

4723 Core Mathematics 3

<p>1 <u>Either</u>: Obtain $x = 0$ Form linear equation with signs of $4x$ and $3x$ different State $4x - 5 = -3x + 5$ Obtain $\frac{10}{7}$ and no other non-zero value(s)</p>	<p>B1 ignoring errors in working M1 ignoring other sign errors A1 or equiv without brackets A1 or exact equiv</p>												
<p><u>Or</u>: Obtain $16x^2 - 40x + 25 = 9x^2 - 30x + 25$ Attempt solution of quadratic equation Obtain $\frac{10}{7}$ and no other non-zero value(s) Obtain 0</p>	<p>B1 or equiv M1 at least as far as factorisation or use of formula A1 or exact equiv B1 ignoring errors in working</p>												
<p>2 (i) Show graph indicating attempt at reflection in $y = x$ Show correct graph with x-coord 2 and y-coord -3 indicated</p>	<p>M1 with correct curvature and crossing negative y-axis and positive x-axis A1 2</p>												
<p>(ii) Show graph indicating attempt at reflection in x-axis Show correct graph with x-coord -3 indicated ... and y-coord -4 indicated [SC: Incorrect curve earning M0 but both correct intercepts indicated</p>	<p>M1 with correct curvature and crossing each negative axis A1 A1 B1] 3</p>												
<p>3 Attempt use of product rule Obtain $2x \ln x + x^2 \cdot \frac{1}{x}$ Substitute e to obtain $3e$ for gradient Attempt eqn of straight line with numerical gradient Obtain $y - e^2 = 3e(x - e)$ Obtain $y = 3ex - 2e^2$</p>	<p>M1 ... + ... form A1 or equiv A1 or exact (unsimplified) equiv M1 allowing approx values A1 or equiv; following their gradient provided obtained by diffn attempt; allow approx values A1 in terms of e now and in requested form 6</p>												
<p>4 (i) Differentiate to obtain form $kx(2x^2 + 9)^n$ Obtain correct $10x(2x^2 + 9)^{\frac{3}{2}}$ Equate to 100 and confirm $x = 10(2x^2 + 9)^{-\frac{3}{2}}$</p>	<p>M1 any constant k; any $n < \frac{5}{2}$ A1 or (unsimplified) equiv A1 AG; necessary detail required 3</p>												
<p>(ii) Attempt relevant calculations with 0.3 and 0.4 Obtain at least one correct value Obtain two correct values and conclude appropriately</p>	<p>M1 A1</p> <table border="1" data-bbox="973 1713 1356 1825"> <thead> <tr> <th>x</th> <th>$f(x)$</th> <th>$x - f(x)$</th> <th>$f'(x)$</th> </tr> </thead> <tbody> <tr> <td>0.3</td> <td>0.3595</td> <td>-0.0595</td> <td>83.4</td> </tr> <tr> <td>0.4</td> <td>0.3515</td> <td>0.0485</td> <td>113.8</td> </tr> </tbody> </table> <p>A1 noting sign change or showing $0.3 < f(0.3)$ and $0.4 > f(0.4)$ or showing gradients either side of 100 3</p>	x	$f(x)$	$x - f(x)$	$f'(x)$	0.3	0.3595	-0.0595	83.4	0.4	0.3515	0.0485	113.8
x	$f(x)$	$x - f(x)$	$f'(x)$										
0.3	0.3595	-0.0595	83.4										
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(iii)	Obtain correct first iterate Carry out correct process Obtain 0.3553	B1 M1 A1	finding at least 3 iterates in all answer required to exactly 4 dp
	[0.3 → 0.35953 → 0.35497 → 0.35534 → 0.35531; 0.35 → 0.35575 → 0.35528 → 0.35532 (→ 0.35531); 0.4 → 0.35146 → 0.35563 → 0.35529 → 0.35532]	3	
5 (a)	Obtain expression of form $\frac{a \tan \alpha}{b + c \tan^2 \alpha}$ State correct $\frac{2 \tan \alpha}{1 - \tan^2 \alpha}$ Attempt to produce polynomial equation in $\tan \alpha$ Obtain at least one correct value of $\tan \alpha$ Obtain 41.8 Obtain 138.2 and no other values between 0 and 180 [SC: Answers only 41.8 or ... B1; 138.2 or ... and no others B1]	M1 A1 M1 A1 A1 A1	any non-zero constants a, b, c or equiv using sound process $\tan \alpha = \pm \sqrt{\frac{4}{5}}$ allow 42 or greater accuracy; allow 0.73 allow 138 or greater accuracy
		6	
(b)(i)	State $\frac{7}{6}$	B1	
		1	
(ii)	Attempt use of identity linking $\cot^2 \beta$ and $\operatorname{cosec}^2 \beta$ Obtain $\frac{13}{36}$	M1 A1	or equiv retaining exactness; condone sign errors or exact equiv
		2	
6	Integrate $k_1 e^{nx}$ to obtain $k_2 e^{nx}$ Obtain correct indefinite integral of their $k_1 e^{nx}$ Substitute limits to obtain $\frac{1}{6} \pi (e^3 - 1)$ or $\frac{1}{6} (e^3 - 1)$ Integrate $k(2x - 1)^n$ to obtain $k'(2x - 1)^{n+1}$ Obtain correct indefinite integral of their $k(2x - 1)^n$ Substitute limits to obtain $\frac{1}{18} \pi$ or $\frac{1}{18}$ Apply formula $\int \pi y^2 dx$ at least once Subtract, correct way round, attempts at volumes y^2 Obtain $\frac{1}{6} \pi e^3 - \frac{2}{9} \pi$	M1 A1 A1 M1 A1 A1 M1 A1	any constants involving π or not; any n or exact equiv perhaps involving e^0 any constants involving π or not; any n or exact equiv for $y = e^{3x}$ and/or $y = (2x - 1)^4$ allow with π missing but must involve or similarly simplified exact equiv
		9	
7 (i)	State $A = 42$ State $k = \frac{1}{9}$ Attempt correct process for finding m Obtain $\frac{1}{9} \ln 2$ or 0.077	B1 B1 M1 A1	or 0.11 or greater accuracy involving logarithms or equiv or 0.08 or greater accuracy
		4	
(ii)	Attempt solution for t using either formula Obtain 11.3	M1 A1	using correct process (log's ms or T&I or ...) or greater accuracy; allow 11.3 ± 0.1
		2	
(iii)	Differentiate to obtain form Be^{mt} Obtain $3.235e^{0.077t}$ Obtain 47.9	M1 A1 A1	where B is different from A or equiv; following their A and m allow 48 or greater accuracy
		3	

<p>8 (i) Show at least correct $\cos \theta \cos 60 + \sin \theta \sin 60$ or $\cos \theta \cos 60 - \sin \theta \sin 60$ Attempt expansion of both with exact numerical values attempted Obtain $\frac{1}{2}\sqrt{3} \sin \theta + \frac{5}{2} \cos \theta$</p>	<p>B1 M1 and with $\cos 60 \neq \sin 60$ A1 or exact equiv</p>
<p>(ii) Attempt correct process for finding R Attempt recognisable process for finding α Obtain $\sqrt{7} \sin(\theta + 70.9)$</p>	<p>M1 whether exact or approx M1 allowing sin / cos muddles A1 allow 2.65 for R; allow 70.9 ± 0.1 for α</p>
<p>(iii) Attempt correct process to find any value of $\theta +$ their α Obtain any correct value for $\theta + 70.9$ Attempt correct process to find $\theta +$ their α in 3rd quadrant Obtain 131 [SC for solutions with no working shown: Correct answer only</p>	<p>M1 $-158, -22, 202, 338, \dots$ M1 or several values including this A1 or greater accuracy and no other A1 B4; 131 with other answers B2]</p>
<p>9 (i) Attempt use of quotient rule Obtain $\frac{75 - 15x^2}{(x^2 + 5)^2}$ Equate attempt at first derivative to zero and rearrange to solvable form Obtain $x = \sqrt{5}$ or 2.24 Recognise range as values less than y-coord of st pt Obtain $0 \leq y \leq \frac{3}{2}\sqrt{5}$</p>	<p>*M1 or equiv; allow u / v muddles A1 or (unsimplified) equiv; this M1A1 available at any stage of question M1 dep *M A1 or greater accuracy M1 allowing $<$ here A1 any notation; with \leq now; any exact equiv</p>
<p>(ii) State $\sqrt{5}$</p>	<p>B1 following their x-coord of st pt; condone answer $x \geq \sqrt{5}$ but not inequality with k</p>
<p>(iii) Equate attempt at first derivative to -1 and attempt simplification Obtain $x^4 - 5x^2 + 100 = 0$ Attempt evaluation of discriminant or equiv Obtain -375 or equiv and conclude appropriately</p>	<p>*M1 and dependent on first M in part (i) A1 or equiv involving 3 non-zero terms M1 dep *M A1</p>