Oxford Cambridge and RSA

GCE

Mathematics

Unit **4721**: Core Mathematics 1

Advanced Subsidiary GCE

Mark Scheme for June 2014

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners’ meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates’ scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

1. **Annotations and abbreviations**

Annotation in scoris	**Meaning**
BP	Blank Page – this annotation must be used on all blank pages within an answer booklet (structured or unstructured) and on each page of an additional object where there is no candidate response.
✓ and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

2. Subject-specific Marking Instructions for GCE Mathematics Pure strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

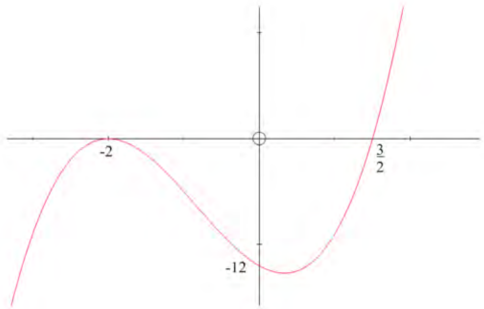
Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance	
1			$5x^2 + 10x + 2 = 5(x^2 + 2x) + 2$ $= 5[(x + 1)^2 - 1] + 2$ $= 5(x + 1)^2 - 3$	B1 B1 M1 A1 [4]	$p = 5$ $q = 1$ $2 - 5$ "their q"^{2} or $\frac{2}{5} -$ "their q"^{2} **Must be evidence of squaring** $r = -3$	If p, q and r found correctly, then **ISW** slips in format. $5(x + 1)^2 + 3$ **B1 B1 M0 A0** $5(x + 1) - 3$ **B1 B1 M1 A1 (BOD)** $5(x + 1x)^2 - 3$ **B1 B0 M1 A0** $5(x^2 + 1)^2 - 3$ **B1 B0 M1 A0** $5(x - 1)^2 - 3$ **B1 B0 M1 A0** $5x(x + 1)^2 - 3$ **B0 B1M1A0**
2	i)		$2\sqrt{3}$	B1 [1]	**cao**	Do not accept $\frac{6\sqrt{3}}{3}$
	ii)		$10\sqrt{3} - 18\sqrt{3}$ $-8\sqrt{3}$	B1 B1 [2]	$\sqrt{27} = 3\sqrt{3}$ **soi**, not just $\sqrt{9}\sqrt{3}$	
	iii)		$3^{\frac{5}{2}} = 3^2 \times 3^{\frac{1}{2}}$ $9\sqrt{3}$	B1 B1 [2]	Separate $\sqrt{3}$ from $3^{\frac{5}{2}}$	Allow **only** $3 \times 3 \times 3^{\frac{1}{2}}$, $3^2 \times \sqrt{3}$, $3 \times 3 \times \sqrt{3}$, or $\sqrt{81}\sqrt{3}$, $3\sqrt{9}\sqrt{3}$ for first mark
3			$k = x^2$ $4k^2 + 3k - 1 = 0$ $(4k - 1)(k + 1) = 0$ $k = \frac{1}{4}, k = -1$ $x = \pm\sqrt{\frac{1}{4}}$ $x = \pm\frac{1}{2}$	M1* M1dep* A1 M1 A1 [5]	Substitute for x^2 Attempt to solve resulting quadratic Correct values of k **soi** Attempt to square root Final answers correct, no extras	**No marks** if whole equation square rooted etc. **No marks** if straight to formula with no evidence of substitution at start and no square rooting/squaring at end. <u>If factorising into two brackets:</u> $(4x^2 - 1)(x^2 + 1) = 0$ **M1 A1** $(2x + 1)(2x - 1)(x^2 + 1) = 0$ **M1 A1** **A1** as before <u>Spotted solutions:</u> If **M0 DM0** or **M1 DM0** **SR B1** $x = \frac{1}{2}$ **www** **SC B1** $x = -\frac{1}{2}$ **www** (Can then get 5/5 if both found **www** and

Question			Answer	Marks	Guidance	
						exactly two solutions justified)
4	i)		(2, 7)	**B1** **[1]**		
	ii)		(1, 5)	**B1** **[1]**		
4	iii)		Translation – 4 units parallel to the x axis	**B1** **B1** **[2]**	Translation Correct description e.g. correct vector (not as a coordinate), "4 units to the left" **Do not allow second B1 after incorrect type of transformation e.g. stretch/rotation etc. but allow after shift/move etc.**	Do **not** accept shift/move etc. for first **B1** For "parallel to the x axis" allow "horizontally", "in the x direction". **Do not accept** "in/on/across/up/along/to/towards the x axis". **Do not accept** "factor 4" etc. Allow extra if not incorrect.
5	i)		$5–3< 6x < 14–3$ $2< 6x < 11$ $\frac{1}{3} < x < \frac{11}{6}$	**M1** **A1** **A1** **[3]**	Attempt to solve two equations/inequalities **each involving all 3 terms** 2, 11 seen from correct inequalities **www** Award full marks if initially working with equations but final answer correct.	Allow "$\frac{1}{3} < x$ **and** $x < \frac{11}{6}$" "$\frac{1}{3} < x$, $x < \frac{11}{6}$" but do not allow "$\frac{1}{3} < x$ **or** $x < \frac{11}{6}$"
	ii)		$3x^2 - 13x - 10 \geq 0$ $(3x+2)(x-5) \geq 0$ $x \leq -\frac{2}{3}, x \geq 5$	**M1*** **M1dep*** **A1** **M1** **A1** **[5]**	Expands and rearranges to collect all terms on one side Correct method to find roots $-\frac{2}{3}, 5$ seen as roots Chooses "outside region" for their roots of their quadratic Do not allow strict inequalities for final mark	**See guidance at end of mark scheme** e.g. $-\frac{2}{3} \geq x \geq 5$ scores **M1A0** Allow "$x \leq -\frac{2}{3}, x \geq 5$", "$x \leq -\frac{2}{3}$ **or** $x \geq 5$" but do not allow "$x \leq -\frac{2}{3}$

Question			Answer	Marks	Guidance	
						and $x \geq 5$" **SC** If question "misread" as $x(3x-13) \geq 0$ Roots found as 0, $\frac{13}{3}$ **B1** $x \leq 0, x \geq \frac{13}{3}$ etc. as above **B1, max 2/5**
6	i)		$y = 6x^3 + 4x^{-\frac{1}{2}} + 5x$ $\frac{dy}{dx} = 18x^2 - 2x^{-\frac{3}{2}} + 5$	B1 M1 A1 A1 [4]	$\frac{4}{\sqrt{x}} = 4x^{-\frac{1}{2}}$ **soi** Attempt to differentiate, any term correct Two correct terms Fully correct, no "+c"	
6	ii)		$\frac{d^2y}{dx^2} = 36x + 3x^{-\frac{5}{2}}$	M1 A1 [2]	Attempt to differentiate their $\frac{dy}{dx}$ **cao www in either part**	Any term still involving x correct – follow through from their expression for the **M** mark only
7	i)		$\left(\frac{5+-1}{2}, \frac{7+-5}{2}\right)$ (2, 1)	M1 A1 [2]	Correct method to find midpoint of line	At least 3 out of 4 terms correctly substituted
	ii)		Gradient of AB = $\frac{7--5}{5--1} = 2$ Perpendicular gradient $= -\frac{1}{2}$ $y - 7 = -\frac{1}{2}(x-5)$ $x + 2y - 19 = 0$	B1 B1ft M1 A1ft A1 [5]	Gradient of AB correctly found as 2 Fully processed $\frac{-1}{\text{their gradient}}$ Equation of straight line through A or B, any non-zero gradient Equation of straight line through A **only**, their perpendicular gradient, in any form Correct equation in given form	i.e. $k(x+2y-19)=0$ for integer k. Must have "=0".

Question			Answer	Marks	Guidance	
8	i)		$\frac{dy}{dx} = 9x^2 - 7 - 2x^{-2}$ When $x = 1$, $\frac{dy}{dx} = 9 - 7 - 2 = 0$ Therefore a stationary point	**M1*** **A1** **A1** **M1dep** **A1** **[5]**	Attempt to differentiate, any term correct Two correct terms Fully correct Substitute $x = 1$ into their derivative Correctly obtain zero **www** and state conclusion **AG**	 <u>Alternative for the last two marks:</u> Sets derivative to zero and makes valid attempt to solve resulting quartic **M1dep** Correctly establishes $x = 1$ as solution and draws clear conclusion **A1www**
8	ii)		$\frac{d^2y}{dx^2} = 18x + 4x^{-3}$ When $x = 1$, $\frac{d^2y}{dx^2} > 0$ so minimum	**M1** **A1** **[2]**	Correct method to find nature of stationary point e.g. substituting $x = 1$ into second derivative (at least one term correct from their first derivative in (i)) No incorrect working seen in this part i.e. if second derivate is evaluated, it must be 22.	**Alternate valid methods include:** 1) Evaluating gradient at either side of 1($x > 0$) 2) Evaluating y at 1 and either side of 1 ($x > 0$) If using alternatives, working must be fully correct to obtain the **A** mark
8	iii)		When $x = 1$, $y = -2$ $(0, -2)$	**B1** **B1** **[2]**	Finding $y = -2$ at $x = 1$ Correct coordinate **www**	
9	i)		y coordinate of the centre is -5 Radius = 5 Centre is five units below x axis and radius is five, so just touches the x-axis	**B1** **B1** **B1** **[3]**	Correct y value Correct radius Correct explanation based on the above – allow clear diagram **www**	<u>Alt</u> Shows only meets x axis at one point **B1** Correct y value for the centre **B1** Correct explanation **B1 www**
9	ii)		$CP^2 = (6-2)^2 + (k+5)^2$ $CP^2 < 25 \Rightarrow 16 + k^2 + 10k + 25 < 25$ $k^2 + 10k + 16 < 0$ $(k+2)(k+8) < 0$ $-8 < k < -2$	**M1** **A1** **A1** **M1** **A1** **[5]**	Attempt to find CP or CP^2 Correct three term quadratic expression* $k = -2$ and $k = -8$ found Chooses "inside region" for their roots of their quadratic Must be strict inequalities for the A mark * Or $(k+5)^2 < 9$	<u>Alternative</u> Puts $x = 6$ to into equation of circle **M1** Correct three term quadratic equation*, could be in terms of y **A1** $k = -2$ and $k = -8$ found (allow y) **A1** Then as main scheme * Or $(k+5)^2 = 9$ **SC** Trial and improvement **B2** if final answer correct (**B1** if inequalities are not strict)

Question			Answer	Marks	Guidance	
						Can only get 5/5 if fully explained
	iii)		$(2y-2)^2+(y+5)^2=25$ $5y^2+2y+4=0$ $b^2-4ac=4-4\times5\times4$ $=-76$ <0 ,so line and circle do not meet	**M1*** **A1** **M1dep*** **A1** **[4]**	Attempts to eliminate x or y from equation of circle Correct three term quadratic obtained Correct method to establish quadratic has no roots e.g. considers value of b^2-4ac, tries to find roots from quadratic formula Correct clear conclusion **www AG**	If y eliminated: $5x^2+4x+16=0$ $b^2-4ac=16-4\times5\times16$ $=-304$ **No marks for purely graphical attempts**
10	i)			**B1** **B1** **B1** **[3]**	Positive cubic with max and min Correct y intercept – graph must be drawn Double root shown at $x=-2$ and single root at $x=\frac{3}{2}$ with no extras – graph must be drawn	For first mark must clearly be a cubic – must not stop at either axis, do not allow straight line sections/tending to extra turning points etc.
	ii)		x^2+4x+4 or $2x^2+x-6$ $2x^3+5x^2-4x-12$ $\frac{dy}{dx}=6x^2+10x-4$ When $x=-1$, gradient $=-8$ When $x=-1$, $y=-5$ $y+5=-8(x+1)$ $8x+y+13=0$	**B1** **M1** **A1** **M1*** **M1dep*** **A1ft** **B1** **M1** **A1**	Obtain one quadratic factor Multiply their three term quadratic by linear factor to obtain at least 5 term cubic If simplified, must be correct Attempt to differentiate (power of at least one term involving x reduced by one) Substitutes to find gradient at $x=-1$ Correct gradient found **ft** their derivative, differentiation of their expression must be fully correct to earn this mark Correct y value Correct equation of straight line through $(-1$, their $y)$, their gradient from differentiation	**Check for working for this in 10 (i)** Alternative using product rule: Clear attempt at product rule **M1*** Differentiates $(x+2)^2$ correctly **A1** Both expressions fully correct **A2 (1 each)**, then as main scheme y must have been found, do not allow use of gradient of normal instead of tangent i.e. $k(8x+y+13)=0$. Must have "=0".

Question			Answer	Marks	Guidance	
				[9]	Correct answer in correct form	<u>Note</u> If $x = 1$ used instead of $x = -1$, then max possible from last 5 marks is **M1 M1** only

APPENDIX 1

Solving a quadratic

This is particularly important to mark correctly as it features several times on the paper.
Consider the equation: $3x^2 - 13x - 10 = 0$

1) If the candidate attempts to solve by factorisation, their attempt when expanded must produce the **correct quadratic term** and **one other correct term** (with correct sign):

$(3x + 5)(x - 2)$	**M1**	$3x^2$ and -10 obtained from expansion
$(3x - 4)(x - 3)$	**M1**	$3x^2$ and $-13x$ obtained from expansion
$(3x + 5)(x + 2)$	**M0**	only $3x^2$ term correct

2) If the candidate attempts to solve by using the formula

a) If the formula is quoted incorrectly then **M0**.

b) If the formula is quoted correctly then one **sign** slip is permitted. Substituting the wrong numerical value for a or b or c scores **M0**

$$\frac{-13 \pm \sqrt{(-13)^2 - 4\times 3\times -10}}{2\times 3}$$ earns **M1** (minus sign incorrect at start of formula)

$$\frac{13 \pm \sqrt{(-13)^2 - 4\times 3\times 10}}{2\times 3}$$ earns **M1** (10 for c instead of -10 is the only sign slip)

$$\frac{-13 \pm \sqrt{(-13)^2 - 4\times 3\times 10}}{2\times 3}$$ **M0** (2 sign errors: initial sign and c incorrect)

$$\frac{13 \pm \sqrt{(-13)^2 - 4\times 3\times -10}}{2\times -10}$$ **M0** ($2c$ on the denominator instead of $2a$)

Notes – for equations such as $3x^2 - 13x - 10 = 0$, then $b^2 = 13^2$ would be condoned in the discriminant and would not be counted as a sign error. Repeating the sign error for a in both occurrences in the formula would be two sign errors and score **M0**.

c) If the formula is not quoted at all, substitution must be completely correct to earn the **M1**

3) If the candidate attempts to complete the square, they must get to the "square root stage" involving ±; we are looking for evidence that the candidate knows a quadratic has two solutions!

$$3x^2 - 13x - 10 = 0$$

$$3\left(x^2 - \frac{13}{3}x\right) - 10 = 0$$

$$3\left[\left(x - \frac{13}{6}\right)^2 - \frac{169}{36}\right] - 10 = 0$$

$$\left(x - \frac{13}{6}\right)^2 = \frac{289}{36}$$

This is where the **M1** is awarded – arithmetical errors may be condoned provided $x - \frac{13}{6}$ seen or implied

$$x - \frac{13}{6} = \pm\sqrt{\frac{289}{36}}$$

If a candidate makes repeated attempts (e.g. fails to factorise and then tries the formula), mark only what you consider to be their last full attempt.

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