RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 4724

Core Mathematics 4

Monday **23 JANUARY 2006** Afternoon 1 hour 30 minutes

Additional materials:
- 8 page answer booklet
- Graph paper
- List of Formulae (MF1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 3 printed pages and 1 blank page.

[Turn over

1 Simplify $\dfrac{x^3 - 3x^2}{x^2 - 9}$. [3]

2 Given that $\sin y = xy + x^2$, find $\dfrac{dy}{dx}$ in terms of x and y. [5]

3 (i) Find the quotient and the remainder when $3x^3 - 2x^2 + x + 7$ is divided by $x^2 - 2x + 5$. [4]

(ii) Hence, or otherwise, determine the values of the constants a and b such that, when $3x^3 - 2x^2 + ax + b$ is divided by $x^2 - 2x + 5$, there is no remainder. [2]

4 (i) Use integration by parts to find $\int x \sec^2 x \, dx$. [4]

(ii) Hence find $\int x \tan^2 x \, dx$. [3]

5 A curve is given parametrically by the equations $x = t^2$, $y = 2t$.

(i) Find $\dfrac{dy}{dx}$ in terms of t, giving your answer in its simplest form. [2]

(ii) Show that the equation of the tangent to the curve at $(p^2, 2p)$ is

$$py = x + p^2.$$ [2]

(iii) Find the coordinates of the point where the tangent at $(9, 6)$ meets the tangent at $(25, -10)$. [4]

6 (i) Show that the substitution $x = \sin^2 \theta$ transforms $\int \sqrt{\dfrac{x}{1-x}} \, dx$ to $\int 2\sin^2 \theta \, d\theta$. [4]

(ii) Hence find $\displaystyle\int_0^1 \sqrt{\frac{x}{1-x}} \, dx$. [5]

7 The expression $\dfrac{11 + 8x}{(2-x)(1+x)^2}$ is denoted by $f(x)$.

(i) Express $f(x)$ in the form $\dfrac{A}{2-x} + \dfrac{B}{1+x} + \dfrac{C}{(1+x)^2}$, where A, B and C are constants. [5]

(ii) Given that $|x| < 1$, find the first 3 terms in the expansion of $f(x)$ in ascending powers of x. [5]

8 **(i)** Solve the differential equation

$$\frac{dy}{dx} = \frac{2-x}{y-3},$$

giving the particular solution that satisfies the condition $y = 4$ when $x = 5$. [5]

(ii) Show that this particular solution can be expressed in the form

$$(x-a)^2 + (y-b)^2 = k,$$

where the values of the constants a, b and k are to be stated. [3]

(iii) Hence sketch the graph of the particular solution, indicating clearly its main features. [3]

9 Two lines have vector equations

$$\mathbf{r} = \begin{pmatrix} 4 \\ 2 \\ -6 \end{pmatrix} + t\begin{pmatrix} -8 \\ 1 \\ -2 \end{pmatrix} \quad \text{and} \quad \mathbf{r} = \begin{pmatrix} -2 \\ a \\ -2 \end{pmatrix} + s\begin{pmatrix} -9 \\ 2 \\ -5 \end{pmatrix},$$

where a is a constant.

(i) Calculate the acute angle between the lines. [5]

(ii) Given that these two lines intersect, find a and the point of intersection. [8]