

Mathematics

Advanced GCE

Unit **4723**: Core Mathematics 3

Mark Scheme for June 2011

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1 (i)	Obtain integral of form ke^{2x+1}	M1	any non-zero constant k different from 6; using substitution $u = 2x + 1$ to obtain ke^u earns M1 (but answer to be in terms of x)
	Obtain correct $3e^{2x+1}$	A1	or equiv such as $\frac{6}{2}e^{2x+1}$
(ii)	Obtain integral of form $k_1 \ln(2x+1)$	M1	any non-zero constant k_1 ; allow if brackets absent; $k_1 \ln u$ (after sub'n) earns M1
	Obtain correct $5\ln(2x+1)$	A1	or equiv such as $\frac{10}{2}\ln(2x+1)$; condone brackets rather than modulus signs but brackets or modulus signs must be present (so that $5\ln 2x+1$ earns A0)
	Include $\dots + c$ at least once	B1	5 anywhere in the whole of question 1; this mark available even if no marks awarded for integration
5			

2	Apply one of the transformations correctly to their equation	B1	
	Obtain correct $-3\ln x + \ln 4$	B1	or equiv
	Show at least one logarithm property	M1	correctly applied to their equation of resulting curve (even if errors have been made earlier)
	Obtain $y = \ln(4x^{-3})$	A1	4 or equiv of required form; $\ln 4x^{-3}$ earns A1; correct answer only earns 4/4; condone absence of $y =$
4			

3 (a)	State $14\sin \alpha \cos \alpha = 3\sin \alpha$	B1	or unsimplified equiv such as $7(2\sin \alpha \cos \alpha) = 3\sin \alpha$
	Attempt to find value of $\cos \alpha$	M1	by valid process; may be implied
	Obtain $\frac{3}{14}$	A1	3 exact answer required; ignore subsequent work to find angle

(b)	Attempt use of identity for $\cos 2\beta$	M1	of form $\pm 2\cos^2 \beta \pm 1$; initial use of $\cos^2 \beta - \sin^2 \beta$ needs attempt to express $\sin^2 \beta$ in terms of $\cos^2 \beta$ to earn M1
	Obtain $6\cos^2 \beta + 19\cos \beta + 10$	A1	or unsimplified equiv or equiv involving $\sec \beta$
	Attempt solution of 3-term quadratic eqn	M1	for $\cos \beta$ or (after adjustment) for $\sec \beta$
	Use $\sec \beta = \frac{1}{\cos \beta}$ at some stage	M1	or equiv
	Obtain $-\frac{3}{2}$	A1	5 or equiv; and (finally) no other answer
8			

4 (i)	Draw sketch of $y = (x-2)^4$	*B1	touching positive x -axis and extending at least as far as the y -axis; no need for 2 or 16 to be marked; ignore wrong intercepts
	Draw straight line with positive gradient	*B1	at least in first quadrant and reaching positive y -axis; assess the two graphs independently of each other
	Indicate two roots	B1	3 AG; dep *B *B and two correct graphs which meet on the y -axis; indicated in words or by marks on sketch
	[SC: Draw sketch of $y = (x-2)^4 - x - 16$ and indicate the two roots : B1 (i.e. max 1 mark)]		

(ii)	State 0 or $x = 0$	B1	1 not merely for coordinates (0, 16)

(iii)	Obtain correct first iterate	B1	to at least 3 dp; any starting value (> -16)
	Show correct iteration process	M1	producing at least 3 iterates in all; may be implied by plausible converging values
	Obtain at least 3 correct iterates	A1	allowing recovery after error; iterates given to only 3 d.p. acceptable; values may be rounded or truncated
	Obtain 4.118	A1	4 answer required to exactly 3 dp; A0 here if number of iterates is not enough to justify 4.118; attempt consisting of answer only earns 0/4
	[0 \rightarrow 4 \rightarrow 4.114743 \rightarrow 4.117769 \rightarrow 4.117849 ; 1 \rightarrow 4.030543 \rightarrow 4.115549 \rightarrow 4.117790 \rightarrow 4.117849 ; 2 \rightarrow 4.059767 \rightarrow 4.116321 \rightarrow 4.117811 \rightarrow 4.117850 ; 3 \rightarrow 4.087798 \rightarrow 4.117060 \rightarrow 4.117830 \rightarrow 4.117850 ; 4 \rightarrow 4.114743 \rightarrow 4.117769 \rightarrow 4.117849 \rightarrow 4.117851 ; 5 \rightarrow 4.140695 \rightarrow 4.118452 \rightarrow 4.117867 \rightarrow 4.117851]		
			8

5	Attempt use of product rule	*M1	to produce $k_1 x \ln(4x-3) + \frac{k_2 x^2}{4x-3}$ form
	Obtain $2x \ln(4x-3)$	A1	
	Obtain $\dots + \frac{4x^2}{4x-3}$	A1	or equiv
	Attempt second use of product rule	*M1	
	Attempt use of quotient (or product) rule	*M1	allow numerator the wrong way round
	Obtain		
	$2 \ln(4x-3) + \frac{8x}{4x-3} + \frac{8x(4x-3) - 16x^2}{(4x-3)^2}$	A1	or equiv
	Substitute 2 into attempt at second deriv	M1	dep *M *M *M
	Obtain $2 \ln 5 + \frac{96}{25}$	A1	8 or exact equiv consisting of two terms

8

6 Method 1: (Differentiation; assume value $\frac{10}{3}$; eqn of tangent; through origin)

Differentiate to obtain $k(3x-5)^{-\frac{1}{2}}$	M1	any constant k
Obtain $\frac{3}{2}(3x-5)^{-\frac{1}{2}}$	A1	or equiv
Attempt to find equation of tangent at P and attempt to show tangent passing through origin	M1	assuming value $\frac{10}{3}$; or equiv
Obtain $y = \frac{3}{2\sqrt{5}}x$ and confirm that tangent passes through O	A1	AG; necessary detail needed

Method 2: (Differentiation; equate $\frac{y \text{ change}}{x \text{ change}}$ to deriv; solve for x)

Differentiate to obtain $k(3x-5)^{-\frac{1}{2}}$	M1	any constant k
Obtain $\frac{3}{2}(3x-5)^{-\frac{1}{2}}$	A1	or equiv
Equate $\frac{y \text{ change}}{x \text{ change}}$ to deriv and attempt solution	M1	
Obtain $\frac{\sqrt{3x-5}}{x} = \frac{3}{2}(3x-5)^{-\frac{1}{2}}$ and solve to obtain $\frac{10}{3}$ only	A1	

Method 3: (Differentiation; find x from $y = f'(x) \cdot x$ and $y = \sqrt{3x-5}$)

Differentiate to obtain $k(3x-5)^{-\frac{1}{2}}$	M1	any constant k
Obtain $\frac{3}{2}(3x-5)^{-\frac{1}{2}}$	A1	or equiv
State $y = \frac{3}{2}(3x-5)^{-\frac{1}{2}}x$, $y = \sqrt{3x-5}$, eliminate y and attempt solution	M1	condone this attempt at 'eqn of tangent'
Obtain $\frac{10}{3}$ only	A1	

Method 4: (No differentiation; general line through origin to meet curve at one point only)

Eliminate y from equations $y = kx$ and $y = \sqrt{3x-5}$ and attempt formation of quadratic eqn	M1	
Obtain $k^2x^2 - 3x + 5 = 0$	A1	or equiv
Equate discriminant to zero to find k	M1	
Obtain $k = \frac{3}{2\sqrt{5}}$ or equiv and confirm $x = \frac{10}{3}$	A1	

Method 5: (No differentiation; use coords of P to find eqn of OP ; confirm meets curve once)

Use coordinates $(\frac{10}{3}, \sqrt{5})$ to obtain $y = \frac{3\sqrt{5}}{10}x$ or equiv as equation of OP	B1	
Eliminate y from this eqn and eqn of curve and attempt quadratic eqn	M1	should be $9x^2 - 60x + 100 = 0$ or equiv
Attempt solution or attempt discriminant	M1	
Confirm $\frac{10}{3}$ only or discriminant = 0	A1	

Either:

Integrate to obtain $k(3x-5)^{\frac{3}{2}}$	*M1	any constant k
Obtain correct $\frac{2}{9}(3x-5)^{\frac{3}{2}}$	A1	
Apply limits $\frac{5}{3}$ and $\frac{10}{3}$	M1	dep *M; the right way round
Make sound attempt at triangle area and calculate (triangle area) minus (their area under curve)	M1	or equiv
Obtain $\frac{10}{6}\sqrt{5} - \frac{10}{9}\sqrt{5}$ and hence $\frac{5}{9}\sqrt{5}$	A1	9 or exact equiv involving single term

Or:

Arrange to $x = \dots$ and integrate to obtain $k_1y^3 + k_2y$ form	*M1	
Obtain $\frac{1}{9}y^3 + \frac{5}{3}y$	A1	
Apply limits 0 and $\sqrt{5}$	M1	dep *M; the right way round
Make sound attempt at triangle area and calculate (their area from integration) minus (triangle area)	M1	
Obtain $\frac{20}{9}\sqrt{5} - \frac{5}{3}\sqrt{5}$ and hence $\frac{5}{9}\sqrt{5}$	A1	(9) or exact equiv involving single term

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7 (i) <u>Either:</u> Attempt solution of at least one linear eq'n of form $ax + b = 12$	M1	
Obtain $\frac{1}{3}$	A2	3 and (finally) no other answer
<u>Or:</u> Attempt solution of 3-term quadratic eq'n obtained by squaring attempt at $g(x+2)$ on LHS and squaring 12 or -12 on RHS	M1	
Obtain $\frac{1}{3}$	A2	(3) and (finally) no other answer

(ii) <u>Either:</u> Obtain $3(3x+5)+5$ for h	B1	
Attempt to find inverse function	M1	of function of form $ax + b$
Obtain $\frac{1}{9}(x-20)$	A1	3 or equiv in terms of x
<u>Or:</u> State or imply g^{-1} is $\frac{1}{3}(x-5)$	B1	
Attempt composition of g^{-1} with g^{-1}	M1	
Obtain $\frac{1}{9}(x-5) - \frac{5}{3}$	A1	(3) or more simplified equiv in terms of x

(iii) State $x \leq 0$	B2	2 give B1 for answer $x < 0$
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8 (i)	Differentiate to obtain form $ke^{-0.014t}$	M1	any constant k different from 400
	Obtain $5.6e^{-0.014t}$ or $-5.6e^{-0.014t}$	A1	or (unsimplified) equiv
	Obtain 4.9 or -4.9 or 4.87 or -4.87	A1	3 but not greater accuracy; allow if final statement seems contradictory; answer only earns 0/3 – differentiation is needed

(ii)	<u>Either</u> : State or imply $M_2 = 75e^{kt}$	B1	or equiv
	Attempt to find formula for M_2	M1	
	Obtain $M_2 = 75e^{0.047t}$	A1	or equiv such as $75e^{(\frac{1}{10}\ln\frac{8}{5})t}$
	Equate masses and attempt rearrangement	M1	as far as equation with e appearing once
	Obtain $e^{0.061t} = \frac{16}{3}$	A1	5 or equiv of required form which might involve 5.33 or greater accuracy on RHS; final two marks might be earned in part iii
<u>Or</u> :	State or imply $M_2 = 75 \times r^{0.1t}$	B1	for positive value r
	Obtain $75 \times 1.6^{0.1t}$	B1	
	Attempt to find M_2 in terms of e	M1	
	Equate masses and attempt rearrangement	M1	
	Obtain $e^{0.061t} = \frac{16}{3}$	A1	5 or equiv of required form which might involve 5.33 or greater accuracy on RHS; final two marks might be earned in part iii

(iii)	Attempt solution involving logarithm of any equation of form $e^{mt} = c_1$	M1	whether the conclusion of part ii or not
	Obtain 27.4	A1	2 or greater accuracy 27.4422...; correct answer only earns both marks

9 (i)	Use at least one identity correctly Attempt use of relevant identities in single rational expression	B1	angle-sum or angle-difference identity
		M1	not earned if identities used in expression where step equiv to $\frac{A+B+C}{D+E+F} = \frac{A}{D} + \frac{B}{E} + \frac{C}{F}$ or similar has been carried out; condone (for M1A1) if signs of identities apparently switched (so that, for example, denominator appears as $\cos \theta \cos \alpha - \sin \theta \sin \alpha +$ $3 \cos \theta + \cos \theta \cos \alpha + \sin \theta \sin \alpha$)
	Obtain $\frac{2 \sin \theta \cos \alpha + 3 \sin \theta}{2 \cos \theta \cos \alpha + 3 \cos \theta}$	A1	or equiv but with the other two terms from each of num'r and den'r absent
	Attempt factorisation of num'r and den'r	M1	
	Obtain $\frac{\sin \theta}{\cos \theta}$ and hence $\tan \theta$	A1	5 AG; necessary detail needed

(ii)	State or imply form $k \tan 150^\circ$	M1	obtained without any wrong method seen
	State or imply $\frac{4}{3} \tan 150^\circ$	A1	or equiv such as $\frac{12 \sin 150^\circ}{9 \cos 150^\circ}$
	Obtain $-\frac{4}{9}\sqrt{3}$	A1	3 or exact equiv (such as $-\frac{4}{3\sqrt{3}}$); correct answer only earns 3/3

(iii)	State or imply $\tan 6\theta = k$	B1	
	State $\frac{1}{6} \tan^{-1} k$	B1	
	Attempt second value of θ	M1	using $6\theta = \tan^{-1} k + (\text{multiple of } 180)$
	Obtain $\frac{1}{6} \tan^{-1} k + 30^\circ$	A1	4 and no other value

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