## OXFORD CAMBRIDGE AND RSA EXAMINATIONS

## Advanced Subsidiary General Certificate of Education Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS

## 4762

Mechanics 2
Friday 27 JANUARY 2006 Afternoon 1 hour 30 minutes

Additional materials:
8 page answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

## TIME

 1 hour 30 minutes
## INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer all the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $\mathrm{g} \mathrm{m} \mathrm{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $\mathrm{g}=9.8$.


## INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is 72 .

1 When a stationary firework P of mass 0.4 kg is set off, the explosion gives it an instantaneous impulse of 16 Ns vertically upwards.
(i) Calculate the speed of projection of P .

While travelling vertically upwards at $32 \mathrm{~m} \mathrm{~s}^{-1}$, P collides directly with another firework Q , of mass 0.6 kg , that is moving directly downwards with speed $u \mathrm{~m} \mathrm{~s}^{-1}$, as shown in Fig. 1. The coefficient of restitution in the collision is 0.1 and Q has a speed of $4 \mathrm{~m} \mathrm{~s}^{-1}$ vertically upwards immediately after the collision.


Fig. 1
(ii) Show that $u=18$ and calculate the speed and direction of motion of P immediately after the collision.

Another firework of mass 0.5 kg has a velocity of $(-3.6 \mathbf{i}+5.2 \mathbf{j}) \mathrm{m} \mathrm{s}^{-1}$, where $\mathbf{i}$ and $\mathbf{j}$ are horizontal and vertical unit vectors, respectively. This firework explodes into two parts, C and D. Part C has mass 0.2 kg and velocity $(3 \mathbf{i}+4 \mathbf{j}) \mathrm{m} \mathrm{s}^{-1}$ immediately after the explosion.
(iii) Calculate the velocity of D immediately after the explosion in the form $a \mathbf{i}+b \mathbf{j}$. Show that C and D move off at $90^{\circ}$ to one another.

2 A uniform beam, AB , is 6 m long and has a weight of 240 N .
Initially, the beam is in equilibrium on two supports at C and D, as shown in Fig. 2.1. The beam is horizontal.


Fig. 2.1
(i) Calculate the forces acting on the beam from the supports at C and D .

A workman tries to move the beam by applying a force $T \mathrm{~N}$ at A at $40^{\circ}$ to the beam, as shown in Fig. 2.2. The beam remains in horizontal equilibrium but the reaction of support C on the beam is zero.


Fig. 2.2
(ii) (A) Calculate the value of $T$.
(B) Explain why the support at D cannot be smooth.

The beam is now supported by a light rope attached to the beam at A, with B on rough, horizontal ground. The rope is at $90^{\circ}$ to the beam and the beam is at $30^{\circ}$ to the horizontal, as shown in Fig. 2.3. The tension in the rope is $P \mathrm{~N}$. The beam is in equilibrium on the point of sliding.


Fig. 2.3
(iii) (A) Show that $P=60 \sqrt{3}$ and hence, or otherwise, find the frictional force between the beam and the ground.
(B) Calculate the coefficient of friction between the beam and the ground.

3 (a) A uniform lamina made from rectangular parts is shown in Fig. 3.1. All the dimensions are centimetres. All coordinates are referred to the axes shown in Fig. 3.1.


Fig. 3.1
(i) Show that the $x$-coordinate of the centre of mass of the lamina is 6.5 and find the $y$-coordinate.

A square of side 2 cm is to be cut from the lamina. The sides of the square are to be parallel to the coordinate axes and the centre of the square is to be chosen so that the $x$-coordinate of the centre of mass of the new shape is 6.4.
(ii) Calculate the $x$-coordinate of the centre of the square to be removed.

The $y$-coordinate of the centre of the square to be removed is now chosen so that the $y$-coordinate of the centre of mass of the final shape is as large as possible.
(iii) Calculate the $y$-coordinate of the centre of mass of the lamina with the square removed, giving your answer correct to three significant figures.
(b) Fig. 3.2 shows a framework made from light rods of length 2 m freely pin-jointed at A, B, C, D and E . The framework is in a vertical plane and is supported at A and C . There are loads of 120 N at B and at E . The force on the framework due to the support at A is $R \mathrm{~N}$.

each rod is
2 m long

Fig. 3.2
(i) Show that $R=150$.
(ii) Draw a diagram showing all the forces acting at the points $\mathrm{A}, \mathrm{B}, \mathrm{D}$ and E , including the forces internal to the rods.

Calculate the internal forces in rods AE and EB, and determine whether each is a tension or a thrust. [You may leave your answers in surd form.]
(iii) Without any further calculation of the forces in the rods, explain briefly how you can tell that rod ED is in thrust.

4 A block of mass 20 kg is pulled by a light, horizontal string over a rough, horizontal plane. During 6 seconds, the work done against resistances is 510 J and the speed of the block increases from $5 \mathrm{~m} \mathrm{~s}^{-1}$ to $8 \mathrm{~m} \mathrm{~s}^{-1}$.
(i) Calculate the power of the pulling force.

The block is now put on a rough plane that is at an angle $\alpha$ to the horizontal, where $\sin \alpha=\frac{3}{5}$. The frictional resistance to sliding is $11 g \mathrm{~N}$. A light string parallel to the plane is connected to the block. The string passes over a smooth pulley and is connected to a freely hanging sphere of mass $m \mathrm{~kg}$, as shown in Fig. 4.


Fig. 4
In parts (ii) and (iii), the sphere is pulled downwards and then released when travelling at a speed of $4 \mathrm{~m} \mathrm{~s}^{-1}$ vertically downwards. The block never reaches the pulley.
(ii) Suppose that $m=5$ and that after the sphere is released the block moves $x \mathrm{~m}$ up the plane before coming to rest.
(A) Find an expression in terms of $x$ for the change in gravitational potential energy of the system, stating whether this is a gain or a loss.
(B) Find an expression in terms of $x$ for the work done against friction.
(C) Making use of your answers to parts $(A)$ and $(B)$, find the value of $x$.
(iii) Suppose instead that $m=15$. Calculate the speed of the sphere when it has fallen a distance 0.5 m from its point of release.

