

ADVANCED GCE

MATHEMATICS (MEI)

Methods for Advanced Mathematics (C3)

4753/01

Candidates answer on the Answer Booklet

OCR Supplied Materials:

- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:

None

Friday 5 June 2009
Afternoon

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

Section A (36 marks)

1 Evaluate $\int_0^{\frac{1}{6}\pi} \sin 3x \, dx$. [3]

2 A radioactive substance decays exponentially, so that its mass M grams can be modelled by the equation $M = Ae^{-kt}$, where t is the time in years, and A and k are positive constants.

(i) An initial mass of 100 grams of the substance decays to 50 grams in 1500 years. Find A and k . [5]

(ii) The substance becomes safe when 99% of its initial mass has decayed. Find how long it will take before the substance becomes safe. [3]

3 Sketch the curve $y = 2 \arccos x$ for $-1 \leq x \leq 1$. [3]

4 Fig. 4 shows a sketch of the graph of $y = 2|x - 1|$. It meets the x - and y -axes at $(a, 0)$ and $(0, b)$ respectively.

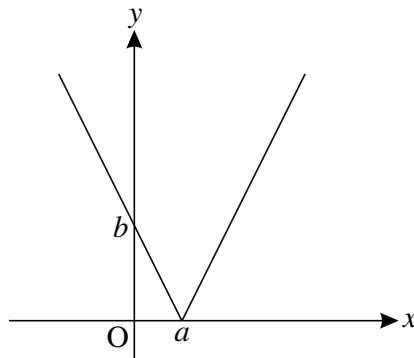


Fig. 4

Find the values of a and b . [3]

5 The equation of a curve is given by $e^{2y} = 1 + \sin x$.

(i) By differentiating implicitly, find $\frac{dy}{dx}$ in terms of x and y . [3]

(ii) Find an expression for y in terms of x , and differentiate it to verify the result in part (i). [4]

6 Given that $f(x) = \frac{x+1}{x-1}$, show that $ff(x) = x$.

Hence write down the inverse function $f^{-1}(x)$. What can you deduce about the symmetry of the curve $y = f(x)$? [5]

7 (i) Show that

$$(A) (x - y)(x^2 + xy + y^2) = x^3 - y^3,$$

$$(B) \left(x + \frac{1}{2}y\right)^2 + \frac{3}{4}y^2 = x^2 + xy + y^2. \quad [4]$$

(ii) Hence prove that, for all real numbers x and y , if $x > y$ then $x^3 > y^3$. [3]

Section B (36 marks)

8 Fig. 8 shows the line $y = x$ and parts of the curves $y = f(x)$ and $y = g(x)$, where

$$f(x) = e^{x-1}, \quad g(x) = 1 + \ln x.$$

The curves intersect the axes at the points A and B, as shown. The curves and the line $y = x$ meet at the point C.

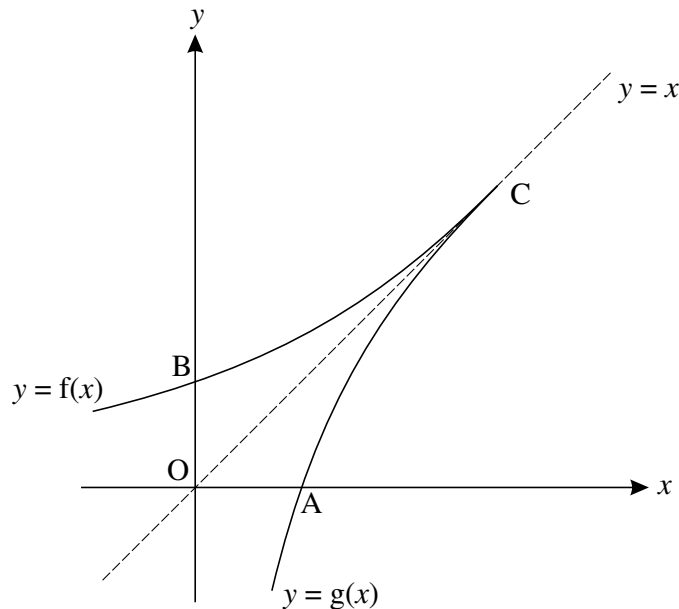


Fig. 8

(i) Find the exact coordinates of A and B. Verify that the coordinates of C are (1, 1). [5]

(ii) Prove algebraically that $g(x)$ is the inverse of $f(x)$. [2]

(iii) Evaluate $\int_0^1 f(x) dx$, giving your answer in terms of e . [3]

(iv) Use integration by parts to find $\int \ln x dx$.

Hence show that $\int_{e^{-1}}^1 g(x) dx = \frac{1}{e}$. [6]

(v) Find the area of the region enclosed by the lines OA and OB, and the arcs AC and BC. [2]

- 9 Fig. 9 shows the curve $y = \frac{x^2}{3x-1}$.

P is a turning point, and the curve has a vertical asymptote $x = a$.

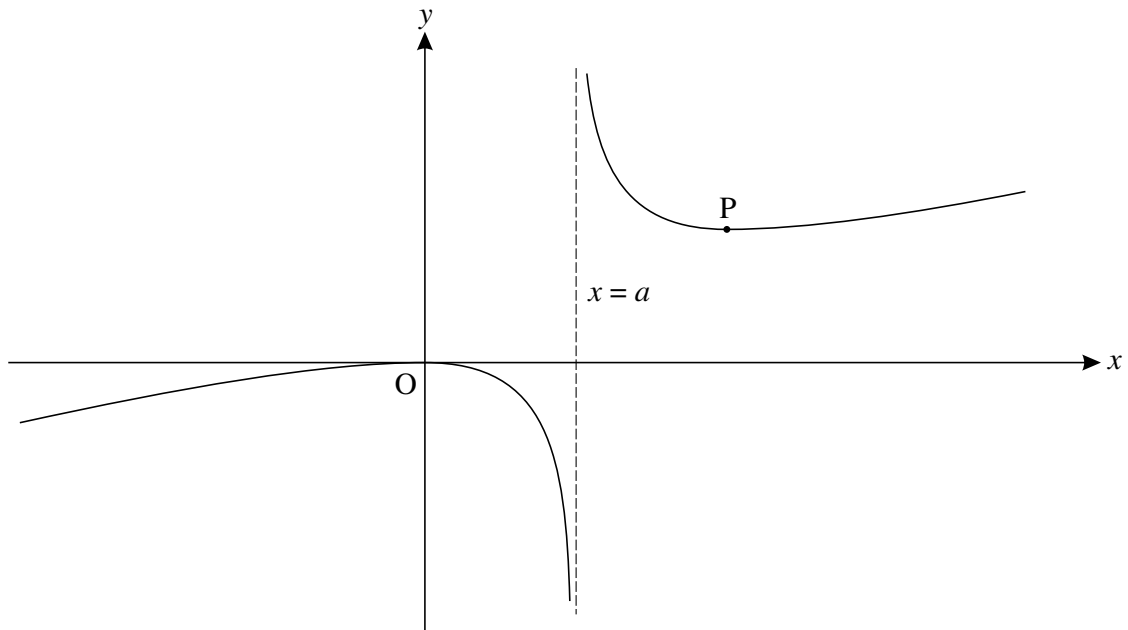


Fig. 9

- (i) Write down the value of a . [1]
- (ii) Show that $\frac{dy}{dx} = \frac{x(3x-2)}{(3x-1)^2}$. [3]
- (iii) Find the exact coordinates of the turning point P.

Calculate the gradient of the curve when $x = 0.6$ and $x = 0.8$, and hence verify that P is a minimum point. [7]

- (iv) Using the substitution $u = 3x - 1$, show that $\int \frac{x^2}{3x-1} dx = \frac{1}{27} \int \left(u + 2 + \frac{1}{u}\right) du$.

Hence find the exact area of the region enclosed by the curve, the x -axis and the lines $x = \frac{2}{3}$ and $x = 1$. [7]

Copyright Information

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations, is given to all schools that receive assessment material and is freely available to download from our public website (www.ocr.org.uk) after the live examination series.

If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact the Copyright Team, First Floor, 9 Hills Road, Cambridge CB2 1PB.

OCR is part of the Cambridge Assessment Group; Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.