OCR
Oxford Cambridge and RSA

Monday 24 June 2019 – Morning

A Level Further Mathematics A

Y545/01 Additional Pure Mathematics

Time allowed: 1 hour 30 minutes

You must have:
- Printed Answer Booklet
- Formulae A Level Further Mathematics A

You may use:
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION
- The total mark for this paper is **75**.
- The marks for each question are shown in brackets **[ ]**.
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **4** pages.

Answer **all** the questions.

1 The sequence $\{u_n\}$ is defined by $u_0 = 2,\ u_1 = 5$ and $u_n = \dfrac{1+u_{n-1}}{u_{n-2}}$ for $n \geqslant 2$.

Prove that the sequence is periodic with period 5. **[4]**

2 A surface has equation $z = \mathrm{f}(x, y)$ where $\mathrm{f}(x, y) = x^2 \sin y + 2y \cos x$.

(a) Determine f_x , f_y , f_{xx} , f_{yy} , f_{xy} and f_{yx} . **[5]**

(b) **(i)** Verify that z has a stationary point at $\left(\frac{1}{2}\pi, \frac{1}{2}\pi, \frac{1}{4}\pi^2\right)$. **[3]**

(ii) Determine the nature of this stationary point. **[3]**

3 **(a)** Solve $7x \equiv 6 \pmod{19}$. **[2]**

(b) Show that the following simultaneous linear congruences have no solution.

$x \equiv 3 \pmod 4$, $x \equiv 4 \pmod 6$. **[2]**

4 **(a)** Solve the second-order recurrence relation $T_{n+2} + 2T_n = -87$ given that $T_0 = -27$ and $T_1 = 27$. **[8]**

(b) Determine the value of T_{20}. **[2]**

5 The group G consists of a set S together with $\times_{80}$, the operation of multiplication modulo 80. It is given that S is the smallest set which contains the element 11.

(a) By constructing the Cayley table for G, determine all the elements of S. **[5]**

The Cayley table for a second group, H, also with the operation $\times_{80}$, is shown below.

$\times_{80}$	1	9	31	39
1	1	9	31	39
9	9	1	39	31
31	31	39	1	9
39	39	31	9	1

(b) Use the two Cayley tables to explain why G and H are not isomorphic. **[2]**

(c) **(i)** List

- all the proper subgroups of G,
- all the proper subgroups of H. **[3]**

(ii) Use your answers to **(c) (i)** to give another reason why G and H are not isomorphic. **[1]**

6 **(a)** For the vectors $\mathbf{p} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $\mathbf{q} = \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} 2 \\ -4 \\ 5 \end{pmatrix}$, calculate

- $\mathbf{p} \bullet \mathbf{q} \times \mathbf{r}$,
- $\mathbf{p} \times (\mathbf{q} \times \mathbf{r})$,
- $(\mathbf{p} \times \mathbf{q}) \times \mathbf{r}$. **[6]**

(b) State whether the vector product is associative for three-dimensional column vectors with real components. Justify your answer. **[1]**

It is given that **a**, **b** and **c** are three-dimensional column vectors with real components.

(c) Explain geometrically why the vector $\mathbf{a} \times (\mathbf{b} \times \mathbf{c})$ must be expressible in the form $\lambda\mathbf{b} + \mu\mathbf{c}$, where λ and μ are scalar constants. **[2]**

It is given that the following relationship holds for **a**, **b** and **c**.

$$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \bullet \mathbf{c})\,\mathbf{b} - (\mathbf{a} \bullet \mathbf{b})\,\mathbf{c} \qquad (*)$$

(d) Find an expression for $(\mathbf{a} \times \mathbf{b}) \times \mathbf{c}$ in the form of **(*)**. **[3]**

7 The points $P\left(\frac{1}{2}, \frac{13}{24}\right)$ and $Q\left(\frac{3}{2}, \frac{31}{24}\right)$ lie on the curve $y = \frac{1}{3}x^3 + \frac{1}{4x}$.

The area of the surface generated when arc PQ is rotated completely about the x-axis is denoted by A.

(a) Find the exact value of A. Give your answer as a rational multiple of π. **[4]**

Student X finds an approximation to A by modelling the arc PQ as the straight line segment PQ, then rotating this line segment completely about the x-axis to form a surface.

(b) Find the approximation to A obtained by student X. Give your answer as a rational multiple of π. **[4]**

Student Y finds a second approximation to A by modelling the original curve as the line $y = M$, where M is the mean value of the function $\mathrm{f}(x) = \frac{1}{3}x^3 + \frac{1}{4x}$, then rotating this line completely about the x-axis to form a surface.

(c) Find the approximation to A obtained by student Y. Give your answer correct to four decimal places. **[4]**

8 **In this question you must show detailed reasoning.**

(a) Prove that $2(p-2)^{p-2} \equiv -1 \pmod{p}$, where p is an odd prime. **[4]**

(b) Find two odd prime factors of the number $N = 2 \times 34^{34} - 2^{15}$. **[7]**

END OF QUESTION PAPER