Wednesday 18 June 2014 – Afternoon

A2 GCE MATHEMATICS (MEI)

4754/01A Applications of Advanced Mathematics (C4) Paper A

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4754/01A
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.
- This paper will be followed by **Paper B: Comprehension**.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

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Section A (36 marks)

1 Express $\dfrac{3x}{(2-x)(4+x^2)}$ in partial fractions. **[5]**

2 Find the first three terms in the binomial expansion of $(4+x)^{\frac{3}{2}}$. State the set of values of x for which the expansion is valid. **[5]**

3 Fig. 3 shows the curve $y = x^3 + \sqrt{(\sin x)}$ for $0 \leqslant x \leqslant \dfrac{\pi}{4}$.

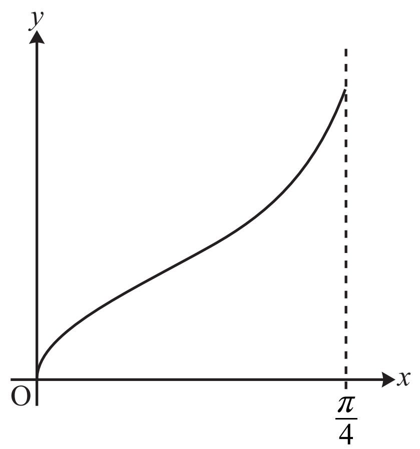

Fig. 3

(i) Use the trapezium rule with 4 strips to estimate the area of the region bounded by the curve, the x-axis and the line $x = \dfrac{\pi}{4}$, giving your answer to 3 decimal places. **[4]**

(ii) Suppose the number of strips in the trapezium rule is increased. Without doing further calculations, state, with a reason, whether the area estimate increases, decreases, or it is not possible to say. **[1]**

4 **(i)** Show that $\cos(\alpha+\beta) = \dfrac{1-\tan\alpha\tan\beta}{\sec\alpha\sec\beta}$. **[3]**

(ii) Hence show that $\cos 2\alpha = \dfrac{1-\tan^2\alpha}{1+\tan^2\alpha}$. **[2]**

(iii) Hence or otherwise solve the equation $\dfrac{1-\tan^2\theta}{1+\tan^2\theta} = \dfrac{1}{2}$ for $0° \leqslant \theta \leqslant 180°$. **[3]**

5 A curve has parametric equations $x = \mathrm{e}^{3t}, y = t\mathrm{e}^{2t}$.

(i) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t. Hence find the exact gradient of the curve at the point with parameter $t = 1$. **[4]**

(ii) Find the cartesian equation of the curve in the form $y = ax^b \ln x$, where a and b are constants to be determined. **[3]**

6 Fig. 6 shows the region enclosed by the curve $y = (1 + 2x^2)^{\frac{1}{3}}$ and the line $y = 2$.

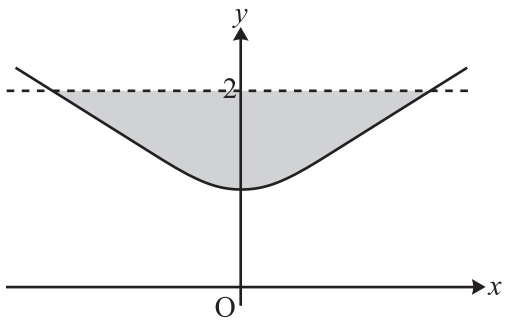

Fig. 6

This region is rotated about the y-axis. Find the volume of revolution formed, giving your answer as a multiple of π. **[6]**

Question 7 begins on page 4.

Section B (36 marks)

7 Fig. 7 shows a tetrahedron ABCD. The coordinates of the vertices, with respect to axes Oxyz, are A(–3, 0, 0), B(2, 0, –2), C(0, 4, 0) and D(0, 4, 5).

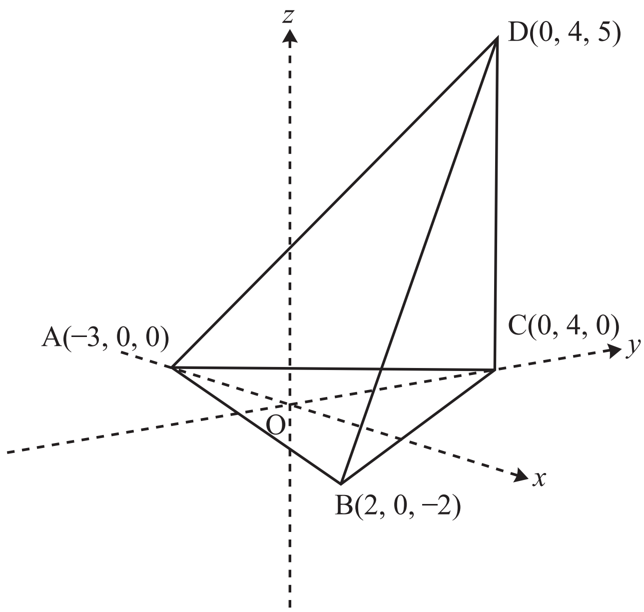

Fig. 7

(i) Find the lengths of the edges AB and AC, and the size of the angle CAB. Hence calculate the area of triangle ABC. **[7]**

(ii) (A) Verify that $4\mathbf{i} - 3\mathbf{j} + 10\mathbf{k}$ is normal to the plane ABC. **[2]**

(B) Hence find the equation of this plane. **[2]**

(iii) Write down a vector equation for the line through D perpendicular to the plane ABC. Hence find the point of intersection of this line with the plane ABC. **[5]**

The volume of a tetrahedron is $\frac{1}{3}$ × area of base × height.

(iv) Find the volume of the tetrahedron ABCD. **[2]**

8 Fig. 8.1 shows an upright cylindrical barrel containing water. The water is leaking out of a hole in the side of the barrel.

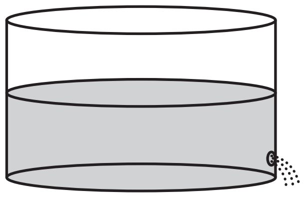

Fig. 8.1

The height of the water surface above the hole t seconds after opening the hole is h metres, where

$$\frac{dh}{dt} = -A\sqrt{h}$$

and where A is a positive constant. Initially the water surface is 1 metre above the hole.

(i) Verify that the solution to this differential equation is

$$h = \left(1 - \tfrac{1}{2}At\right)^2.$$ **[3]**

The water stops leaking when $h = 0$. This occurs after 20 seconds.

(ii) Find the value of A, and the time when the height of the water surface above the hole is 0.5 m. **[4]**

Fig. 8.2 shows a similar situation with a different barrel; h is in metres.

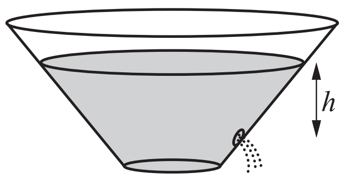

Fig. 8.2

For this barrel,

$$\frac{dh}{dt} = -B\frac{\sqrt{h}}{(1+h)^2},$$

where B is a positive constant. When $t = 0$, $h = 1$.

(iii) Solve this differential equation, and hence show that

$$h^{\frac{1}{2}}(30 + 20h + 6h^2) = 56 - 15Bt.$$ **[7]**

(iv) Given that $h = 0$ when $t = 20$, find B.

Find also the time when the height of the water surface above the hole is 0.5 m. **[4]**

END OF QUESTION PAPER

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Wednesday 18 June 2014 – Afternoon

A2 GCE MATHEMATICS (MEI)

4754/01B Applications of Advanced Mathematics (C4) Paper B: Comprehension

QUESTION PAPER

Candidates answer on the Question Paper.

OCR supplied materials:
- Insert (inserted)
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator
- Rough paper

Duration: Up to 1 hour

3092753766

475401B

Candidate forename		Candidate surname	

Centre number						Candidate number				

INSTRUCTIONS TO CANDIDATES

- The Insert will be found inside this document.
- Write your name, centre number and candidate number in the boxes above. Please write clearly and in capital letters.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Write your answer to each question in the space provided. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- The insert contains the text for use with the questions.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You may find it helpful to make notes and do some calculations as you read the passage.
- You are **not** required to hand in these notes with your question paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **18**.
- This document consists of **8** pages. Any blank pages are indicated.

1 The example illustrated in Table 1 in the article referred to the members of a racing club.

This question is about a similar club and the equivalent table is given below. The club has 150 members and each of them belongs to one of the groups P, Q, R or S.

For each group the probability of an accident and the average cost is given in the table below.

Group	**P**	**Q**	**R**	**S**
Proportion of people	10%	20%	40%	30%
Probability of an accident	0.1	0.15	0.05	0.2
Average cost per accident	£5000	£2000	£1000	£500

(i) Complete the table below. **[3]**

(ii) Calculate the premium if each member of the club pays the same amount and the total of the premiums is equal to the total average cost of accidents per year. **[2]**

(iii) The club management decides that the total of the premiums should be 50% more than the total average cost of accidents per year, and that each group should pay an amount proportional to its contribution to the total average cost.

Calculate the premium for a member of group S. **[2]**

1 (i)

Group	**P**	**Q**	**R**	**S**
Number of people	15			
Average number of accidents in a year		4.5		
Average cost of accidents per year	£7500	£9000		

1 (ii)	
1 (iii)	

PLEASE DO NOT WRITE IN THIS SPACE

2 In lines 97 to 100, the article says

"Most insurance companies have a maximum no-claims discount of 65%. One way of interpreting this practice is that the figure arrived at by applying the maximum no-claims discount is actually the basic cost of the insurance, and that drivers who have not earned the so-called discount are actually paying a surcharge."

A new driver without any no-claims discount pays k times "the basic cost of the insurance".

Find the value of k. **[2]**

2	

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3 On June 1st 2007 Louise paid her first car insurance premium. She did not have any no-claims discount.

She retained her policy with the same insurance company. She had no accidents and so did not make any claim on her insurance. Her no-claims discount followed the pattern in Table 7. Apart from her no-claims discount, her basic premium remained the same.

On May 31st 2014, Louise calculated that so far her no-claims discount had saved her £3875.

What premium did Louise pay on June 1st 2007? **[3]**

3

4 In lines 68 to 69, the article says

"Clearly the role of inexperience goes down with age. A possible mathematical model is that it decays exponentially."

This question investigates this in the case of male drivers. The relevant part of Table 4 is reproduced here.

Age (in years)	**18.0**	**20.5**	**23.0**	**27.5**	**45.0**
Male %	36	15	6	2.5	1

A model is proposed in which

$$y = a + be^{-k(x-17)},$$

where

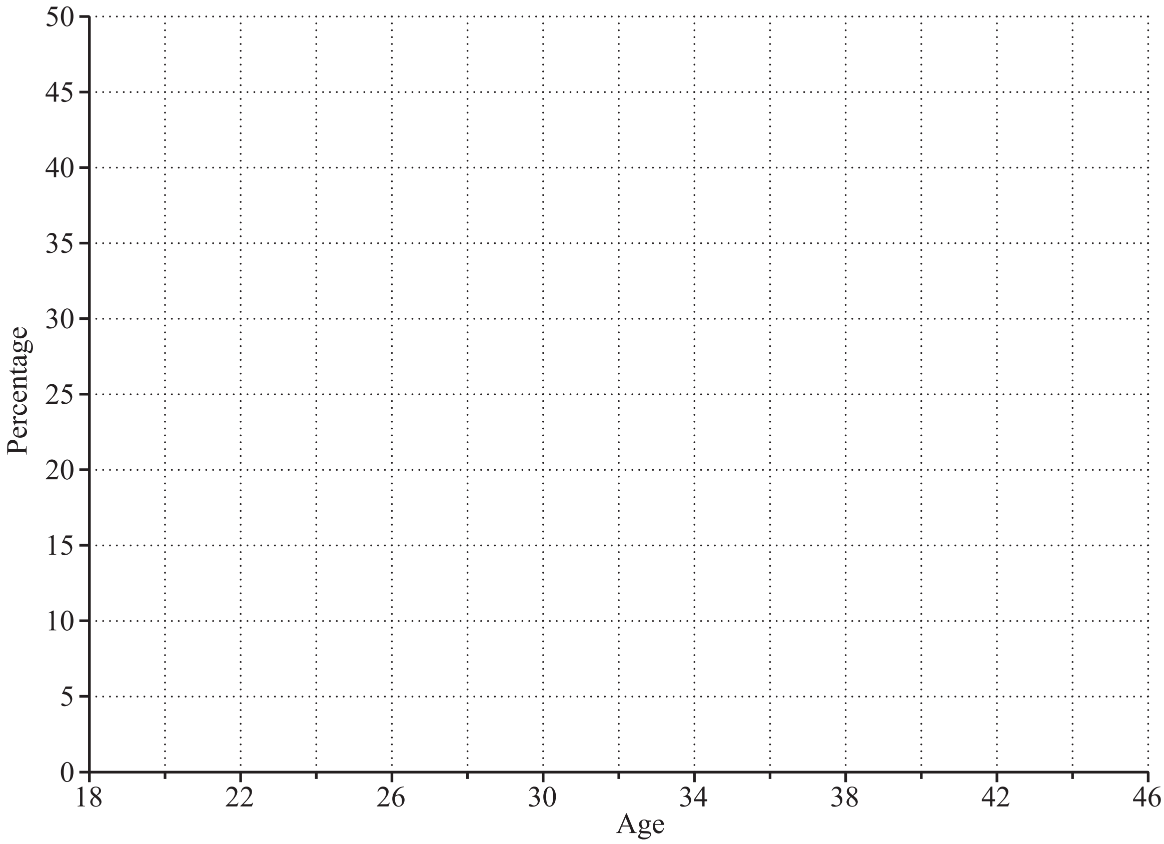
- x is the age of the driver,
- y is the percentage of accidents in which inexperience played a role,
- a, b and k are constants to be determined.

(i) Draw a sketch graph of y against x for values of x between 18 and 45. **[1]**

(ii) Explain why the graph justifies putting $a = 1$. **[1]**

(iii) Taking $a = 1$ and using the first two data points, it is estimated that $b = 50.5$ and $k = 0.3665$. Investigate whether these values are consistent with the point (23, 6). **[2]**

4 (i)



4 (ii)

4 (iii)

5 In this question the number of points on a driver's licence for motoring offences is denoted by n.

When calculating a driver's premium, a particular insurance company takes account of such offences by multiplying the premium by an amount M, where

$$M = 1 \quad \text{if } n \leqslant 3,$$

$$M = 2^{\frac{n}{6}} \quad \text{if } 3 < n < 12.$$

A driver who is insured with this company is paying a premium of £520. Initially he has no points on his licence.

He is convicted of a speeding offence and so receives 3 points on his licence. He is then convicted for dangerous driving and receives a further 6 points on his licence.

Calculate his premium

(*A*) after his first offence,

(*B*) after his second offence. **[2]**

5 (*A*)	
5 (*B*)	

Wednesday 18 June 2014 – Afternoon

A2 GCE MATHEMATICS (MEI)

4754/01B Applications of Advanced Mathematics (C4) Paper B: Comprehension

INSERT

Duration: Up to 1 hour

INFORMATION FOR CANDIDATES

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Insurance for young drivers

Introduction

Most people would regard being able to drive as an essential skill for adults in today's world. Passing the driving test is an important event for many young people; some see it as a passport into adult life.

The test is not particularly easy but passing it is often only the start of your worries. For example, you then have to pay for motor insurance and this can be very expensive for young inexperienced drivers. This article looks at why insurance costs are so high, and what you can do about it.

Insurance: a simple model

Insurance is about sharing the cost associated with risk.

Imagine, for example, that 100 people belong to a racing club; they have regular races using a particular type of vehicle. They provide their own vehicles. On average, 10 people have an accident each year and the cost of repairs per accident is £5000. So, in any year there is a probability of $\frac{1}{10}$ that a particular person will have an accident. The members of the club agree to share the risk by each paying an annual amount of $\frac{1}{10} \times £5000 = £500$ into a fund to provide enough money to compensate those who have accidents. This simplified example illustrates the idea of sharing risk which underpins insurance; the £500 payment is called the *premium*.

Insurance is virtually always run by companies and they add an amount to any premium to cover their own costs, and to allow them a profit margin. They also need to ensure that they have sufficient money to cover a bad run of accidents. These factors are not taken into account in this example.

Refining the model

In the example above, everyone was assigned an equal probability of an accident of $\frac{1}{10}$ and paid the same premium. In practice, insurance companies try to identify high-risk groups and charge them higher premiums, and low-risk groups who can pay lower premiums. They also vary premiums according to the likely cost of an accident. Thus expensive and fast cars tend to attract higher premiums.

Imagine that, in the example of the racing club, three distinct groups of people, denoted by A, B and C, can be identified. Table 1 gives the proportion of people in each of these groups, the probabilities of their having an accident in any year and the average cost of an accident.

Group	**A**	**B**	**C**
Proportion of people	10%	30%	60%
Probability of an accident	0.4	0.1	0.05
Average cost per accident	£9500	£3000	£1000

Table 1 Probabilities (per year) and average costs of accidents for different groups

So among the 100 people in the club, 10 are in Group A, 30 in Group B and the rest in Group C.

The average number of accidents per year is 4 for those in Group A, 3 for those in Group B and 3 for those in Group C.

The average cost of these accidents is £38 000 for Group A, £9000 for Group B and £3000 for Group C, making £50 000 in total. So, although only 10% of the people are in Group A, on average they account for 76% of the total accident costs.

In fact, the average cost per year of the accidents per person in Group A is £3800; in Group B it is £300 and in Group C it is £50.

Clearly it would not be fair for those in the three groups to pay the same premiums; those in Group A should pay very much more.

Insurance for car drivers

Although the example of the racing club, with just three groups, is very much simpler than a real-life situation, it does highlight the two basic elements involved in calculations of insurance premiums: the probability and the cost of an accident.

In the example, the premium was determined by sharing the estimated cost of accidents among a specific group of people. In real motor insurance, it is not possible to define such a closed group, and so premiums are calculated purely on the basis of estimates of the probabilities and costs of accidents.

In motor insurance, important considerations when estimating the probability of an accident include the driver's age, gender, experience and past record. Those for the cost of an accident include the power of the car (and so its speed) and its value.

The driver's age and gender

The age of the driver is an important factor in determining the risk. Table 2, which is based on data from the Association of British Insurers relating to serious accidents, compares the accident rates of drivers, by age group and by gender. The group with the lowest rate, female drivers aged 60 to 69, has been assigned a value of 1, and all the others are compared to that. Thus a male driver in the 17 to 20 age group is 13.4 times as likely to have a serious accident as a female in the 60 to 69 age group.

In the rest of this article it is assumed that this general pattern can be applied to all accidents that give rise to insurance claims.

Age group	**17–20**	**21–29**	**30–39**	**40–59**	**60–69**	**70+**
Male	13.4	4.8	2.9	1.9	1.2	1.4
Female	11.3	4.6	2.8	1.9	1.0	1.2

Table 2 Relative accident rates for car drivers, classified by age group and by gender (Source ABI)

Table 2 shows that young drivers are much more likely to be involved in accidents than older drivers; figures like these make it inevitable that insurance premiums for young drivers are high. Table 2 also shows that the accident rate for male drivers is higher than that for females for most age groups.

Some people start learning to drive on the road on their 17th birthday, the first possible opportunity, whereas others wait until they are quite a lot older. So while years of driving experience and age are related, the relationship is not a simple one. Table 3 shows the age and gender distribution of those holding full UK driving licences; figures are given in thousands, to the nearest 1000; it does not include those with provisional licences, many of whom are in the 17 to 20 age group.

Age group	**17–20**	**21–29**	**30–39**	**40–59**	**60–69**	**70+**
Male	538	2444	3430	8277	3467	2333
Female	482	2200	3037	7109	2726	1499

Table 3 Distribution by age and gender of UK licence holders (× 1000) (Source DVLA)

Table 3 shows that in all age groups male drivers outnumber female drivers.

The most striking feature of the figures in Table 2 is the difference in accident rate according to age. An obvious cause is inexperience of driving. Table 4 gives estimates from the insurance industry of the percentage of accidents to which inexperience contributes, for drivers of certain ages.

Age (in years)	**18.0**	**20.5**	**23.0**	**27.5**	**45.0**
Male %	36	15	6	2.5	1
Female %	39	18.5	7.5	3.5	1.5

Table 4 Estimated role of inexperience in accidents (Source ABI)

Clearly the role of inexperience goes down with age. A possible mathematical model is that it decays exponentially.

Other data show that young drivers are more likely to have accidents at night and this is particularly so for males. One possible explanation for this is lack of experience of driving in the dark; if so, the driving test could be changed so as to include some night-time driving but that might not be easy to implement. Another explanation is that the drivers have been at parties; the worst times for accidents are Friday and Saturday nights.

While inexperience is obviously an important consideration, it clearly is not the only cause. Fig. 5 provides another part of the explanation. It shows the estimated percentages of accidents in which excessive speed is a contributory factor. There are large differences between male and female drivers for all the age groups concerned, particularly among the youngest drivers.

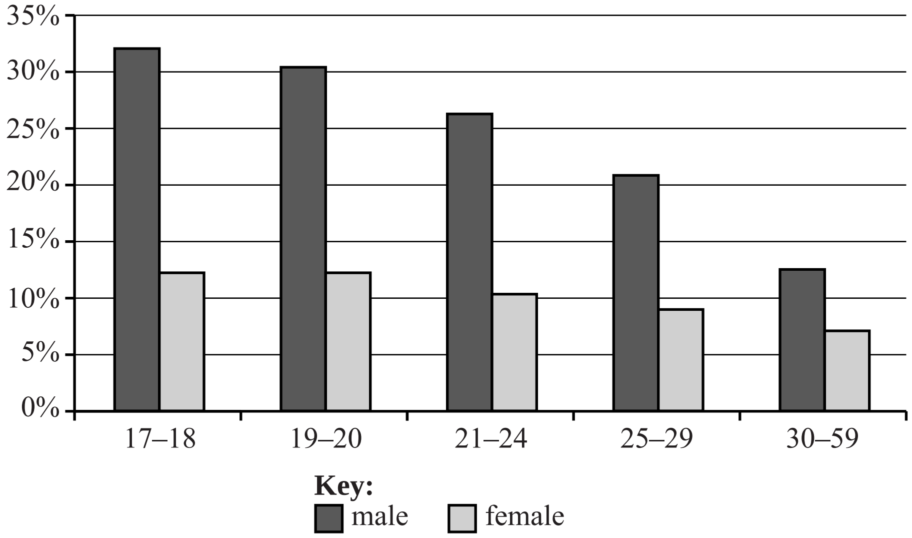

Fig. 5 Percentage of accidents with excessive speed as a contributory factor (Source DfT, STATS19)

The faster you are driving at the time of an accident, the more serious the accident is likely to be and so the greater the claim on the insurance company. Fig. 6 shows the average costs of claims for different groups of drivers. They are highest for young drivers; for almost all age groups they are higher for males than females.

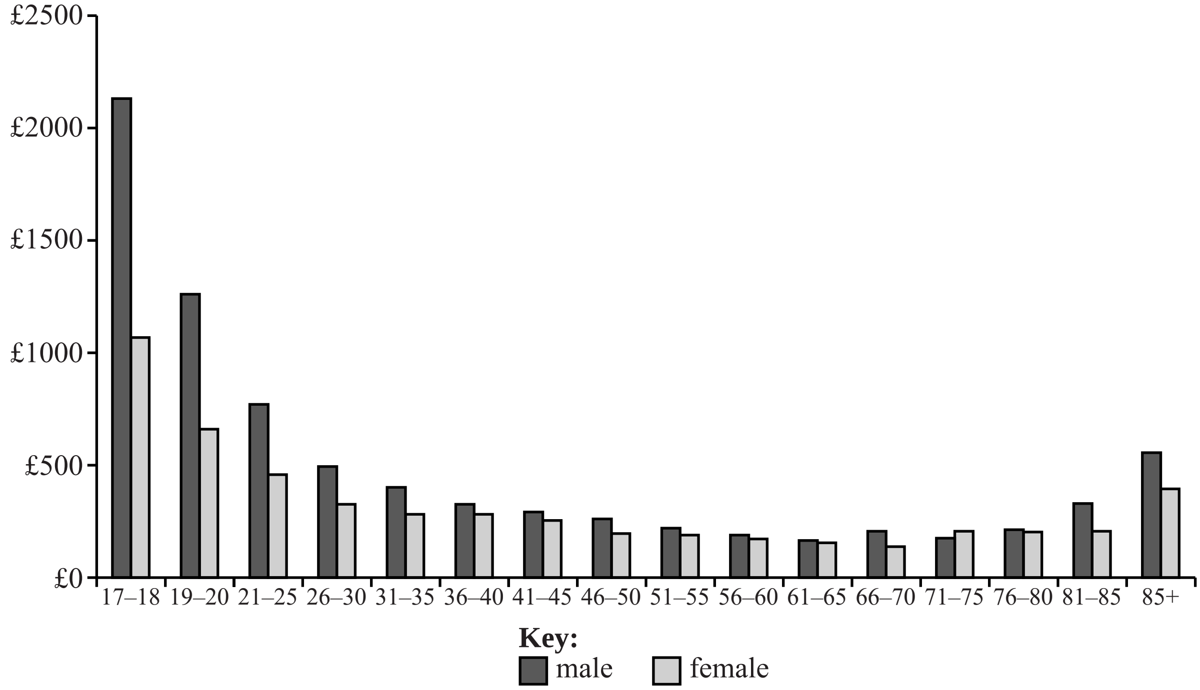

Fig. 6 Average costs of claims for different groups (Data source DfT, 2009)

Reducing insurance costs

The two major considerations when an insurance premium is being calculated are the probability of an accident and the average cost if it happens. Both of these are highest for young drivers and higher for males than females.

So what can young drivers do to reduce their insurance premiums?

Probability of an accident

When you are first insured, a company has no information about how careful a driver you will be, and so it is assumed that you will be as likely to have an accident as anyone else in your situation. It is up to you to prove otherwise. If you avoid accidents, the insurance company will usually reduce your future premiums.

Nearly all insurance companies reduce the premium of a driver who does not make any claims. This reduction is called a *no-claims discount*. Reductions vary from one insurance company to another but typical values are shown in Table 7. A good no-claims discount saves a lot of money. A no-claims discount has to be earned by claim-free driving and so is never given to a new driver.

Years of claim-free driving	% discount from the full premium
1	30%
2	40%
3	50%
4	60%
5+	65%

Table 7 Typical no-claims discounts

Most insurance companies have a maximum no-claims discount of 65%. One way of interpreting this practice is that the figure arrived at by applying the maximum no-claims discount is actually the basic cost of the insurance, and that drivers who have not earned the so-called discount are actually paying a surcharge.

If you are a very careful young driver it may seem unfair that you have to pay the same insurance premium as others who are reckless; you are paying extra for their bad driving. One way to prove that you really are a good driver is to make use of *telematics technology*. A small device is fitted to your car and transmits information to your insurance company, typically covering five aspects of your driving: cornering, swerving, braking, speed and acceleration. Feedback is provided in the form of advice on how to improve your driving and so reduce your insurance premium.

There is no discount for not breaking the law but if you are convicted for an offence like speeding or using your mobile phone while driving, an insurance company will see this as evidence of bad driving and so will probably increase your premium. Convictions result in points on your licence on a scale of 0 (a clean licence) to 12 (the level at which you are normally disqualified from driving).

Another risk factor is the number of passengers in the car. The figures in Table 8 show that the percentage of serious accidents increases very greatly with the number of passengers drivers are carrying in their cars. Among the possible explanations are that drivers are distracted or that they try to show off.

Number of passengers	0	1	2	3+
% of serious accidents	14	20	26	40

Table 8 Serious accidents classified according to the number of passengers in the car

The data in this article show that young female drivers have a lower probability of accidents than young male drivers; so you would expect lower premiums for females. Until recently that was the case, but a ruling of the European Court of Justice has now made this illegal.

Cost of an accident

The cost of an accident is obviously likely to increase with the value of the car. Not only will an expensive car be more expensive to repair, but it is likely to be more powerful and so able to go faster.

Insurance companies place cars in bands between 1 and 50 with the most expensive in the groups with the highest numbers. The lower the band of car you have, the cheaper the premium. Table 9 lists the insurance groups for a few new cars at the time of writing this article.

Car	Insurance group
Skoda Citigo 5 1.0	1
Peugeot 107 Access 1.0	3
Fiat 500 1.2 Pop	5
Nissan NOTE 1.4 Acenta	10
Volvo S40 D2 ES	20
Renault Clio Sport 200	30
Chrysler 300C Executive	40
Jaguar XKR 5.0 Coupé	50

Table 9 Insurance groups for some new cars

Calculating insurance premiums

Each insurance company has its own way of calculating a driver's premium. That is why the premium varies from one company to another. A company has to strike a balance between two different demands.

- The premium must be low enough to be attractive to customers.
- It must be high enough to ensure the company does not make a long-term loss.

In addition insurance companies offer benefits, such as a reduced premium in exchange for agreement to pay the first £200 (say) of any claim, so the premium is not the only consideration when choosing between companies.