

General Certificate of Education  
January 2006  
Advanced Subsidiary Examination



**MATHEMATICS**  
**Unit Pure Core 1**

**MPC1**

Tuesday 10 January 2006 1.30 pm to 3.00 pm

**For this paper you must have:**

- an 8-page answer book
  - the **blue** AQA booklet of formulae and statistical tables
- You must **not** use a calculator.



Time allowed: 1 hour 30 minutes

**Instructions**

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC1.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The use of calculators (scientific and graphics) is **not** permitted.

**Information**

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

**Advice**

- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

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Answer **all** questions.

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- 1 (a) Simplify  $(\sqrt{5} + 2)(\sqrt{5} - 2)$ . *(2 marks)*
- (b) Express  $\sqrt{8} + \sqrt{18}$  in the form  $n\sqrt{2}$ , where  $n$  is an integer. *(2 marks)*
- 2 The point  $A$  has coordinates  $(1, 1)$  and the point  $B$  has coordinates  $(5, k)$ .
- The line  $AB$  has equation  $3x + 4y = 7$ .
- (a) (i) Show that  $k = -2$ . *(1 mark)*
- (ii) Hence find the coordinates of the mid-point of  $AB$ . *(2 marks)*
- (b) Find the gradient of  $AB$ . *(2 marks)*
- (c) The line  $AC$  is perpendicular to the line  $AB$ .
- (i) Find the gradient of  $AC$ . *(2 marks)*
- (ii) Hence find an equation of the line  $AC$ . *(1 mark)*
- (iii) Given that the point  $C$  lies on the  $x$ -axis, find its  $x$ -coordinate. *(2 marks)*
- 3 (a) (i) Express  $x^2 - 4x + 9$  in the form  $(x - p)^2 + q$ , where  $p$  and  $q$  are integers. *(2 marks)*
- (ii) Hence, or otherwise, state the coordinates of the minimum point of the curve with equation  $y = x^2 - 4x + 9$ . *(2 marks)*
- (b) The line  $L$  has equation  $y + 2x = 12$  and the curve  $C$  has equation  $y = x^2 - 4x + 9$ .
- (i) Show that the  $x$ -coordinates of the points of intersection of  $L$  and  $C$  satisfy the equation
- $$x^2 - 2x - 3 = 0 \quad \text{span style="float: right;">*(1 mark)*$$
- (ii) Hence find the coordinates of the points of intersection of  $L$  and  $C$ . *(4 marks)*

4 The quadratic equation  $x^2 + (m + 4)x + (4m + 1) = 0$ , where  $m$  is a constant, has equal roots.

(a) Show that  $m^2 - 8m + 12 = 0$ . (3 marks)

(b) Hence find the possible values of  $m$ . (2 marks)

5 A circle with centre  $C$  has equation  $x^2 + y^2 - 8x + 6y = 11$ .

(a) By completing the square, express this equation in the form

$$(x - a)^2 + (y - b)^2 = r^2 \quad (3 \text{ marks})$$

(b) Write down:

(i) the coordinates of  $C$ ; (1 mark)

(ii) the radius of the circle. (1 mark)

(c) The point  $O$  has coordinates  $(0, 0)$ .

(i) Find the length of  $CO$ . (2 marks)

(ii) Hence determine whether the point  $O$  lies inside or outside the circle, giving a reason for your answer. (2 marks)

6 The polynomial  $p(x)$  is given by

$$p(x) = x^3 + x^2 - 10x + 8$$

(a) (i) Using the factor theorem, show that  $x - 2$  is a factor of  $p(x)$ . (2 marks)

(ii) Hence express  $p(x)$  as the product of three linear factors. (3 marks)

(b) Sketch the curve with equation  $y = x^3 + x^2 - 10x + 8$ , showing the coordinates of the points where the curve cuts the axes.

(You are not required to calculate the coordinates of the stationary points.) (4 marks)

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7 The volume,  $V \text{ m}^3$ , of water in a tank at time  $t$  seconds is given by

$$V = \frac{1}{3}t^6 - 2t^4 + 3t^2, \quad \text{for } t \geq 0$$

(a) Find:

(i)  $\frac{dV}{dt}$ ; *(3 marks)*

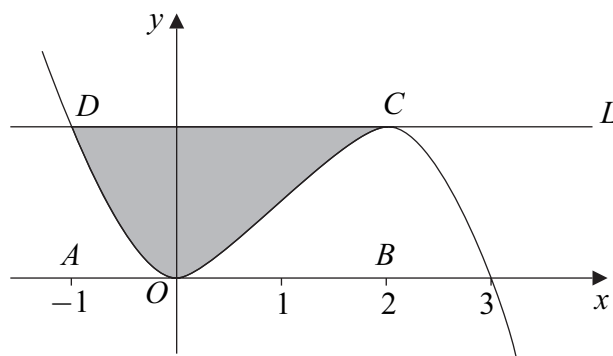
(ii)  $\frac{d^2V}{dt^2}$ . *(2 marks)*

(b) Find the rate of change of the volume of water in the tank, in  $\text{m}^3 \text{ s}^{-1}$ , when  $t = 2$ . *(2 marks)*

(c) (i) Verify that  $V$  has a stationary value when  $t = 1$ . *(2 marks)*

(ii) Determine whether this is a maximum or minimum value. *(2 marks)*

- 8 The diagram shows the curve with equation  $y = 3x^2 - x^3$  and the line  $L$ .



The points  $A$  and  $B$  have coordinates  $(-1, 0)$  and  $(2, 0)$  respectively. The curve touches the  $x$ -axis at the origin  $O$  and crosses the  $x$ -axis at the point  $(3, 0)$ . The line  $L$  cuts the curve at the point  $D$  where  $x = -1$  and touches the curve at  $C$  where  $x = 2$ .

- (a) Find the area of the rectangle  $ABCD$ . (2 marks)
- (b) (i) Find  $\int (3x^2 - x^3) dx$ . (3 marks)
- (ii) Hence find the area of the shaded region bounded by the curve and the line  $L$ . (4 marks)
- (c) For the curve above with equation  $y = 3x^2 - x^3$ :
- (i) find  $\frac{dy}{dx}$ ; (2 marks)
- (ii) hence find an equation of the tangent at the point on the curve where  $x = 1$ ; (3 marks)
- (iii) show that  $y$  is decreasing when  $x^2 - 2x > 0$ . (2 marks)
- (d) Solve the inequality  $x^2 - 2x > 0$ . (2 marks)

**END OF QUESTIONS**

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