OCR
RECOGNISING ACHIEVEMENT

GCE

Mathematics (MEI)

Advanced GCE

Unit **4753:** Methods for Advanced Mathematics

Mark Scheme for June 2012

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All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

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Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	

Other abbreviations in mark scheme	Meaning
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand

a. Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c. The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d. When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep *’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e. The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f. Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g. Rules for replaced work

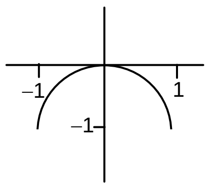
If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.
If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h. For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance	
1			$\int_1^2 \frac{1}{\sqrt{3x-2}}\,\mathrm{d}x = \left[\frac{2}{3}(3x-2)^{1/2}\right]_1^2$ $= \frac{2}{3}.2 - \frac{2}{3}.1$ = 2/3* OR $u = 3x - 2 \Rightarrow \mathrm{d}u/\mathrm{d}x = 3$ $\Rightarrow \int_1^2 \frac{1}{\sqrt{3x-2}}\,\mathrm{d}x = \int_1^4 \frac{1}{\sqrt{u}}.\frac{1}{3}\,\mathrm{d}u$ $= \left[\frac{2}{3}u^{1/2}\right]_1^4 = \frac{2}{3}.2 - \frac{2}{3}.1$ = 2/3*	M1 A2 M1dep A1 M1 A1 A1 M1dep A1 **[5]**	$[k(3x-2)^{1/2}]$ $k = 2/3$ substituting limits dep 1st M1 NB **AG** $\int \frac{1}{\sqrt{u}}$ × 1/3 (du) $\left[\frac{2}{3}u^{1/2}\right]$ o.e. substituting correct limits dep 1st M1 NB **AG**	or $w^2 = 3x - 2 \Rightarrow \int \frac{1}{w}$ × 2/3 w (dw) $\left[\frac{2}{3}w\right]$ upper – lower, 1 to 4 for u or 1 to 2 for w or substituting back (correctly) for x and using 1 to 2
2			$\lvert 2x + 1 \rvert > 4$ $\Rightarrow$ $2x + 1 > 4$, $x > 3/2$ or $2x + 1 < -4$, $x < -2\frac{1}{2}$	B1 M1 A1 **[3]**	$x > 3/2$ mark final ans; if from $\lvert 2x \rvert > 3$ B0 o.e. , e.g. $-(2x + 1) > 4$ (or $2x + 1 = -4$) if $\lvert 2x + 1 \rvert < -4$, M0 $x < -2\frac{1}{2}$ mark final ans allow 3 for correct unsupported answers	by squaring: $4x^2 + 4x - 15 >$ (or =) 0 M1 $x > 3/2$ A1 $x < -2\frac{1}{2}$ A1 Penalise ≥ and ≤ once only $3/2 < x < -2\frac{1}{2}$ SC2 (mark final ans)
3			$\mathrm{e}^{2y} = 5 - \mathrm{e}^{-x}$ $\Rightarrow 2\mathrm{e}^{2y}\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{e}^{-x}$ $\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{e}^{-x}}{2\mathrm{e}^{2y}}$ At (0, ln2) $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{e}^{0}}{2\mathrm{e}^{2\ln 2}}$ $= \frac{1}{8}$	B1 B1 M1dep A1cao **[4]**	$2\mathrm{e}^{2y}\frac{\mathrm{d}y}{\mathrm{d}x} = ...$ $= \mathrm{e}^{-x}$ substituting $x = 0$, $y = \ln 2$ into their dy/dx dep 1st B1 – allow one slip	or $y = \ln\sqrt{(5 - \mathrm{e}^{-x})}$ o.e (e.g. $\frac{1}{2}\ln(5 - \mathrm{e}^{-x})$)B1 $\Rightarrow \mathrm{d}y/\mathrm{d}x = \mathrm{e}^{-x}/[2(5 - \mathrm{e}^{-x})]$ o.e. B1 (but must be correct) or substituting $x = 0$ into their correct dy/dx

Question			Answer	Marks	Guidance	
4	(i)		$1 - 9a^2 = 0$ $\Rightarrow a^2 = 1/9 \Rightarrow a = 1/3$	M1 A1 **[2]**	or $1 - 9x^2 = 0$ or 0.33 or better $\sqrt{(1/9)}$ is A0	$\sqrt{(1 - 9a^2)} = 1 - 3a$ is M0 not $a = \pm 1/3$ nor $x = 1/3$
4	(ii)		Range $0 \le y \le 1$	B1 **[1]**	or $0 \le f(x) \le 1$ or $0 \le f \le 1$, not $0 \le x \le 1$ $0 \le y \le \sqrt{1}$ is B0	allow also [0,1], or 0 to 1 inclusive, but not 0 to 1 or (0,1)
4	(iii)			M1 M1 A1 **[3]**	curve goes from $x = -3a$ to $x = 3a$ (or −1 to 1) vertex at origin curve, 'centre' (0,−1), from (−1, −1) to (1, −1) (y-coords of −1 can be inferred from vertex at O and correct scaling)	must have evidence of using s.f. 3 allow also if s.f.3 is stated and stretch is reasonably to scale allow from $(-3a, -1)$ to $(3a, -1)$ provided $a = 1/3$ or $x = [\pm]\ 1/3$ in (i) A0 for badly inconsistent scale(s)
5	(i)		When $t = 0$, $P = 7 - 2 = 5$, so 5 (million) In the long term $e^{-kt} \to 0$ So long-term population is 7 (million)	B1 M1 A1 **[3]**	allow substituting a large number for t (for both marks)	allow 7 unsupported
5	(ii)		$P = 7 - 2e^{-kt}$ When $t = 1$, $P = 5.5$ $\Rightarrow \quad 5.5 = 7 - 2e^{-k}$ $\Rightarrow \quad e^{-k} = (7 - 5.5)/2 = 0.75$ $\Rightarrow \quad -k = \ln((7 - 5.5)/2)$ $\Rightarrow \quad k = 0.288$ (3 s.f.)	 M1 M1 A1 **[3]**	re-arranging and anti-logging – allow 1 slip (e.g. arith of 7 – 5.5, or k for $-k$) or $\ln 2 - k = \ln 1.5$ o.e. 0.3 or better allow $\ln(4/3)$ or $-\ln(3/4)$ if final ans	but penalise negative lns, e.g. $\ln(-1.5) = \ln(-2) - k$ rounding from a correct value of $k = 0.2876820725\ldots$, penalise truncation, and incorrect work with negatives

Question			Answer	Marks	Guidance	
6	(i)		$y = 2 \arcsin \frac{1}{2} = 2 \times \pi/6$ $= \pi/3$	M1 A1 **[2]**	$y = 2 \arcsin \frac{1}{2}$ must be in terms of π – can isw approximate answers	1.047… implies M1
6	(ii)		$y = 2 \arcsin x$ $\quad x \leftrightarrow y$ $\Rightarrow x = 2 \arcsin y$ $\Rightarrow x/2 = \arcsin y$ $\Rightarrow y = \sin(x/2)$ [so $g(x) = \sin(x/2)$] $\Rightarrow dy/dx = \frac{1}{2}\cos(\frac{1}{2}x)$ At Q, $x = \pi/3$ $\Rightarrow dy/dx = \frac{1}{2}\cos \pi/6 = \frac{1}{2}\sqrt{3}/2 = \sqrt{3}/4$ $\Rightarrow$ gradient at P $= 4/\sqrt{3}$	M1 A1 A1cao M1 A1 B1 ft **[6]**	or $y/2 = \arcsin x$ but must interchange x and y at some stage substituting their $\pi/3$ into their derivative must be exact, with their $\cos(\pi/6)$ evaluated o.e. e.g. $4\sqrt{3}/3$ but must be exact ft their $\sqrt{3}/4$ unless 1	or $f'(x) = 2/\sqrt{(1-x^2)}$ $f'(\frac{1}{2}) = 2/\sqrt{\frac{3}{4}} = 4/\sqrt{3}$ cao
7	(i)		$s(-x) = f(-x) + g(-x)$ $= -f(x) + -g(x)$ $= -(f(x) + g(x))$ $= -s(x)$ (so s is odd)	M1 A1 **[2]**	must have $s(-x) = \ldots$	
7	(ii)		$p(-x) = f(-x)g(-x)$ $=(-f(x)) \times (-g(x))$ $= f(x)g(x) = p(x)$ so p is even	M1 A1 **[2]**	must have $p(-x) = \ldots$ Allow SC1 for showing that $p(-x) = p(x)$ using two specific odd functions, but in this case they must still show that p is even	e.g. $f(x) = x$, $g(x) = x^3$, $p(x) = x^4$ $p(-x) = (-x)^4 = x^4 = p(x)$, so p even condone f and g being the same function

Question			Answer	Marks	Guidance	
8	(i)		$\frac{dy}{dx} = \sin 2x + 2x\cos 2x$ dy/dx = 0 when sin 2x + 2x cos 2x = 0 $\Rightarrow \frac{\sin 2x + 2x\cos 2x}{\cos 2x} = 0$ $\Rightarrow$ tan 2x + 2x = 0 *	M1 A1 M1 A1 **[4]**	d/dx(sin 2x) = 2cos 2x soi cao, mark final answer equating their derivative to zero, **provided it has two terms** must show evidence of division by cos 2x	can be inferred from dy/dx = 2x cos 2x e.g. dy/dx = tan 2x + 2x is A0
8	(ii)		At P, x sin 2x = 0 $\Rightarrow$ sin 2x = 0, 2x = (0), $\pi \Rightarrow x = \pi/2$ At P, dy/dx = sin π + 2(π/2) cos π = $-\pi$ Eqn of tangent: $y - 0 = -\pi(x - \pi/2)$ $\Rightarrow y = -\pi x + \pi^2/2$ $\Rightarrow 2\pi x + 2y = \pi^2$ * When x = 0, $y = \pi^2/2$, so Q is (0, $\pi^2/2$)	M1 A1 B1 ft M1 A1 M1A1 **[7]**	$x = \pi/2$ ft their $\pi/2$ and their derivative substituting 0, their $\pi/2$ and their $-\pi$ into $y - y_1 = m(x - x_1)$ **NB AG** can isw inexact answers from $\pi^2/2$	Finding $x = \pi/2$ using the given line equation is M0 or their $-\pi$ into $y = mx+c$, and then evaluating c: $y = (-\pi)x + c$, $0 = (-\pi)(\pi/2) + c$ M1 $\Rightarrow c = \pi^2/2$ $\Rightarrow y = -\pi x + \pi^2/2 \Rightarrow 2\pi x + 2y = \pi^2$ *A1
8	(iii)		Area = triangle OPQ – area under curve Triangle OPQ = ½ × π/2 × π^2/2 [= π^3/8] Parts: $u = x$, dv/dx = sin 2x du/dx = 1, v = –½ cos2x $\int_0^{\pi/2} x\sin 2x\,dx = \left[-\frac{1}{2}x\cos 2x\right]_0^{\pi/2} - \int_0^{\pi/2} -\frac{1}{2}\cos 2x\,dx$ $= \left[-\frac{1}{2}x\cos 2x + \frac{1}{4}\sin 2x\right]_0^{\pi/2}$ $= -\frac{1}{4}\pi\cos\pi + \frac{1}{4}\sin\pi - (-0\cos 0 + \frac{1}{4}\sin 0) = \frac{1}{4}\pi[\ -0]$ So shaded area = $\pi^3/8 - \pi/4 = \pi(\pi^2 - 2)/8$*	M1 B1cao M1 A1ft A1 A1cao A1 **[7]**	soi (or area under PQ – area under curve allow art 3.9 condone $v = k$ cos2x soi ft their v = –½ cos2x, ignore limits [– ½ x cos 2x + ¼ sin 2x] o.e., must be correct at this stage, ignore limits (so dep previous A1) **NB AG** must be from fully correct work	area under line may be expressed in integral form or using integral: $\int_0^{\pi/2}(\frac{1}{2}\pi^2 - \pi x)\,dx = \left[\frac{1}{2}\pi^2 x - \frac{1}{2}\pi x^2\right]_0^{\pi/2} = \frac{\pi^3}{4} - \frac{\pi^3}{8} [= \frac{\pi^3}{8}]$ v can be inferred from their ‘uv’

Question			Answer	Marks	Guidance	
9	(i)		(*A*) (0, 6) and (1, 4) (*B*) (−1, 5) and (0, 4)	B1B1 B1B1 **[4]**	Condone P and Q incorrectly labelled (or unlabelled)	
9	(ii)		$f'(x)=\frac{(x+1).2x-(x^2+3).1}{(x+1)^2}$ $f'(x) = 0 \Rightarrow 2x(x+1) - (x^2+3) = 0$ $\Rightarrow x^2 + 2x - 3 = 0$ $\Rightarrow (x-1)(x+3) = 0$ $\Rightarrow x = 1$ or $x = -3$ When $x = -3$, $y = 12/(-2) = -6$ so other TP is (−3, −6)	M1 A1 M1 A1dep B1B1cao **[6]**	Quotient or product rule consistent with their derivatives, condone missing brackets correct expression their derivative = 0 obtaining correct quadratic equation (soi) dep 1st M1 but withhold if denominator also set to zero must be from correct work (but see note re quadratic)	PR: $(x^2+3)(-1)(x+1)^{-2} + 2x(x+1)^{-1}$ If formula stated correctly, allow one substitution error. condone missing brackets if subsequent working implies they are intended Some candidates get $x^2 + 2x + 3$, then realise this should be $x^2 + 2x - 3$, and correct back, but not for every occurrence. Treat this sympathetically. Must be supported, but −3 could be verified by substitution into correct derivative
9	(iii)		$f(x-1) = \frac{(x-1)^2+3}{x-1+1}$ $=\frac{x^2-2x+1+3}{x-1+1}$ $=\frac{x^2-2x+4}{x} = x-2+\frac{4}{x}$ *	M1 A1 A1 **[3]**	substituting $x - 1$ for both *x's* in f NB **AG**	allow 1 slip for M1
9	(iv)		$\int_a^b (x-2+\frac{4}{x})\,dx = \left[\frac{1}{2}x^2-2x+4\ln x\right]_a^b$ $=(\frac{1}{2}b^2-2b+4\ln b)-(\frac{1}{2}a^2-2a+4\ln a)$ Area is $\int_0^1 f(x)\,dx$ So taking $a = 1$ and $b = 2$ area = (2 − 4 + 4ln 2) − (½ − 2 + 4ln 1) = 4 ln 2 − ½	B1 M1 A1 M1 A1 cao **[5]**	$\left[\frac{1}{2}x^2-2x+4\ln x\right]$ F(*b*) – F(*a*) condone missing brackets oe (mark final answer) must be simplified with ln 1 = 0	F must show evidence of integration of at least one term or $f(x) = x + 1 - 2 + 4/(x+1)$ $A = \int_0^1 f(x)\,dx = \left[\frac{1}{2}x^2-x+4\ln(1+x)\right]_0^1$ M1 = ½ − 1 + 4ln 2 = 4ln2 − ½ A1