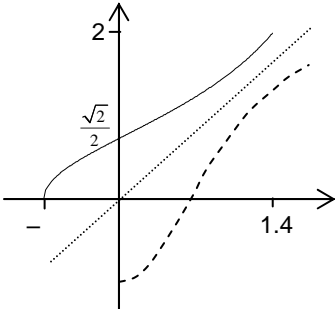


4753 (C3) Methods for Advanced Mathematics

<p>1 $e^{2x} - 5e^x = 0$ $\Rightarrow e^x(e^x - 5) = 0$ $\Rightarrow e^x = 5$ $\Rightarrow x = \ln 5$ or 1.6094</p>	<p>M1 M1 A1 A1 [4]</p>	<p>factoring out e^x or dividing $e^{2x} = 5e^x$ by e^x $e^{2x} / e^x = e^x$ $\ln 5$ or 1.61 or better, mark final answer -1 for additional solutions, e.g. $x = 0$</p>
<p>or $\ln(e^{2x}) = \ln(5e^x)$ $\Rightarrow 2x = \ln 5 + x$ $\Rightarrow x = \ln 5$ or 1.6094</p>	<p>M1 A1 A1 A1 [4]</p>	<p>taking lns on $e^{2x} = 5e^x$ $2x, \ln 5 + x$ $\ln 5$ or 1.61 or better, mark final answer -1 for additional solutions, e.g. $x = 0$</p>
<p>2 (i) When $t = 0, T = 100$ $\Rightarrow 100 = 20 + b$ $\Rightarrow b = 80$ When $t = 5, T = 60$ $\Rightarrow 60 = 20 + 80 e^{-5k}$ $\Rightarrow e^{-5k} = 1/2$ $\Rightarrow k = \ln 2 / 5 = 0.139$</p>	<p>M1 A1 M1 A1 [4]</p>	<p>substituting $t = 0, T = 100$ cao substituting $t = 5, T = 60$ $1/5 \ln 2$ or 0.14 or better</p>
<p>(ii) $50 = 20 + 80 e^{-kt}$ $\Rightarrow e^{-kt} = 3/8$ $\Rightarrow t = \ln(8/3) / k = 7.075$ mins</p>	<p>M1 A1 [2]</p>	<p>Re-arranging and taking lns correctly – ft their b and k answers in range 7 to 7.1</p>
<p>3(i) $\frac{dy}{dx} = \frac{1}{3}(1+3x^2)^{-2/3} \cdot 6x$ $= 2x(1+3x^2)^{-2/3}$</p>	<p>M1 B1 A1 [3]</p>	<p>chain rule $1/3 u^{-2/3}$ or $\frac{1}{3}(1+3x^2)^{-2/3}$ o.e but must be '2' (not 6/3) mark final answer</p>
<p>(ii) $3y^2 \frac{dy}{dx} = 6x$ $\Rightarrow dy/dx = 6x/3y^2$ $= \frac{2x}{(1+3x^2)^{2/3}} = 2x(1+3x^2)^{-2/3}$</p>	<p>M1 A1 A1 E1 [4]</p>	<p>$3y^2 \frac{dy}{dx}$ $= 6x$ if deriving $2x(1+3x^2)^{-2/3}$, needs a step of working</p>

4(i) $\int_0^1 \frac{2x}{x^2+1} dx = [\ln(x^2+1)]_0^1$ $= \ln 2$	M2 A1 [3]	$[\ln(x^2+1)]$ cao (must be exact)
<i>or</i> let $u = x^2 + 1, du = 2x dx$ $\Rightarrow \int_0^1 \frac{2x}{x^2+1} dx = \int_1^2 \frac{1}{u} du$ $= [\ln u]_1^2$ $= \ln 2$	M1 A1 A1 [3]	$\int \frac{1}{u} du$ or $[\ln(1+x^2)]_0^1$ with correct limits cao (must be exact)
(ii) $\int_0^1 \frac{2x}{x+1} dx = \int_0^1 \frac{2x+2-2}{x+1} dx = \int_0^1 (2 - \frac{2}{x+1}) dx$ $= [2x - 2\ln(x+1)]_0^1$ $= 2 - 2\ln 2$	M1 A1, A1 A1 A1 [5]	dividing by $(x+1)$ 2, $-2/(x+1)$
<i>or</i> $\int_0^1 \frac{2x}{x+1} dx$ let $u = x + 1, \Rightarrow du = dx$ $= \int_1^2 \frac{2(u-1)}{u} du$ $= \int_1^2 (2 - \frac{2}{u}) du$ $= [2u - 2\ln u]_1^2$ $= 4 - 2\ln 2 - (2 - 2\ln 1)$ $= 2 - 2\ln 2$	M1 B1 M1 A1 A1 [5]	substituting $u = x + 1$ and $du = dx$ (or $du/dx = 1$) and correct limits used for u or x $2(u-1)/u$ dividing through by u $2u - 2\ln u$ allow ft on $(u-1)/u$ (i.e. with 2 omitted) o.e. cao (must be exact)
5 (i) $a = 0, b = 3, c = 2$	B(2,1,0)	or $a = 0, b = -3, c = -2$
(ii) $a = 1, b = -1, c = 1$ or $a = 1, b = 1, c = -1$	B(2,1,0) [4]	
6 $f(-x) = -f(x), g(-x) = g(x)$ $g f(-x) = g [-f(x)]$ $= g f(x)$ \Rightarrow $g f$ is even	B1B1 M1 E1 [4]	condone f and g interchanged forming $g f(-x)$ or $g f(x)$ and using $f(-x) = -f(x)$ www
7 Let $\arcsin x = \theta$ $\Rightarrow x = \sin \theta$ $\theta = \arccos y \Rightarrow y = \cos \theta$ $\sin^2 \theta + \cos^2 \theta = 1$ $\Rightarrow x^2 + y^2 = 1$	M1 M1 E1 [3]	

<p>9(i) $f'(x) = \frac{(x^2+1)4x - (2x^2-1)2x}{(x^2+1)^2}$ $= \frac{4x^3+4x-4x^3+2x}{(x^2+1)^2} = \frac{6x}{(x^2+1)^2}$*</p> <p>When $x > 0$, $6x > 0$ and $(x^2+1)^2 > 0$ $\Rightarrow f'(x) > 0$</p>	<p>M1 A1 E1</p> <p>M1 E1</p> <p>[5]</p>	<p>Quotient or product rule correct expression www</p> <p>attempt to show or solve $f'(x) > 0$ numerator > 0 and denominator > 0 or, if solving, $6x > 0 \Rightarrow x > 0$</p>
<p>(ii) $f(2) = \frac{8-1}{4+1} = 1\frac{2}{5}$</p> <p>Range is $-1 \leq y \leq 1\frac{2}{5}$</p>	<p>B1</p> <p>B1 [2]</p>	<p>must be \leq, y or f(x)</p>
<p>(iii) $f'(x)$ max when $f''(x) = 0$ $\Rightarrow 6 - 18x^2 = 0$ $\Rightarrow x^2 = 1/3, x = 1/\sqrt{3}$ $\Rightarrow f'(x) = \frac{6/\sqrt{3}}{(1/\sqrt{3})^2} = \frac{6}{\sqrt{3}} \cdot \frac{9}{16} = \frac{9\sqrt{3}}{8} = 1.95$</p>	<p>M1 A1 M1 A1 [4]</p>	<p>$(\pm)1/\sqrt{3}$ oe (0.577 or better) substituting $1/\sqrt{3}$ into $f'(x)$ $9\sqrt{3}/8$ o.e. or 1.95 or better (1.948557..)</p>
<p>(iv) Domain is $-1 < x < 1\frac{2}{5}$ Range is $0 \leq y \leq 2$</p> 	<p>B1</p> <p>B1</p> <p>M1 A1 cao</p> <p>[4]</p>	<p>ft their 1.4 but not $x \geq -1$</p> <p>or $0 \leq g(x) \leq 2$ (not f)</p> <p>Reasonable reflection in $y = x$ from $(-1, 0)$ to $(1.4, 2)$, through $(0, \sqrt{2}/2)$ allow omission of one of $-1, 1.4, 2, \sqrt{2}/2$</p>
<p>(v) $y = \frac{2x^2-1}{x^2+1} \quad x \leftrightarrow y$ $x = \frac{2y^2-1}{y^2+1}$</p> <p>$\Rightarrow xy^2 + x = 2y^2 - 1$ $\Rightarrow x + 1 = 2y^2 - xy^2 = y^2(2-x)$ $\Rightarrow y^2 = \frac{x+1}{2-x}$ $\Rightarrow y = \sqrt{\frac{x+1}{2-x}}$*</p>	<p>M1 M1 M1</p> <p>E1 [4]</p>	<p>(could start from g)</p> <p>Attempt to invert clearing fractions collecting terms in y^2 and factorising</p> <p>www</p>

NOTE

For the domain of $g(x)$ in 9(iv) the answer is $-1 < x < 1.4$. The published markscheme gives $-1 < x < 1.4$; this is because, at the time, the published markscheme was not the same as the final one used by the examiners and so the published markscheme contains a printing error. This did not affect marking as the examiners used a correct version.