## 4753 (C3) Methods for Advanced Mathematics

$\Rightarrow \Rightarrow \Rightarrow$	$e^{2x} - 5e^x = 0$ $e^x (e^x - 5) = 0$ $e^x = 5$ $x = \ln 5$ or 1.6094	M1 M1 A1 A1 [4]	factoring out $e^x$ or dividing $e^{2x} = 5e^x$ by $e^x$ $e^{2x} / e^x = e^x$ In 5 or 1.61 or better, mark final answer $-1$ for additional solutions, e.g. $x = 0$
	$\ln(e^{2x}) = \ln(5e^{x})$ 2x = \ln 5 + x x = \ln 5 \text{ or } 1.6094	M1 A1 A1 A1 [4]	taking lns on $e^{2x} = 5e^x$ $2x$ , $\ln 5 + x$ $\ln 5$ or 1.61 or better, mark final answer -1 for additional solutions, e.g. $x = 0$
$\Rightarrow \Rightarrow $	When $t = 0$ , $T = 100$ 100 = 20 + b b = 80 When $t = 5$ , $T = 60$ $60 = 20 + 80 e^{-5k}$ $e^{-5k} = \frac{1}{2}$ $k = \ln 2 / 5 = 0.139$	M1 A1 M1 A1 [4]	substituting $t = 0$ , $T = 100$ cao substituting $t = 5$ , $T = 60$ $1/5 \ln 2$ or $0.14$ or better
$\Rightarrow$	$50 = 20 + 80 e^{-kt}$ $e^{-kt} = 3/8$ $t = \ln(8/3) / k = 7.075 \text{ mins}$	M1 A1 [2]	Re-arranging and taking lns correctly – ft their $b$ and $k$ answers in range 7 to 7.1
3(i)	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{3}(1+3x^2)^{-2/3}.6x$ $= 2x(1+3x^2)^{-2/3}$	M1 B1 A1 [3]	chain rule $1/3 u^{-2/3} \text{ or } \frac{1}{3} (1+3x^2)^{-2/3}$ o.e but must be '2' (not 6/3) mark final answer
(ii) ⇒	$3y^{2} \frac{dy}{dx} = 6x$ $dy/dx = 6x/3y^{2}$ $= \frac{2x}{(1+3x^{2})^{2/3}} = 2x(1+3x^{2})^{-2/3}$	M1 A1 A1 E1 [4]	$3y^{2} \frac{dy}{dx}$ = 6x  if deriving $2x(1+3x^{2})^{-2/3}$ , needs a step of working

4(i) $\int_0^1 \frac{2x}{x^2 + 1} dx = \left[ \ln(x^2 + 1) \right]_0^1$ = \ln 2	M2 A1 [3]	$[\ln(x^2 + 1)]$ cao (must be exact)
or let $u = x^2 + 1$ , $du = 2x dx$ $\Rightarrow \int_0^1 \frac{2x}{x^2 + 1} dx = \int_1^2 \frac{1}{u} du$ $= [\ln u]_1^2$ $= \ln 2$	M1 A1 A1 [3]	$\int \frac{1}{u} du$ or $\left[\ln(1+x^2)\right]_0^1$ with correct limits cao (must be exact)
(ii) $\int_0^1 \frac{2x}{x+1} dx = \int_0^1 \frac{2x+2-2}{x+1} dx = \int_0^1 (2-\frac{2}{x+1}) dx$ $= \left[ 2x - 2\ln(x+1) \right]_0^1$ $= 2 - 2\ln 2$	M1 A1, A1 A1 A1 [5]	dividing by $(x + 1)$ 2, $-2/(x+1)$
or $\int_{0}^{1} \frac{2x}{x+1} dx  \text{let } u = x+1, \Rightarrow du = dx$ $= \int_{1}^{2} \frac{2(u-1)}{u} du$ $= \int_{1}^{2} (2 - \frac{2}{u}) du$ $= [2u - 2\ln u]_{1}^{2}$ $= 4 - 2\ln 2 - (2 - 2\ln 1)$ $= 2 - 2\ln 2$	M1 B1 M1 A1 A1 [5]	substituting $u = x + 1$ and $du = dx$ (or $du/dx = 1$ ) and correct limits used for $u$ or $x$ $2(u - 1)/u$ dividing through by $u$ $2u - 2\ln u$ allow ft on $(u - 1)/u$ (i.e. with 2 omitted) o.e. cao (must be exact)
<b>5 (i)</b> $a = 0, b = 3, c = 2$	B(2,1,0)	or $a = 0$ , $b = -3$ , $c = -2$
(ii) $a = 1, b = -1, c = 1$ or $a = 1, b = 1, c = -1$	B(2,1,0) [4]	
6 $f(-x) = -f(x), g(-x) = g(x)$ g f(-x) = g [-f (x)] = g f (x) $\Rightarrow g f \text{ is even}$	B1B1 M1 E1 [4]	condone f and g interchanged forming $gf(-x)$ or $gf(x)$ and using f(-x) = -f(x) www
7 Let $\arcsin x = \theta$ $\Rightarrow x = \sin \theta$ $\theta = \arccos y \Rightarrow y = \cos \theta$ $\sin^2 \theta + \cos^2 \theta = 1$ $\Rightarrow x^2 + y^2 = 1$	M1 M1 E1 [3]	

8(i) At P, $x \cos 3x = 0$ $\Rightarrow \cos 3x = 0$ $\Rightarrow 3x = \pi/2, 3\pi/2$ $\Rightarrow x = \pi/6, \pi/2$ So P is $(\pi/6, 0)$ and Q is $(\pi/2, 0)$	M1 M1 A1 A1 [4]	or verification $3x = \pi/2$ , $(3\pi/2)$ dep both Ms condone degrees here
(ii) $\frac{dy}{dx} = -3x \sin 3x + \cos 3x$ $At P, \frac{dy}{dx} = -\frac{\pi}{2} \sin \frac{\pi}{2} + \cos \frac{\pi}{2} = -\frac{\pi}{2}$ $At TPs \frac{dy}{dx} = -3x \sin 3x + \cos 3x = 0$ $\Rightarrow \cos 3x = 3x \sin 3x$ $\Rightarrow 1 = 3x \sin 3x / \cos 3x = 3x \tan 3x$ $\Rightarrow x \tan 3x = 1/3 *$	M1 B1 A1 M1 A1cao M1	Product rule $d/dx (\cos 3x) = -3 \sin 3x$ cao (so for $dy/dx = -3x\sin 3x$ allow B1) mark final answer substituting their $-\pi/6$ (must be rads) $-\pi/2$ $dy/dx = 0$ and $\sin 3x / \cos 3x = \tan 3x$ used
(iii) $A = \int_0^{\pi/6} x \cos 3x dx$ Parts with $u = x$ , $dv/dx = \cos 3x$ $du/dx = 1$ , $v = 1/3 \sin 3x$	B1 M1	Correct integral and limits (soi) – ft their P, but must be in radians
$\Rightarrow A = \left[\frac{1}{3}x\sin 3x\right]_0^{\frac{\pi}{6}} - \int_0^{\pi/6} \frac{1}{3}\sin 3x dx$	A1	can be without limits
$= \left[ \frac{1}{3} x \sin 3x + \frac{1}{9} \cos 3x \right]_0^{\frac{\pi}{6}}$ $= \frac{\pi}{18} - \frac{1}{9}$	A1 M1dep A1 cao [6]	dep previous A1. substituting correct limits, dep 1 <sup>st</sup> M1: ft their P provided in radians o.e. but must be exact

9(i) ⇒	$f'(x) = \frac{(x^2 + 1)4x - (2x^2 - 1)2x}{(x^2 + 1)^2}$ $= \frac{4x^3 + 4x - 4x^3 + 2x}{(x^2 + 1)^2} = \frac{6x}{(x^2 + 1)^2} *$ When $x > 0$ , $6x > 0$ and $(x^2 + 1)^2 > 0$ $f'(x) > 0$	M1 A1 E1 M1 E1	Quotient or product rule correct expression www attempt to show or solve $f'(x) > 0$ numerator $> 0$ and denominator $> 0$ or, if solving, $6x > 0 \Rightarrow x > 0$
(ii)	$f(2) = \frac{8-1}{4+1} = 1\frac{2}{5}$ Range is $-1 \le y \le 1\frac{2}{5}$	B1 B1 [2]	must be ≤, $y$ or $f(x)$
$\Rightarrow$	f'(x) max when f''(x) = 0 6 - 18 x <sup>2</sup> = 0 x <sup>2</sup> = 1/3, x = 1/\sqrt{3} f'(x) = $\frac{6/\sqrt{3}}{(1\frac{1}{3})^2} = \frac{6}{\sqrt{3}} \cdot \frac{9}{16} = \frac{9\sqrt{3}}{8} = 1.95$	M1 A1 M1 A1 [4]	(±)1/ $\sqrt{3}$ oe (0.577 or better) substituting 1/ $\sqrt{3}$ into f'(x) $9\sqrt{3}/8$ o.e. or 1.95 or better (1.948557)
(iv)	Domain is $-1 < x < 1\frac{2}{5}$ Range is $0 \le y \le 2$	B1 B1 M1 A1 cao	ft their 1.4 but not $x \ge -1$ or $0 \le g(x) \le 2$ (not f) Reasonable reflection in $y = x$ from $(-1, 0)$ to $(1.4, 2)$ , through $(0, \sqrt{2}/2)$ allow omission of one of $-1$ , 1.4, 2, $\sqrt{2}/2$
	$y = \frac{2x^2 - 1}{x^2 + 1}  x \leftrightarrow y$ $x = \frac{2y^2 - 1}{y^2 + 1}$ $xy^2 + x = 2y^2 - 1$ $x + 1 = 2y^2 - xy^2 = y^2(2 - x)$ $y^2 = \frac{x + 1}{2 - x}$ $y = \sqrt{\frac{x + 1}{2 - x}} *$	M1 M1 M1	(could start from g)  Attempt to invert clearing fractions collecting terms in y² and factorising  www

## **NOTE**

For the domain of g(x) in 9(iv) the answer is  $-1 \pounds x \pounds 1.4$ . The published markscheme gives -1 < x < 1.4; this is because, at the time, the published markscheme was not the same as the final one used by the examiners and so the published markscheme contains a printing error. This did not affect marking as the examiners used a correct version.