Oxford Cambridge and RSA Examinations
Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS

4754

APPLICATIONS OF ADVANCED MATHEMATICS, C4
PAPER A

MARK SCHEME

Qu	Answer	Mark	Comment
Section A			
1	$(1+2x)^{\frac{1}{2}}$	M1	Handling $\sqrt{\ }$
	$1+\frac{1}{2}(2x)+\frac{1}{2}(\frac{1}{2}-1)\frac{(2x)^2}{2!}+\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}-2)\frac{(2x)^3}{3!}+\ ...$	M1,A1	Expansion of right form
	$1+x-\frac{1}{2}x^2+\frac{1}{2}x^3+...$	A1	
	$-\frac{1}{2}<x<\frac{1}{2}$	B1 **[5]**	
2	$\tan P\hat{S}Q=\frac{5}{12}$ and $\tan Q\hat{S}R=\frac{16}{12}$	B1,B1	
	Let $P\hat{S}Q=A,\ Q\hat{S}R=B$		
	$\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}$	M1	Use of formula
	$\tan(A+B)=\frac{\frac{5}{12}+\frac{16}{12}}{1-\frac{5}{12}\times\frac{16}{12}}$	A1	
	$\sin P\hat{S}R=\frac{\frac{21}{12}}{\frac{144-80}{144}}=\frac{63}{16}$	E1 **[5]**	
3	$5(\sin\theta\times\frac{3}{5}+\cos\theta\times\frac{4}{5})$	M1	Correct form
	$5\sin(\theta+53.1°)$, $R=5,\ \alpha=53.13...°$	A1,A1	
	$5\sin(\theta+53.1°)=1$		
	$\sin(\theta+53.1°)=0.2$, $\arcsin(0.2)=11.536...°$		
	$\theta+53.1°=...\ 11.5°,\ 168.5°,\ 371.5°,\ 528.5°,\ ...$	M1	Search for many roots
	In range, $\theta=115.3°,\ \ 318.4°$	A1,A1 **[6]**	
4(i)	$x^2=\sin^2\theta+\cos^2\theta-2\sin\theta\cos\theta+2\sin\theta-2\cos\theta+1$	M1,A1	
	$x^2=-\sin 2\theta+2\sin\theta-2\cos\theta+2$	B1	
	$x^2=-y+2x$		
	$y=-x^2+2x$	E1 **[4]**	
4(ii)	Sketch graph of $y=-x^2+2x$	B1	
	Part between approx. (-0.4, -1) and (2.4, -1) highlighted.	B1 **[2]**	

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Qu	Answer	Mark	Comment
Section A (continued)			
5	$1-\frac{1}{x+1}=\frac{x+1-1}{x+1}=\frac{x}{x+1}$	B1	
	Volume = $\pi\int_0^2\left(\frac{x}{x+1}\right)^2 dx$	M1	Volume of revolution procedure
	$=\pi\int_0^2\left(1-\frac{2}{x+1}+\frac{1}{(x+1)^2}\right)dx$	A1	
	$=\pi\left[x-2\ln\|x+1\|-\frac{1}{x+1}\right]_0^2$	A1	logarithm
		A1	$-\frac{1}{x+1}$
	$=[2-2\ln 3-\frac{1}{3}]\pi-[-1]\pi$	M1	Use of limits
	$=\left(\frac{8}{3}-2\ln 3\right)\pi$	A1 **[7]**	
6	Gradient of line is $-\frac{1}{3}$	M1	
	For curve $\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{-\frac{4}{t^2}}{3}=-\frac{4}{3t^2}$	M1,A1	Procedure for finding gradient
	$-\frac{4}{3t^2}=-\frac{1}{3}$ when $t=2$ or -2	M1 A1	Equating gradient to $-\frac{1}{3}$
	When $t=2$, the curve is at (6, 2) and (6, 2) lies on the line $x+3y=12$	M1 E1 **[7]**	
			Section A Total: 36
Section B			
7(i)(*A*)	$P=Ae^{kt}$		
	$\Rightarrow\frac{dP}{dt}=kAe^{kt}$	M1	Differentiating
	$=kP$	E1	Replacing by P
	when $t=0$, $P=1$, $\Rightarrow 1=Ae^0=A$	B1	Verifying $A=1$ (may come first)
	when $t=1$, $P=1.24=1.e^k$	M1	Substituting $t=1$
	$\Rightarrow k=\ln 1.24=0.215$	A1 **[5]**	0.215… accept ln1.24 or 0.22 or better
7(i)(*B*)	As $t\to\infty, P\to\infty$, so population grows without limit	B1 **[1]**	Unlimited growth

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Qu	Answer	Mark	Comment
Section B (continued)			
7(ii)(*A*)	$\frac{4}{P(2-P)} \equiv \frac{A}{P} + \frac{B}{2-P}$ $\Rightarrow 4 \equiv A(2-P) + BP$ $P = 0 \;\Rightarrow 4 = 2A$ $\Rightarrow A = 2$ $P = 2 \;\Rightarrow 4 = 2B$ $\Rightarrow B = 2$ so $\frac{4}{P(2-P)} \equiv \frac{2}{P} + \frac{2}{2-P}$	M1 A1 A1 **[3]**	$\frac{A}{P} + \frac{B}{2-P}$ $A = 2$ $B = 2$
7(ii)(*B*)	$\int \frac{4}{P(2-P)} \mathrm{d}P = \int \mathrm{d}t$ $\Rightarrow 2\int (\frac{1}{P} + \frac{1}{2-P}) \mathrm{d}P = \int \mathrm{d}t$ $\Rightarrow 2[\ln P - \ln(2-P)] = t + c$ $\Rightarrow \ln \frac{P}{2-P} = \frac{1}{2}t + c$ when $t = 0$, $P = 1$, $\Rightarrow \ln 1 = c = 0$ $\Rightarrow \frac{P}{2-P} = e^{\frac{1}{2}t}$ *	M1 B1,B1 DM1 E1 **[5]**	$\int \frac{4}{P(2-P)} \mathrm{d}P = \int \mathrm{d}t$ LHS $= 2[\ln P - \ln(2-P)]$ Evaluating c at any stage Deriving*
7(ii)(*C*)	$\frac{P}{2-P} = e^{\frac{1}{2}t}$ $\Rightarrow P = (2-P)e^{\frac{1}{2}t} = 2e^{\frac{1}{2}t} - Pe^{\frac{1}{2}t}$ $\Rightarrow P(1 + e^{\frac{1}{2}t}) = 2e^{\frac{1}{2}t}$ $\Rightarrow P = \frac{2e^{\frac{1}{2}t}}{1 + e^{\frac{1}{2}t}}$ when $t = 1.24$, $P = 1.2449 \approx 1.24$	M1 A1 E1 **[3]**	Multiplying through by $2-P$ and expanding Collecting Ps cao $P = \frac{2e^{\frac{1}{2}t}}{1 + e^{\frac{1}{2}t}}$ or $P = \frac{2}{e^{-\frac{1}{2}t} + 1}$ P = 1.2449 or 1.245 accept 1.24 or better SC putting t = 1 and verifying P = 1.24 (B1)
7(ii)(*D*)	As $t \to \infty, P \to 2$	B1 **[1]**	$P \to 2$

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Qu	Answer	Mark	Comment
Section B (continued)			
8(i)	G is (1, 0.5, 0)	B1 **[1]**	(1, 0.5, 0) accept: $\begin{pmatrix}1\\0.5\\0\end{pmatrix}$, $\mathbf{i}+\frac{1}{2}\mathbf{j}$, (1, 0.5)
8(ii)	Direction of GH is $\mathbf{i}+2\mathbf{j}+2\mathbf{k}$	M1	Direction of GH
	$\tan\theta=\frac{2}{\sqrt{5}}\Rightarrow\theta=42°$	A1	$\tan\theta=\frac{2}{\sqrt{5}}$ or equivalent
		A1 **[3]**	42°
8(iii)	Direction of GF is $\mathbf{i}+2\mathbf{j}$	M1	$\mathbf{i}+2\mathbf{j}$ or $\arctan\frac{1}{2}$ seen anywhere
	Angle with north is $\arctan\frac{1}{2}=27°$	A1	27° or 027°
	Bearing is 027°	**[2]**	
8(iv)	$z=2$ when $t=1$, $r=2\mathbf{i}+2.5\mathbf{j}+2\mathbf{k}$ coordinates are (2, 2.5, 2)	M1 A1 A1 **[3]**	$z=2$ $\Rightarrow t=1$ (2, 2.5, 2)
8(v)	$\overrightarrow{HM}=5\mathbf{i}+4.5\mathbf{j}+3\mathbf{k}-[(1+t)\mathbf{i}+(0.5+2t)\mathbf{j}+2t\mathbf{k}]$ $=(4-t)\mathbf{i}+(4-2t)\mathbf{j}+(3-2t)\mathbf{k}$	M1	$\overrightarrow{HM}=(4-t)\mathbf{i}+(4-2t)\mathbf{j}+(3-2t)\mathbf{k}$
	perpendicular when $\overrightarrow{HM}.\overrightarrow{GH}=0$	M1	$\overrightarrow{HM}.\overrightarrow{GH}=0$ allow this (M1) for $\overrightarrow{HM}$.(their $\overrightarrow{GH}$)
	$\Rightarrow[(4-t)\mathbf{i}+(4-2t)\mathbf{j}+(3-2t)\mathbf{k}].[\mathbf{i}+2\mathbf{j}+2\mathbf{k}]=0$ $\Rightarrow 4-t+8-4t+6-4t=0$	A1	
	$\Rightarrow 18-9t=0\Rightarrow t=2$	A1	$t=2$ f.t. their equation
	at this time $\overrightarrow{HM}=2\mathbf{i}+\mathbf{k}$, $\lvert HM\rvert=\sqrt{5}$ km	A1 **[5]**	cao $\sqrt{5}=2.24$ km
8(vi)	$\overrightarrow{GM}=(5\mathbf{i}+4.5\mathbf{j}+3\mathbf{k})-(\mathbf{i}+0.5\mathbf{j})$ $=4\mathbf{i}+4\mathbf{j}+3\mathbf{k}$	M1	
	So line GM is $\mathbf{r}=(\mathbf{i}+0.5\mathbf{j})+\lambda(4\mathbf{i}+4\mathbf{j}+3\mathbf{k})$	A1	
	Angle MGH is between vectors $(4\mathbf{i}+4\mathbf{j}+3\mathbf{k})$ and $(\mathbf{i}+2\mathbf{j}+2\mathbf{k})$	M1	
	$(4\mathbf{i}+4\mathbf{j}+3\mathbf{k})\bullet(\mathbf{i}+2\mathbf{j}+2\mathbf{k})$ $=\sqrt{4^2+4^2+3^2}\sqrt{1^2+2^2+2^2}\cos\theta$		
	$\Rightarrow\theta=20.4°$	A1 **[4]**	
			Section B Total: 36
			Total: 72

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AO	Range	Total	Paper A Question Number								Paper B Comprehension
			1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	
1	27-32	27	2	2	2	2	4	3	6	6	-
2	27-32	29	2	2	2	4	3	4	8	4	-
3	9-18	10	-	-	-	-	-	-	4	6	-
4	13-23	19	1	-	-	-	-	-	-	-	18
5	4-14	5	-	1	2	-	-	-	-	2	-
	Totals	**72**	5	5	6	6	7	7	18	18	18

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PAPER B: COMPREHENSION

4754

MARK SCHEME

Qu	Answer	Mark
1	ht is the energy expended when hunting. $r(24-t)$ is the energy expended when not hunting.	B1 B1 **[2]**
2	Equation **(1)** is $E = ht + r(24-t)$ $t = 3.45, E = 15.3, r = 0.22$ $\Rightarrow$ $15.3 = 3.45h + 0.22(24-3.45)$ $\Rightarrow$ $h = (15.3-4.521)/3.45$ $= 3.1243$	 M1 A1 **[2]**
3	Equation **(2)** is $ct = ht + r(24-t)$ $\Rightarrow$ $ct + rt - ht = 24r$ $\Rightarrow$ $(c+r-h)t = 24r$ $\Rightarrow$ Equation **(3)**	 M1 E1 **[2]**
4	Equation **(3)** is $t = \dfrac{24r}{c+r-h}$ This has an asymptote when $c + r - h = 0$ $\Rightarrow$ $c = h - r$ $\Rightarrow$ $C_0 = h - r$ Hence $C_0 = 3.12 - 0.22 = 2.90$	 M1 A1 E1 **[3]**
5	If 20% of the meat is stolen, $p = 0.2$ Equation **(4)** is $t = \dfrac{24r}{(1-p)c + r - h}$ $\Rightarrow$ $t = 24x0.22/((1-0.2)4.43\ldots + 0.22 - 3.12\ldots$ $= 8.2$	 M1 A1 **[2]**
6	From equation **(4)** the asymptote occurs when $(1-p)c + r - h = 0$ $(1 - p = (h-r)/c$ $= (3.12-0.22)/4.43$ $= 0.655(0.6546\ldots)$ $p = 0.345$	 M1 A1 **[2]**

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7(i)	One sensible comment such as: The sample size of six dogs was very small; The formula for calculating *r* applies to domestic dogs and so may not be accurate for wild dogs.	B1 **[1]**
7(ii)	Any sensible answer, such as: Give the various inputs somewhat different values, say by 10%, and repeat the calculation to find the corresponding error in the value of *p*.	B1 **[1]**
8	Energy output 15.3 Mj per day Energy from 1 kg of meat = 4.4 Mj Meat consumed $\frac{15.3}{4.4} \simeq 3.5$ kg	B1 B1 B1
		Total: 18

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