Level 2 Certificate

FURTHER MATHEMATICS

8365/1

Paper 1 Non-Calculator

Mark scheme

June 2023

Version: 1.0 Final

236G8365/1/MS

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Glossary for Mark Schemes

GCSE examinations are marked in such a way as to award positive achievement wherever possible. Thus, for GCSE Mathematics papers, marks are awarded under various categories.

If a student uses a method which is not explicitly covered by the mark scheme the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

M	Method marks are awarded for a correct method which could lead to a correct answer.
M dep	A method mark dependent on a previous method mark being awarded.
A	Accuracy marks are awarded when following on from a correct method. It is not necessary to always see the method. This can be implied.
B	Marks awarded independent of method.
B dep	A mark that can only be awarded if a previous independent mark has been awarded.
ft	Follow through marks. Marks awarded following a mistake in an earlier step.
SC	Special case. Marks awarded within the scheme for a common misinterpretation which has some mathematical worth.
oe	Or equivalent. Accept answers that are equivalent. eg accept 0.5 as well as $\frac{1}{2}$
[a, b]	Accept values between a and b inclusive.
3.14…	Accept answers which begin 3.14 eg 3.14, 3.142, 3.1416

Examiners should consistently apply the following principles.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Responses which appear to come from incorrect methods

Whenever there is doubt as to whether a candidate has used an incorrect method to obtain an answer, as a general principle, the benefit of doubt must be given to the candidate. In cases where there is no doubt that the answer has come from incorrect working then the candidate should be penalised.

Questions which ask candidates to show working

Instructions on marking will be given but usually marks are not awarded to candidates who show no working.

Questions which do not ask candidates to show working

As a general principle, a correct response is awarded full marks.

Misread or miscopy

Candidates often copy values from a question incorrectly. If the examiner thinks that the candidate has made a genuine misread, then only the accuracy marks (A or B marks), up to a maximum of 2 marks are penalised. The method marks can still be awarded.

Further work

Once the correct answer has been seen, further working may be ignored unless it goes on to contradict the correct answer.

Choice

When a choice of answers and/or methods is given, mark each attempt. If both methods are valid then M marks can be awarded but any incorrect answer or method would result in marks being lost.

Work not replaced

Erased or crossed out work that is still legible should be marked.

Work replaced

Erased or crossed out work that has been replaced is not awarded marks.

Premature approximation

Rounding off too early can lead to inaccuracy in the final answer. This should be penalised by 1 mark unless instructed otherwise.

Continental notation

Accept a comma used instead of a decimal point (for example, in measurements or currency), provided that it is clear to the examiner that the candidate intended it to be a decimal point.

Q	Answer	Mark	Comments
1(a)	$2x + 1 = –5$ or $2x = –6$ or $f^{-1}(x) = \frac{x-1}{2}$	M1	
	$x = –3$	A1	
	Additional Guidance		
	Correct answer not from incorrect working		M1 A1
	$f(x) = 5$ followed by $x = 2$		SC1

Q	Answer	Mark	Comments
1(b)	**Alternative method 1**		
	(g(3) =) 9	B1	may be implied by 19 if no other working
	19 or f [their g(3)] correctly evaluated	B1ft	any incorrect value for g(3) used correctly in f. Must be evidence of g to gain this mark (can't be f(anything))
	Alternative method 2		
	(fg(x) =) $2x^2 + 1$ or $2 \times 3^2 + 1$	B1	may be implied by 19 if no other working
	19	B1ft	
	Additional Guidance		
	49 from gf(x) on answer line		SC1
	Common errors from Alt 2 will come from getting fg(x) incorrect and might include (but must include the function clearly stated): $fg(x) = 4x^2 + 1$ followed by fg(3) = 37 $fg(x) = x^2(2x + 1)$ followed by fg(3) = 63		B0 B1ft
	ff(x) doesn't get any marks		B0 B0
	Common errors from Alt 1 will come from getting g(x) incorrect and might include g(3) = 6 then fg(3) = 13 g(3) = 3 then fg(3) = 7		B0 B1ft

Q	Answer	Mark	Comments
2	$3xy(2x + 7)$	B2	B1 for any factor taken out correctly. $3(2x^2y + 7xy)$ or $x(6xy + 21y)$ or $y(6x^2 + 21x)$ or $3x(2xy + 7y)$ or $3y(2x^2 + 7x)$ or $xy(6x + 21)$ B1 for correct highest common factor with one term incorrect in the bracket $3xy(a + 7)$ or $3xy(2x + b)$
	Additional Guidance		
	Condone final bracket missing		
	Condone multiplication signs		
	Condone changing order of terms eg $3(2x + 7)xy$		
	$(3xy + 0)(2x + 7)$ or $(3xy + \quad)(2x + 7)$ or $3xy$ **and** $(2x + 7)$ seen but not together		SC1

Q	Answer	Mark	Comments
3(a)	$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$	B1	
	Additional Guidance		

Q	Answer	Mark	Comments
3(b)	$\begin{pmatrix} 2\times-3+4\times1 & 2\times-4+4\times2 \\ -1\times-3+-3\times1 & -1\times-4+-3\times2 \end{pmatrix}$ or or $\begin{pmatrix} -6+4 & -8+8 \\ 3-3 & 4-6 \end{pmatrix}$ or $\begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}$	M1	allow M1 for any three correct calculations or any three correct terms in the matrix calculations don't need to be in a matrix condone missing brackets
	$-2\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}(= -2\,\mathbf{I})$	A1	may be written as $\mathrm{k}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ where $\mathrm{k} = -2$
	Additional Guidance		
	Errors in identity matrix will be penalised eg $2\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ or $\begin{pmatrix} \mathrm{k} & 0 \\ 0 & \mathrm{k} \end{pmatrix}$ are insufficient for A1		
	Must be evidence of identity matrix for A mark eg $\begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}$ and $\mathrm{k} = -2$		M1 A0

Q	Answer	Mark	Comments
4(a)	$\frac{2 - -4}{7-5}$ or $\frac{6}{2}$ or $\frac{-4-2}{5-7}$ or $\frac{-6}{-2}$	M1	oe these calculations may appear in a diagram but there needs to be evidence of subtraction and division taking place to gain M1 if A0
	3	A1	3 only (no x)
	Additional Guidance		
	Could use $y = mx + c$ then simultaneous equations in m and c. If completed correctly then full marks		M1 A1
	If the simultaneous equations method is used incorrectly or only partially		M0 A0
	Correct answer from graphical method or without working		M1 A1
	$(y=)$ $3x$ or $3x - k$ on answer line		M1 A0

Q	Answer	Mark	Comments
4(b)	$(7-5)^2 + (2 - -4)^2$ or $2^2 + 6^2$ or $4 + 36$	M1	may be seen on a diagram ft differences from Q4(a) for this mark only (must be numbers used in Q4a and not equivalents – see Additional Guidance)
	$\sqrt{40}$ or $\sqrt{4} \times \sqrt{10}$	M1dep	
	$2\sqrt{10}$	A1	just $a = 2$ and $b = 10$ would not be enough for A1 (unless $2\sqrt{10}$ in working)
	Additional Guidance		
	Penalise incorrect further working		
	Example of ft from Q4(a) would be $m = \frac{6}{12}$ so $[(ST)^2 =]\ 6^2 + 12^2$		M1 M0 A0
	But $m = \frac{6}{12}$ then cancelled and so $[(ST)^2 =]\ 1^2 + 2^2$		M0 M0 A0

Q	Answer	Mark	Comments
5	$(n+1)(.........)$ or $n^2 + n - n - 1$ or $n^2 - 1$ or $n^2 + 2n + n + 2$ or $n^2 + 3n + 2$	M1	ie common factor of $(n + 1)$ taken out and attempt to subtract oe
	$(n+1)(n+2-(n-1))$ or $n^2+2n+n+2-(n^2-n+n-1)$	M1dep	oe eg could be subtracted vertically
	$(n+1)(n+2-n+1) = 3(n+1)$ or $3n+3$ or $n^2+3n+2-n^2+1 = 3n+3$ or $3(n+1)$	A1	no statement needed at end of proof
	Additional Guidance		
	$X_n - Y_n$ can be awarded method marks but A0 unless rescued at the end eg recovery $-3(n + 1)$ so $Y_n - X_n$ is a multiple of 3	M2 A0 M2 A1	
	Missing brackets can be recovered later in the proof (but if not recovered then second M should not be awarded). Must see $3n + 3$ to show recovery eg $n^2 + 3n + 3 - n^2 - 1$ (error in X_n but Y_n correct with missing bracket)	M2 A1 M1 M0 A0	
	Condone use of x (or other letter) instead of n. A mixture of letters would only gain M1. However $Y_n - X_n = 3(n + 1) = 3x$ would be acceptable to show a multiple of 3		

Q	Answer	Mark	Comments
6	$6x^5 + 8x$	M1	either term correct ignore LHS
	$6 \times 1^5 + 8 \times 1$ or 14	M1dep	all needs to be correct (no ft)
	$-2 =$ (their 14) $\times 1 + c$ or $y - -2 =$ (their 14) $(x - 1)$	M1	from $y =$ (their 14) $x + c$ must be evidence of differentiation to get this mark
	$y = 14x - 16$	A1	
	Additional Guidance		
	Negative reciprocal of 14 to find the normal rather than the tangent ie. $-2 = \frac{-1}{14} \times 1 + c$ or $y - -2 = \frac{-1}{14}(x - 1)$	M3 A0	
	14 from incorrect differentiation eg $6x^4 + 8$ in differentiation with correct answer then ft correct eg $6x^4 + 8x$ in differentiation with correct answer then ft correct	 M0 M0 M1 A0 M1 M0 M1 A0	

Q	Answer	Mark	Comments
7	(Cone =) $\frac{1}{3}\pi r^2 x$ or (Prism =) $\frac{1}{2}y^2 x$	M1	could appear in M1 dep working height must be x
	$\frac{1}{3}\pi r^2 x = 2y^2 x$	M1dep	for correctly processing the 4 must be a correct equation oe eg $\frac{\frac{1}{3}\times\pi r^2\times x}{2} = \frac{2y^2\times x}{2}$ or $\frac{\pi r^2\times x}{6} = \frac{2y^2\times x}{2}$ $\frac{1}{3}\pi r^2 x = 4\ (\frac{1}{2}y^2 x)$ is not enough for M1 here because the 4 has not been processed
	$\frac{1}{3}\pi r^2 = 2y^2$ or $r^2 = \frac{6y^2}{\pi}$ or $\frac{1}{3}r^2 = \frac{2y^2}{\pi}$ or $\pi r^2 = 6y^2$	M1	oe for eliminating x dependent on both formulae being correct may have been numerical errors in previous M mark allow multiplication signs unprocessed numerical values can be condoned
	$r = \sqrt{\frac{6}{\pi}}\ y$ or $r = \sqrt{\frac{6y^2}{\pi}}$	A1	numbers need to be simplified $\sqrt{\frac{12y^2}{2\pi}}$ or $\sqrt{\frac{2y^2}{\frac{1}{3}\pi}}$ are A0 ensure that root is over the whole fraction (or separate roots over the numerator and denominator)
	Additional Guidance on next page		

	Additional Guidance	
	$r = \sqrt{\dfrac{6y^2x}{\pi x}}$	M1 M1 M0 A0
	$\frac{4}{3}\pi r^2 x = \frac{1}{2}y^2 x$ (multiplying the wrong side by 4) followed by eg $\frac{4}{3}\pi r^2 = \frac{1}{2}y^2$ or $8r^2 = \dfrac{3y^2}{\pi}$ or $\frac{4}{3}r^2 = \dfrac{y^2}{2\pi}$ or $\pi r^2 = \frac{3}{8}y^2$	M1 M0 M1 A0
	Penalise ± on answer line	M3 A0
	Either formula incorrect would result in maximum mark of M1	M1 M0 M0 A0
	Possible restarts could use both correct formulae	

Q	Answer	Mark	Comments
8	**Alternative method 1**		
	$x^2 + y^2 = 25$	B1	oe equation of circle eg $x^2 + y^2 = 5^2$ eg $(x - 0)^2 + (y - 0)^2 = 25$
	$(2y - 5)^2 + y^2$	M1	must be $2y - 5$ substituted into $x^2 + y^2$
	$(4y^2 - 20y + 25) + y^2$	M1	oe ft their $2y - 5$ correctly expanded
	$5y^2 - 20y = 0$ or $y^2 - 4y = 0$ or $5y(y - 4) = 0$ or $y(y - 4) = 0$	M1dep	oe (dependent on B1 M2) must be fully correct (no ft)
	$y = 0$ and $y = 4$ or (–5 , 0) or (3 , 4)	A1	
	(–5 , 0) and (3 , 4)	A1	
	Alternative method 2		
	$x^2 + y^2 = 25$	B1	oe equation of circle eg $x^2 + y^2 = 5^2$ eg $(x - 0)^2 + (y - 0)^2 = 25$
	$x^2 + \left(\frac{x+5}{2}\right)^2$	M1	must be $\left(\frac{x+5}{2}\right)$ substituted into $x^2 + y^2$
	$x^2 + \left(\frac{x^2+10x+25}{4}\right)$ or $4x^2 + x^2 + 10x + 25$ (= 100)	M1	oe ft their $\left(\frac{x+5}{2}\right)$ correctly expanded
	$5x^2 + 10x - 75 = 0$ or $x^2 + 2x - 15 = 0$ or $(x - 3)(x + 5) = 0$	M1dep	oe (dependent on B1 M2) must be fully correct (no ft)
	$x = 3$ and $x = -5$ or (–5 , 0) or (3 , 4)	A1	
	(–5 , 0) and (3 , 4)	A1	
	Additional Guidance on next page		

<table>
<tr><td rowspan="7">8 (cont'd)</td><td colspan="2">Additional Guidance</td></tr>
<tr><td>Answer could be written as $x = -5$, $y = 0$ and $x = 3$, $y = 4$</td><td>B1 M3 A2</td></tr>
<tr><td>Answer with no working (for M3) or from trying to draw an accurate graph or T&I</td><td>B0 M0 A0</td></tr>
<tr><td>Incorrect circle equation can still score 2 marks
eg. $x^2 + y^2 = 10$
$x^2 + \left(\frac{x+5}{2}\right)^2$
$4x^2 + x^2 + 10x + 25 = 40$
$5x^2 + 10x - 15 = 0$
$(x + 3)(x - 1) = 0$
$x = -3$ and $x = 1$
$y = 1$ and $y = 3$</td><td>B0 M2 M0 A0 A0
B0
M1

M1
M0

A0
A0</td></tr>
<tr><td>Correct answer can be gained by using the incorrect substitution
$x = 5 - 2y$</td><td>B1 M0 M1 M0 A0</td></tr>
<tr><td>Candidates that rearrange $x^2 + y^2 = 25$ correctly and substitute into $2y = x + 5$ will gain first M mark. They should then follow main scheme for other marks</td><td>B1 M1 …..</td></tr>
</table>

Q	Answer	Mark	Comments
9	$w(y^2 - 2) = y^2 + 5$	M1	
	$wy^2 - 2w = y^2 + 5$	M1dep	
	$y^2 (w - 1) = 2w + 5$ or $y^2 = \frac{(2w+5)}{(w-1)}$ or $y^2 (-w + 1) = -2w - 5$ or $y^2 = \frac{(-2w-5)}{(-w+1)}$	M1dep	rearranges and factorises
	$y = (\pm)\sqrt{\frac{(2w+5)}{(w-1)}}$ or $y = (\pm)\sqrt{\frac{(-2w-5)}{(-w+1)}}$	A1	
	Additional Guidance		
	Root must cover the whole function for final mark (or separate roots over both numerator and denominator)		
	Must be $y =$ If correct answer in working then $y =$ lost on the answer line then transcription error so A1 can be awarded		
	Penalise incorrect further working		

Q	Answer	Mark	Comments
10	$\frac{(1+\sqrt{5})(3+\sqrt{5})}{(3-\sqrt{5})(3+\sqrt{5})}$	M1	oe $\frac{(1+\sqrt{5})(-3-\sqrt{5})}{(3-\sqrt{5})(-3-\sqrt{5})}$ (if this method is used then follow equivalent MS for –4 and $-3-5-\sqrt{5}-3\sqrt{5}$) could be seen separately or in a grid
	Denominator = 4 or 9 – 5	M1dep	
	Numerator $3+5+\sqrt{5}+3\sqrt{5}$	M1dep	oe any 3 terms out of 4 correct (terms may have already been simplified eg $3+4\sqrt{5}$ just one error so M1dep) do not accept $\sqrt{5}\ \sqrt{5}$ or $\sqrt{25}$ for 5 this would be one error could be implied from $\frac{8+4\sqrt{5}}{4}$ only dependent on first M mark
	$2+\sqrt{5}$	A1	condone $2+1\sqrt{5}$ don't condone $\frac{2+\sqrt{5}}{1}$
	Additional Guidance		
	Untidy mathematical notation can be condoned as long as it's recovered		

Q	Answer	Mark	Comments
11	$\left(\frac{dy}{dx} =\right) \frac{x^3}{3} + 6x$	M1	oe eg $\frac{4}{12}\ x^3$ either term correct ignore LHS
	$\left(\frac{d^2y}{dx^2} =\right) x^2 + 6$ or $x^2 + 6 = 55$	M1dep	oe eg $x^2 = 49$ must come from fully correct first M
	7	A1	do not accept –7 or ±7
	Additional Guidance		
	M1 dep (and hence A1) can only be gained if the first M mark is fully correct (no incorrect terms and no additional terms) eg an answer of $\frac{1}{3}x^3 + 6x + 4$ would be allowed for M1 but as it's not fully correct would lose the following 2 marks		M1 M0 A0

Q	Answer	Mark	Comments
12(a)	$(x =)\ 270°$	B1	
	Additional Guidance		
	Do not accept answers outside of the given range or a choice of answers		
	Condone ° missing		

Q	Answer	Mark	Comments
12(b)	$\tan y = \frac{1}{\sqrt{3}}$ or $\tan y = \frac{\sqrt{3}}{3}$ or $y = \tan^{-1}\frac{\sqrt{3}}{3}$ or $y = \tan^{-1}\frac{1}{\sqrt{3}}$	M1	
	$(y =)\ 30°$ or $210°$	M1dep	could include up to two further incorrect values or 30° may be seen as part of a further calculation or on a diagram or embedded in tan 30° tan 30° embedded in a table with other exact trigonometric values (and not identified) would not gain M1dep
	$(y =)\ 30°$ and $210°$	A1	
	Additional Guidance		
	Correct answer could come from incorrect working		M0 A0
	30° with no working		M2
	30° and 210° with no working		M2 A1

Q	Answer	Mark	Comments
13	Common denominator found and used for all terms at some point during the method and at least one new numerator correct for that denominator	M1	eg $\frac{6x-9}{3x}-\frac{1}{3x}+\frac{3x}{3x}$ or $\frac{3x(2x-3)}{3x^2}-\frac{x}{3x^2}+\frac{3x^2}{3x^2}$ or $\frac{6x-10}{3x}+\frac{3x}{3x}$
	Numerator of $6x-9-1+3x$ for denominator of $3x$ or numerator of $6x^2-9x-x+3x^2$ for denominator of $3x^2$	M1dep	oe – will depend on the denominator but all numerator terms must be correct for the denominator used. could be a single fraction or separate fractions with the same denominator
	$\frac{9x-10}{3x}$	A1	$3-\frac{10}{3x}$ is not a single fraction so is A0
	Additional Guidance		
	Penalise incorrect further working		M2 A0

Q	Answer	Mark	Comments
14	$3x^2 - 10x + 8$ or $-3x^2 + 10x - 8$	M1	
	Factorising $(3x - 4)(x - 2)$ or completing the square $\left(x - \frac{5}{3}\right)^2 - \frac{1}{9}$ or $3\left(x - \frac{5}{3}\right)^2 - \frac{1}{3}$ or formula $\frac{10 \pm \sqrt{4}}{6}$	M1dep	$(3x - 4)(3x - 6)$ is acceptable for M1dep here
	$\frac{4}{3}$ and 2	M1 dep	oe – such as $1.\dot{3}$ or $1\frac{1}{3}$ or $\frac{4}{3}$ or $\frac{8}{6}$ can be seen in any format or inequality (correct or incorrect) eg $\frac{4}{3} > x$
	$\frac{4}{3} \leqslant x \leqslant 2$	A1	oe – could be written as two separate inequalities but do not accept $\frac{8}{6}$ at this stage as an oe $\frac{4}{3} \leqslant x$ **and** $x \leqslant 2$ must see the word 'and' in this format incorrect inequality signs is A0
	Additional Guidance		
	Candidates may presume that there may be a negative solution and reverse the inequality	M3 A0	
	They may however work this through and then realise it is giving the incorrect values when they test it. They can then correct themselves to still gain full marks	M3 A1	
	Students attempting T&I to get the solutions would need to get both solutions to get M3. If they then put that into the correct inequality it will be M3 A1	M3 A0 or M3 A1	

Q	Answer	Mark	Comments
15	$x - x^2 - x^2 + x^3$ $(= x^2 + x)$	M1	allow error in 1 term but must not have missing terms (could be seen in a grid) if they go straight to $x - 2x^2 + x^3$ then M1 if they go straight to $x - kx^2 + x^3$ where k is not 2 then this could be more than one error so M0
	$x^3 - 3x^2 = 0$ or $x^3 = 3x^2$	M1dep	no ft
	$x^2(x - 3)(= 0)$	M1	factorising their equation fully and correctly (must have x^3 plus/minus other term(s) in x^n where $n \neq 0$).
	$(x =)$ 0 and $(x =)$ 3	A1	condone $x = 0$ $x = 0$ $x = 3$

Additional Guidance

First M mark could be done in root form but would still need to end up with $x - x^2 - x^2 + x^3$ $(= x^2 + x)$ for M1	
Candidates may divide by x at some point during the method. If they do this and state that $x = 0$ is a solution at this point, the method can be continued to a correct outcome for full marks or $x^2 - 3x = 0$ or $x^2 = 3x$ would be M1dep and $x(x - 3)(= 0)$ would be the third M mark eg $x - x^2 - x^2 + x^3 = x^2 + x$ $1 - x - x + x^2 = x + 1$ with $x = 0$ $x^2 - 3x = 0$	M1 M1dep
If they do not state $x = 0$ is a solution and continue to a correct solution then award full marks but if it's not stated and they don't get to the correct solution then no further marks available as soon as the division is done eg $x - x^2 - x^2 + x^3 = x^2 + x$ $1 - x - x + x^2 = x + 1$ (no mention of $x = 0$ being a possible solution) $x^2 - 3x = 0$	M1 M0dep
If no other marks awarded then $x = 0$ **or** $x = 3$ seen on answer line (or the last thing they have written)	SC1
$x = 0$ and $x = 3$ not from incorrect working	M3 A1

Q	Answer	Mark	Comments
16	(1 4) 6 (4 1) or (1) 3 (3 1) seen and used correctly (in correct place in expansion) as part of Pascal's triangle or $\begin{pmatrix}4\\2\end{pmatrix}$ or $^4C_2$ or $\begin{pmatrix}3\\1\end{pmatrix}$ or $^3C_1$ or $\begin{pmatrix}3\\2\end{pmatrix}$ or $^3C_2$ seen and used	B1	$6(1^2)12x^2$ would be B1 $6(1^2)12^2x^2$ would be B1 leading onto M1dep $6(1^2) + 12x^2$ would be B0 or equivalent in other expansion
	$6 \times 12 \times 12$ ($\times x^2$) or 864($\times x^2$)	M1dep	oe could be within an expansion dependent on B mark only
	$3 \times a \times 4 \times 4$($\times x^2$) or $48a$($\times x^2$)	M1dep	oe could be within an expansion dependent on B mark only
	18	A1	
	Additional Guidance		
	(1) 3 (3 1) used in $(1 + 12x)^4$ would gain no marks		B0
	Either M mark implies the B mark		
	Candidates attempting to expand the brackets will need to get either M mark to get the B mark. They could have made errors in their expansions as long as they get $864x^2$ or $48ax^2$		

Q	Answer	Mark	Comments
17	$\left(\frac{dy}{dx} =\right) 3ax^2 + 2bx$	M1	either term correct ignore LHS
	$3a(-2)^2 + 2b(-2) = 0$ or $12a - 4b = 0$ or $3a - b = 0$ or $3a(-2)^2 + 2(1+2a)(-2) = 0$ (this would come from doing the third M mark before the differentiation)	M1dep	when $\frac{dy}{dx} = 0$ oe can follow through from an error in first M mark (as long as there are at least two terms in x^n)
	$11 = a(-2)^3 + b(-2)^2 + 7$ or $-8a + 4b = 4$ or $-2a + b = 1$ or $11 = a(-2)^3 + 3a(4) + 7$	M1	oe
	$a = 1$ or $b = 3$	A1	dependent on M3 awarded
	$a = 1$ and $b = 3$	A1	
	Additional Guidance		
	Correct answer could come from incorrect working (follow usual guidance)		
	$a = 1$ and $b = 3$ not from incorrect working will be full marks	M3 A2	
	Only one correct answer without working is 0 marks	M0 A0	

Q	Answer	Mark	Comments
18	**Alternative Method 1 – Elimination of x from the first 2 equations**		
	Correctly setting up two equations to eliminate x with an attempt to add or subtract from first two equations	M1	eg $2(x + 3z) – (2x + y)$
	Correct method and terms to eliminate a second variable	M1dep	eg $(6z – y) – 6(z – 2y)$
	Correctly eliminating a variable from their two correct equations to form an equation in one variable	M1dep	eg $y –12y = 9 – 42$ or $11y = 33$ or $y = 3$
	Two correct values with working seen	A1	eg $y = 3$ and $x = 5$
	$x = 5$ and $y = 3$ and $z = –1$ with correct working seen	A1	oe
	Additional Guidance		
	For first two M marks LHS manipulation only required		M2
	All three M marks can be combined in one equation: ie $2(x + 3z) – (2x + y) – 6(z – 2y) = 11y$ and $11y = 33$ and $y = 3$		M3
	May be multiple attempts. M marks can be awarded for best attempt but A marks would have to be from answers chosen		
	Stop marking after first error on LHS (but check for restarts)		
	Stop marking as soon as T&I/inspection methods begin (but check for restarts)		
	Alternative Method 2 – Elimination of y from 2 equations		
	Correctly setting up two equations to eliminate y with an attempt to add or subtract from two equations	M1	eg $2(2x + y) + (z – 2y)$
	Correct method and terms to eliminate a second variable	M1dep	eg $(4x + z) – 4(x + 3z)$
	Correctly eliminating a variable from their two correct equations to form an equation in one variable	M1dep	eg $z –12z = –7+18$ or $–11z = 11$ or $z = –1$
	Two correct values with working seen	A1	eg $z = –1$ and $x = 5$
	$x = 5$ and $y = 3$ and $z = –1$ with correct working seen	A1	oe

	Additional Guidance	
	For first two M marks LHS manipulation only required	M2
	All three M marks can be combined in one equation: ie $2(x + 3z) - (2x + y) - 6(z - 2y) = 11y$ and $11y = 33$ and $y = 3$	M3
	May be multiple attempts. M marks can be awarded for best attempt but A marks would have to be from answers chosen	
	Stop marking after first error on LHS (but check for restarts)	
	Stop marking as soon as T&I/inspection methods begin (but check for restarts)	

Q	Answer	Mark	Comments
18 (cont'd)	**Alternative Method 3 – Elimination of z from 2 equations**		
	Correctly setting up two equations to eliminate z with an attempt to add or subtract from two equations	M1	eg $(x + 3z) - 3(z - 2y)$
	Correct method and terms to eliminate a second variable	M1dep	eg $2(x + 6y) - (2x + y)$
	Correctly eliminating a variable from their two correct equations to form an equation in one variable	M1dep	eg $y - 12y = 9 - 42$ or $11y = 33$ or $y = 3$
	Two correct values with working seen	A1	eg $y = 3$ and $x = 5$
	$x = 5$ and $y = 3$ and $z = -1$ with correct working seen	A1	oe
	Additional Guidance		
	For first two M marks LHS manipulation only required		M2
	All three M marks can be combined in one equation: ie $2(x + 3z) - (2x + y) - 6(z - 2y) = 11y$ and $11y = 33$ and $y = 3$		M3
	May be multiple attempts. M marks can be awarded for best attempt but A marks would have to be from answers chosen		
	Stop marking after first error on LHS (but check for restarts)		
	Stop marking as soon as T&I/inspection methods begin (but check for restarts)		

	Alternative Method 4 – Substitution		
	One equation rearranged and correctly substituted into a second equation to eliminate an unknown	M1	eg $x = 2 - 3z$ and $2(2 - 3z) + y$
	Two correct equations formed with the same two unknowns in each equation (must be simplified correctly) and one equation rearranged so it can be substituted into the second equation	M1dep	eg $-6z + y = 9$ and $z = 2y - 7$
	Correctly eliminating a variable from their two correct equations to form an equation in one variable	M1dep	eg $-6(2y - 7) + y = 9$ or $-11y = -33$ or $y = 3$
	Two correct values with working seen	A1	eg $z = -1$ and $y = 3$
	$x = 5$ and $y = 3$ and $z = -1$ with correct working seen	A1	oe
	Additional Guidance		
	Could use a combination of elimination and substitution at any point. Stop marking after first error		
	May be multiple attempts. M marks can be awarded for best attempt but A marks would have to be from answers chosen		
	Stop marking as soon as T&I/inspection methods begin (but check for restarts)		

Q	Answer	Mark	Comments
19	(c =) 8 , (d =) $\frac{5}{4}$ or 1.25 and (n =) $\frac{31}{2}$ or $15\frac{1}{2}$ or 15.5	B3	oe B2 any 2 correct on answer line (not embedded) B1 (d =) $\frac{5}{4}$ or 1.25 or (n =) $\frac{31}{2}$ or $15\frac{1}{2}$ or 15.5 on answer line or $cx^2 + 2cdx + cd^2$ (+3) or $8\left(x+\frac{5}{4}\right)^2$ or $8(x+1.25)^2$ in method
	Additional Guidance		
	(c =) 8 on its own is 0 marks		M0 A0

Q	Answer	Mark	Comments
20	Angle PQR = 60°	B1	no reason required (angle at circumference is half angle at centre) could be on diagram
	(PR^2 =) $4^2 + 5^2 - 2(4)(5)\cos(\text{their } 60°)$	M1	cosine rule applied to work out PR
	cos60° = 0.5 or $\frac{1}{2}$ or $\frac{\sqrt{1}}{2}$	B1	must be used as part of an attempted cosine rule that requires 60°
	PR^2 = 21 or PR = $\sqrt{21}$	M1dep	would imply the B marks
	(PO =) $\left(\dfrac{\frac{1}{2}(\text{their PR})}{\sin 60^\circ}\right)$	M1	PR must have a value oe eg PO = $\frac{\sqrt{21}}{2} \div \frac{\sqrt{3}}{2}$ could use cosine rule again or sine rule at this stage but it would need to lead directly in one calculation to $\sqrt{7}$ eg $\left(\dfrac{\text{their}\sqrt{21}\sin 30^\circ}{\sin 120^\circ}\right)$ must be explicit: $\left(\dfrac{PO}{\sin 30^\circ} = \dfrac{\text{their}\sqrt{21}}{\sin 120^\circ}\right)$ is not M1 **but** this then followed by $\left(\dfrac{PO}{\frac{1}{2}} = \dfrac{\text{their}\sqrt{21}}{\frac{1}{3}}\right)$ (error in denominator) and $\left(PO = \dfrac{\frac{1}{2}\text{their}\sqrt{21}}{\frac{1}{3}}\right)$ (rearranged correctly) would be M1
	(PO =) $\sqrt{7}$	A1	
	Additional Guidance on next page		

	Additional Guidance	
	POM = 60° where M is the midpoint of PR	B0
	Final answer coming from double errors in surd manipulation	B2 M3 A0 or B2 M2 A0