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General Certificate of Education

Mathematics 6360

MPC3 Pure Core 3

Mark Scheme

2005 examination – June series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Key to mark scheme and abbreviations used in marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	OE	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
$-x$ EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

Application of Mark Scheme

No method shown:	
Correct answer without working	mark as in scheme
Incorrect answer without working	zero marks unless specified otherwise
More than one method / choice of solution:	
2 or more complete attempts, neither/none crossed out	mark both/all fully and award the mean mark rounded down
1 complete and 1 partial attempt, neither crossed out	award credit for the complete solution only
Crossed out work	do not mark unless it has not been replaced
Alternative solution using a correct or partially correct method	award method and accuracy marks as appropriate

MPC3

Q	Solution	Marks	Total	Comments
1(a)	$y = x\sin 2x$			
	$\frac{dy}{dx} = x2\cos 2x + \sin 2x$	M1		product rule
		A1,A1	3	
(b)(i)	$y = (x^2-6)^4$			
	$\frac{dy}{dx} = 4(x^2-6)^3(2x)$ (or better)	M1A1	2	M1 for $(x^2-6)^3$
(ii)	$\int 8x(x^2-6)^3\,dx = (x^2-6)^4$	M1		for $c(x^2-6)^4$ if correct attempt
	$\int = \frac{1}{8}(x^2-6)^4$ **(+c)**	A1		for $\frac{1}{k}(x^2-6)^4$ at 'by parts' M1A0
		A1	3	for $k=8$ Or $(x^2-6)^3 = x^6 - 18x^4 + 108x^2 - 216$ (M1A1) $\int x(x^2-6)^3 = \frac{x^8}{8} - 3x^6 + 27x^4 - 108x^2$ (A1)
	Total		**8**	
2(a)	$fg = h = \frac{6}{x+3} - 2$	M1		correct order
		A1	2	
	$\left(= \frac{6-2x-6}{x+3} = \frac{-2x}{x+3}\right)$			
(b)(i)	$x = \frac{-2y}{y+3}$			Or: $y = \frac{6}{x+3} - 2$
	$xy + 3x = -2y$			$y + 2 = \frac{6}{x+3}$
	$y(x+2) = -3x$	M1		attempt to isolate x or y $x + 3 = \frac{6}{y+2}$
	$h^{-1}(x) = y = \frac{-3x}{(x+2)}$	M1		$x \Leftrightarrow y$ $\quad x = \frac{6}{y+2} - 3$
		A1	3	$h^{-1}(x) = \frac{6}{x+2} - 3$
(ii)	(Range) $\neq -3$	B1	1	
	Total		**6**	

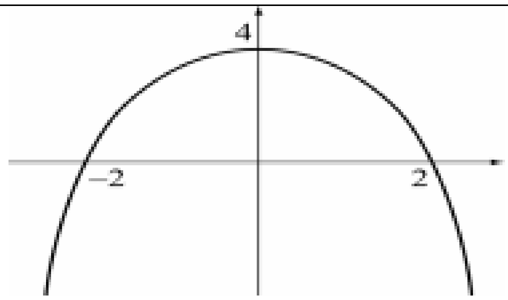
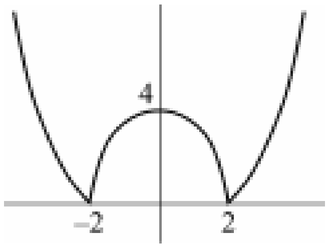
MPC3 (cont)

Q	Solution	Marks	Total	Comments
3(a)	$\frac{1}{4}e^{4x}$	B1	1	
(b)	$\int e^{4x}(2x+1)\,dx$			
	$u = 2x+1$ $\quad dv = e^{4x}$	M1		by parts
	$du = 2$ $\quad v = \frac{1}{4}e^{4x}$			
	$= \frac{1}{4}(2x+1)e^{4x} - \frac{1}{2}\int e^{4x}\,dx$	M1		Their $\left(uv - \int v\,du\right)$
	$= \frac{1}{4}(2x+1)e^{4x} - \frac{1}{8}e^{4x}\ (+c)$	A1	3	
(c)	$u = 1 + \ln x$			
	$\frac{du}{dx} = \frac{1}{x}$ or $\frac{dx}{du} = e^{u-1}$	B1		
	$\int = \int u\,du = \frac{u^2}{2}(+c)$	M1 A1		in terms of u only
	$= \frac{(1+\ln x)^2}{2}(+c)$	A1	4	
	Total		**8**	
4(a)	$\tan^2 x = \sec x + 11$			Or attempt to form quadratic in $\cos^2$
	$\sec^2 x - 1 = \sec x + 11$	M1		$\tan^2 x = \sec^2 x - 1$
	$\sec^2 x - \sec x - 12 = 0$	A1	2	AG
(b)	$(\sec x - 4)(\sec x + 3) = 0$	M1		attempt at solving quadratic
	$\sec x = 4, -3$	A1F		
	$\therefore \cos x = \frac{1}{4}, -\frac{1}{3}$	A1	3	AG; (A0 if no use of $\cos x = \frac{1}{\sec x}$)
(c)	$x = 76°, 284°$	B1		2 correct
	$x = 109°, 251°$ (or better)	B1,B1	3	other answers (–1 each extra in range) If radians $x = 1.32,\ 4.97$ $1.91,\ 4.37$ B1 any 2 correct B1 other 2 correct
	Total		**8**	

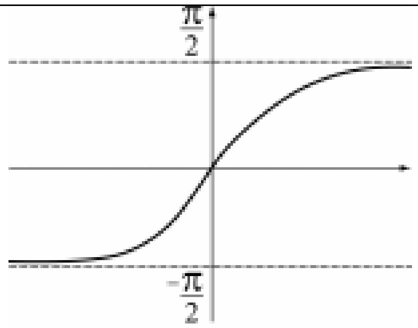
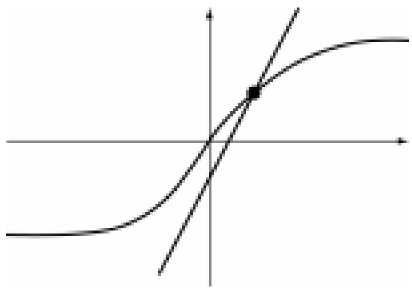
MPC3 (cont)

Q	Solution	Marks	Total	Comments
5(a)	$2e^x = 5$			
	$e^x = \frac{5}{2}$	M1		(exact)
	$x = \ln\frac{5}{2}$ (0.916)	A1	2	(A0 if further wrong work)
(b)(i)	$2e^x + 5e^{-x} = 7$			
	$2e^{2x} + 5 = 7e^x$	M1		Dealing with e^{-x}
	$2y^2 - 7y + 5 = 0$	A1	2	AG
(ii)	$(2y-5)(y-1) = 0$	M1		attempt to solve
				$y = \frac{5}{2}, 1$ (SC B1)
	$x = \ln\frac{5}{2}$	A1		$e^x = \frac{5}{2}$
	$x = 0$ (or ln 1)	A1	3	$e^x = 1$
	Total		**7**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
6(a)(i)		M1 A1	2	Shape symmetrical about y axis all correct
(ii)	$\text{V} = (k)\int (4 - x^2)^2 \,(\text{d}x)$	M1		
	$= (\pi)\int 16 - 8x^2 + x^4 \text{d}x$	B1		expanding bracket
	$= (\pi)\left[16x - \frac{8x^3}{3} + \frac{x^5}{5}\right]$	M1		correctly integrating 2 of their terms
	$= \pi\frac{256}{15}$	A1	4	
(b)(i)		M1 A1	2	modulus graph shape
(ii)	$\left\lvert 4 - x^2 \right\rvert = 3$	M1		attempt at solving a correct equation
	$4 - x^2 = 3 \Rightarrow x = +1, -1$	A1		2 correct
	$4 - x^2 = -3 \Rightarrow x = \pm\sqrt{7}$ (or exact equivalent)	A1	3	2 correct
(iii)	$-\sqrt{7} < x < -1$	B1F		condone $\sqrt{7}$ = 2.6 (or better)
	$1 < x < \sqrt{7}$	B1F	2	
	Total		**13**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
7(a)		B1 B1	 2	shape asymptotes (shown or stated) ($\frac{\pi}{2}$ seen)
(b)(i)		B1 B1	 2	sketch of $2x - 1$ correct
(ii)	$\tan^{-1} x - 2x + 1 = 0$ f(0.8) = 0.07 f(0.9) = -0.07 change of sign $\therefore$ root	 M1 A1	 2	 allow +ve, –ve A0 if f(0.8), f(0.9) wrong
(c)	$(x_1 = 0.8)$ $x_2 = 0.837(37)\ldots$ $x_3 = 0.85$	M1 A1 A1	 3	attempt at x_2 for x_2 for x_3
	Total		**9**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
8(a)	Stretch (parallel) to x-axis	B1		
	Scale factor $\frac{1}{2}$	B1		
	Translate $\begin{pmatrix} 0 \\ 3 \end{pmatrix}$	B1, B1	4	
(b)	x y			
	2.25 93.017	M1		Use of mid-ordinate rule
	2.75 247.692	A1		correct x
	3.25 668.142			
	3.75 1811.042	A1		3 correct y (2 sf)
	Area $= 0.5 \times 2819.893$			
	= 1410	A1	4	CAO
(c)	$A = \int e^{2x} + 3 \ \mathrm{d}x$	M1		(+ attempt to integrate)
	$= \left[\frac{1}{2}e^{2x} + 3x\right]$	A1		(correct)
	$\left(\frac{1}{2}e^{8} + 12\right) - \left(\frac{1}{2}e^{4} + 6\right)$	m1		Substitute 2,4 into their $\int$
	$= \frac{1}{2}\left(e^{8} - e^{4}\right) + 6$	A1	4	$\left(\frac{1}{2}e^{4}\left(e^{4} - 1\right) + 6\right)$
(d)	$x_1 = 2, \quad y_1 = e^{4} + 3 \quad (57.6)$	M1		Attempt at $y(2)$ or $y(4)$
	$x_2 = 4, \quad y_2 = e^{8} + 3 \quad (2980)$	A1		Both correct
	Area of $A + B =$ $2\left(e^{8} - e^{4}\right) + 2\left(e^{8} + 3\right)$	M1		Attempt to find correct area
	Area $B =$ $4e^{8} - 2e^{4} + 6$ $-\frac{1}{2}e^{8} + \frac{1}{2}e^{4} - 6$			
	$= \frac{7}{2}e^{8} - \frac{3}{2}e^{4}$	A1	4	
	Total		**16**	
	Total		**75**	