |
|    |              |   | [3]       |              |   |

### Mark Scheme

|    | Question | Answer  | Marks | AO   | Guidance   |
|----|----------|---|-------|------|--|
| 11 | (b)      | their $k \times \frac{x^{\frac{3}{2}}}{\frac{3}{2}}$ oe                                   | B1    | 3.1a | <b>FT</b> their k  |
|    |          | $10 = \left(\sqrt{4}\right)^3 + c$  | M1    | 1.1  | <b>FT</b> their integration, one term in x with index 1.5  |
|    |          | $c = 2 \text{ or } y = x^{\frac{3}{2}} + 2 \text{ or } y = x\sqrt{x} + 2 \text{ isw}$     | A1    | 1.1  | must see 'y =' at some point   |
|    |          |   |       |      | if <b>B0M0</b> allow <b>SC2</b> for $y = 12x^{\frac{1}{2}} - 14$ or $y = 12\sqrt{x} - 14$                      |
|    |          |   |       |      | or $y = 12x^{\frac{1}{2}} + c$ and $c = -14$   |
|    |          |   |       |      |  |
|    |          |   | [3]   |      |  |
| 12 | (a)      | because it's neither a one-to-one nor a<br>many-to-one (mapping)                          | B1    | 2.4  | allow because it's one-to-many (mapping)<br>allow eg because each value of x is mapped to two values <b>oe</b> |
|    |          | do not allow<br>eg because it's neither a one-to-one nor a<br>many-to-one <b>function</b> |       |      | do not allow<br>eg because it's a one-to-many <b>function</b>  |
|    |          |   | [1]   |      |  |
| 12 | (b)      | $\frac{x^2+2}{x^2+2-3}$   | M1    | 1.1  |  |
|    |          | $\frac{x^2+2}{x^2-1}$ or $\frac{x^2+2}{(x-1)(x+1)}$                                       | A1    | 1.1  |  |
|    |          |   | [2]   |      |  |

|    | Question | Answer   | Marks      | AO   | Guidance   |
|----|----------|--|------------|------|--|
| 12 | (c)      | x  > 1<br>or x < -1 or x > 1<br>or x < -1, x > 1<br>or x < -1 $\cup$ x > 1 | B1         | 1.1  | do not allow<br>eg $x \le -1$ and $x > 1$<br>eg $-1 > x > 1$   |
|    |          |  | [1]        |      |  |
| 13 | (a)      | H <sub>0</sub> : $\mu = 0.14$<br>H <sub>1</sub> : $\mu < 0.14$             | B1         | 1.1  | allow any other symbol except $\bar{x}$ or $\bar{X}$ , as long as it is correctly defined;<br>allow hypotheses stated in words                                 |
|    |          | their $\mu$ is the <b>population mean mass</b> of this variety of apple    | B1         | 2.5  | <ul> <li>allow weight;</li> <li>correct definition of μ may be embedded in hypotheses written out as a sentence;</li> <li>do not allow x̄ or X̄</li> </ul>     |
|    |          |  | [2]        |      |  |
| 13 | (b)      | $[\bar{X}\sim]N\left(0.14,\frac{0.0199^2}{80}\right)$                      | B1         | 3.3  | Normal distribution with correct mean or variance<br>allow variance = <b>awrt</b> $4.95 \times 10^{-6}$ or <b>awrt</b> $0.00222^{2}$                           |
|    |          |  | B1         | 2.2a | all correct, but allow full credit if no symbol used;<br>allow symbol other than $\overline{X}$ if correctly defined as sample mean,<br>but do not allow $\mu$ |
|    |          |  | [2]        |      |  |
| 13 | (c)      | <b>awrt</b> 0.136 seen <b>BC</b>   | B1         | 1.1  |  |
|    |          | $\overline{X} < 0.136$ only or $\overline{X} \le 0.136$ only               | <b>B</b> 1 | 3.4  | FT other correctly defined symbol  |
|    |          |  | [2]        |      |  |

|    | Question | Answer   | Marks | AO   | Guidance   |
|----|----------|--|-------|------|--|
| 13 | (d)      | 0.1316 < 0.136 or 0.1316 is in the critical<br>region (must be correct critical region) oe<br>or p = awrt 0.00008 < 0.05 oe<br>NB 0.0000799<br>or z = awrt -3.78 < -1.645 oe | M1    | 3.4  | condone $p = awrt \ 0.00007 < 0.05$ oe NB $0.0000740$<br>or $z = awrt \ -3.79 < -1.645$ oe from use of $\overline{X} \sim N\left(0.14, \frac{0.0198^2}{80}\right)$ |
|    |          | reject H <sub>0</sub>  | A1    | 1.1  | allow accept H <sub>1</sub> or result is significant   |
|    |          | there is sufficient evidence at the 5% level<br>to <b>suggest</b> that the <b>mean</b> mass of the<br>apples is <b>less than</b> 0.14 kg                                     | A1    | 2.2b | allow weight; do not allow eg conclude / prove / indicate or other<br>assertive statement instead of suggest   |
|    |          |  | [3]   |      |  |
| 14 | (a)      | <ul><li>discard City of London (as part of the data not available)</li><li>or discard any regions where one or more pieces of data are missing oe</li></ul>                  | B1    | 2.4  | LDS advantage<br>do not allow if answer spoiled<br>eg because it's an anomaly,<br>eg because it's an outlier,  |
|    |          |  | [1]   |      |  |
| 14 | (b)      | scatter does not look linear oe  | B1    | 3.4  | ignore extra comments unless they contradict an otherwise correct answer   |
|    |          | pmcc not close to 1 <b>oe</b>  | B1    | 3.4  | ignore extra comments unless they contradict an otherwise correct<br>answer  |
|    |          |  | [2]   |      |  |

### Mark Scheme

|    | Question     | Answer   | Marks | AO  | Guidance  |
|----|--------------|--|-------|-----|---|
| 14 | (c)          | $27216 \pm 2 \times 4177.5 \text{ or } 61.0 \pm 2 \times 5.32$   | M1    | 1.1 | use of 2 standard deviation check for one of the 4 calculations soi |
|    |              | m < 18861 or m > 35 571  | A1    | 1.1 | allow $\leq$ and $\geq$   |
|    |              | percentage < 50.36 or percentage > 71.64   | A1    | 1.1 | allow $\leq$ and $\geq$   |
|    |              |  |       |     | if M1A0A0 allow M1 SCB1 for all 4 correct values seen               |
|    |              | Scatter diagram to show Percentage of Pupils Achieving<br>5 A*-C Grades against Median Income of Taxpayers                 | A1    | 1.1 |   |
|    |              |  | [4]   |     |   |
| 14 | ( <b>d</b> ) | between 0 and 0.3743 since<br>eg outliers gave a false impression of<br>linearity<br>eg scatter will be more like a circle | B1    | 2.4 | need to refer to the shape of the scatter <b>oe</b>                 |
|    |              |  | [1]   |     |   |

| Question | Answer   | Marks      | AO           | Guidance  |
|----------|--|------------|--------------|---|
| 15       | y = 1 then $x = 2$ only  | B1         | <b>3.1</b> a |   |
|          | $\frac{1}{y} \times \frac{\mathrm{d}y}{\mathrm{d}x}$   | <b>B</b> 1 | 2.1          | first term correct; allow y' for $\frac{dy}{dx}$  |
|          | $x^3 \times \frac{\mathrm{d}y}{\mathrm{d}x} + 3x^2y$   | M1         | 1.1          | Product Rule; allow one coefficient error <b>or</b> one index error   |
|          | $\frac{1}{y} \times \frac{\mathrm{d}y}{\mathrm{d}x} + x^3 \times \frac{\mathrm{d}y}{\mathrm{d}x} + 3x^2 y [= 0]$ | A1         | 1.1          |   |
|          | substitution of their x = 2 <b>and</b> y = 1 to<br>obtain numerical value for $\frac{dy}{dx}$                    | M1*        | 1.1          | $NB - \frac{4}{3}$<br>dependent on at least two of 3 terms correct on LHS following differentiation;<br>if expression for $\frac{dy}{dx}$ or evaluation of $\frac{dy}{dx}$ is incorrect, need to see substitution for award of M1 |
|          | $y - 1 = \left(their\frac{3}{4}\right)\left(x - their\ 2\right) \text{ oe}$                                      | M1dep*     | <b>3.1</b> a | <b>FT</b> negative reciprocal of their $-\frac{4}{3}$ and their 2<br>may see eg $1 = \frac{3}{4} \times 2 + c$  |
|          | 3x - 4y - 2 = 0 or $-3x + 4y + 2 = 0$ oe   | A1         | 3.2a         | must be in required form, but coefficients may be fractions   |
|          |  | [7]        |              |   |

|    | Question     | Answer  | Marks | AO           | Guidance  |
|----|--------------|---|-------|--------------|---|
| 16 | (a)          | 0.16×0.84×2   | M1    | 1.1          | condone omission of 2   |
|    |              | or B(2, 0.16) or B(2, 0.84) seen                        |       |              |   |
|    |              | or $1 - (0.84^2 + 0.16^2)$                              |       |              | allow recovery from bracket error   |
|    |              | $\frac{168}{625}$ or 0.2688 or 0.269 or 0.27 <b>cao</b> | A1    | 1.1          | mark the final answer<br>allow <b>SC1</b> for correct answer unsupported                          |
|    |              |   | [2]   |              |   |
| 16 | (b)          | 0.75 - 0.66 = 0.09                                      | M1    | 3.1a         | allow 0.09 embedded in correct place in Venn diagram or contingency table; allow <b>M1</b> for 9% |
|    |              | [0.16 - 0.09 = ] 0.07 isw                               | A1    | 1.1          | allow SC1 for correct answer unsupported  |
|    |              |   | [2]   |              |   |
| 16 | (c)          | $\frac{0.09}{0.75}$                                     | M1    | <b>3.1</b> a | M0 for 0.12 from wrong working  |
|    |              | 0.12  | A1    | 1.1          | allow SC1 for correct answer unsupported  |
|    |              |   | [2]   |              |   |
| 16 | ( <b>d</b> ) | 0.12 ≠ 0.16   | M1    | 2.1          |   |
|    |              | so not independent                                      | A1    | 2.2a         | if <b>M0</b> allow <b>SCB1</b> for $0.16 \times 0.75 \neq 0.09$ so <b>not</b> independent         |
|    |              |   | [2]   |              |   |

| Question | Answer   | Marks  | AO           | Guidance   |
|----------|--|--------|--------------|--|
| 17       | divide through by cos x to obtain  | B1     | 2.1          |  |
|          | $2\tan x + \sec^2 x = 4$   |        |              |  |
|          | $2\tan x + \tan^2 x + 1 = 4$   | M1*    | <b>3.1</b> a | use of Pythagoras to obtain equation in $\tan x$ only; allow 1 sign error  |
|          | $\tan^2 x + 2\tan x - 3[=0]$   | A1     | 1.1          |  |
|          | $\tan x = 1 \text{ or } - 3$   | M1*dep | 1.1          | 2 values obtained for tanx from their quadratic  |
|          | [x =] -1.24905 to -1.249 or -1.25 or -1.2  |        |              |  |
|          | [x =] 1.8925 to 1.893 or 1.89 or 1.9   | A1     | 3.2a         | any two correct  |
|          | $[x =] \frac{\pi}{4}$ or 0.785 to 0.7854 or 0.79   |        |              |  |
|          | $[x = ] -\frac{3\pi}{4} \text{ or } -2.3562 \text{ to } -2.356 \text{ or } -2.36$<br>or -2.4 | A1     | 2.2a         | all four correct and <b>no extra</b> values in range; ignore <b>correct</b> extra values outside range but <b>A0</b> if incorrect values outside range |
|          |  | [6]    |              |  |

| Question | Answer   | Marks  | AO | Guidance   |
|----------|--|--------|----|--|
|          | alternatively<br>multiply through by cos x to obtain                       |        |    |  |
|          | $2\sin x \cos x + 1 = 4\cos^2 x$   | B1     |    |  |
|          | $\sin 2x + 1 = 2\cos 2x + 2$   | M1*    |    | use of double angle formulae, allow 1 sign error   |
|          | $5\cos^2 2x + 4\cos 2x = 0$  | A1     |    | or $\sqrt{5}\cos(2x + 0.4636) = -1$  |
|          | <b>NB</b> square both sides:<br>$\sin^2 2x = 4\cos^2 2x + 4\cos 2x + 1$ oe |        |    | or $\sqrt{5}\sin(2x - 1.1071) = 1$   |
|          | $\cos 2x = 0 \text{ or } -0.8$   | M1dep* |    | $\cos(2x + 0.4636 \dots) = -\frac{1}{\sqrt{5}}$ or $\sin(2x - 1.1071 \dots) = \frac{1}{\sqrt{5}}$  |
|          | 2 values obtained for cos2x from their quadratic                           |        |    | ν5 ν5 ν5   |
|          | [x =] -1.24905 to -1.249 or -1.25 or -1.2                                  |        |    |  |
|          | [x =] 1.8925 to 1.893 or 1.89 or 1.9                                       | A1     |    | any two correct  |
|          | $[x =] \frac{\pi}{4}$ or 0.785 to 0.7854 or 0.79                           |        |    |  |
|          | $[x = ] -\frac{3\pi}{4}$ or -2.3562 to -2.356 or -2.36<br>or -2.4          | A1     |    | all four correct and <b>no extra</b> values in range; ignore <b>correct</b> extra values outside range but <b>A0</b> if incorrect values outside range |
|          |  |        |    |  |

|    | Question     | Answer   | Marks | AO           | Guidance  |
|----|--------------|--|-------|--------------|---|
| 18 | (a)          | 260 <b>cao</b>   | B1    | 1.1          |   |
|    |              |  | [1]   |              |   |
| 18 | ( <b>b</b> ) | 31 cao   | B1    | 1.1          | mark the final answer   |
|    |              |  | [1]   |              |   |
| 18 | (c)          | any 2 distinct reasons<br>eg (approximately) symmetrical (about<br>the mean) | B1    | 3.5a         | ignore extra comments unless they contradict an otherwise correct<br>answer |
|    |              | eg approximately bell-shaped / unimodal                                      | B1    | <b>3.5</b> a |   |
|    |              | eg data is continuous  |       |              |   |
|    |              |  | [2]   |              |   |
| 18 | ( <b>d</b> ) | [variance is] <b>awrt</b> 62.2   | M1    | 3.3          | <b>NB</b> 62.15567may be implied by sd = 7.88 or 7.89                       |
|    |              | or [sd is] 7.89 seen BC  |       |              | <b>NB</b> 0.263047from $\sqrt{62.2}$ or 0.263133 from 7.89                  |
|    |              |  |       |              | <b>NB</b> 0.262871 from 7.88, 0.262973from unrounded sd                     |
|    |              | 0.26287 – 0.263134 or 0.26   | A1    | 3.4          | allow <b>B2</b> for correct answer unsupported                              |
|    |              |  | [2]   | 1            |   |
| 18 | (e)          | B(28, p) used, where p is value calculated in (d)                            | M1    | <b>3.1</b> a |   |
|    |              | $0.888 \le p < 0.896$  | A1    | 1.1          | may be given to 2 sf; allow <b>B2</b> for correct answer unsupported        |
|    |              |  | [2]   |              |   |

|    | Question     |  | Answer                  | Marks     | AO           | Guidance                                |
|----|--------------|--|-------------------------|-----------|--------------|---|
| 18 | ( <b>f</b> ) |  | $7.8 + 0.18 \times 260$ | <b>M1</b> | <b>3.1</b> a |   |
|    |              |  | $0.18^2 \times 62.2$ oe | M1        | 3.5c         | <b>or</b> $0.18 \times \sqrt{62.2}$     |
|    |              |  | N(54.6, 2.0138 – 2.02)  | A1        | 1.1          | allow eg 1.42 <sup>2</sup> for variance |
|    |              |  |                         | [3]       |              |   |

#### APPENDIX

Exemplar responses for Q2(b)

| Response | Mark |
|----------|------|
|          |      |
|          |      |
|          |      |
|          |      |

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